
A SVD Matrices

The SVD matrices and mass singular values for vanishing τ coupling up to $\mathcal{O}(v^2/M_{L,E}^2)$ are listed in this appendix chapter.

A.1 Left SVD Matrix

$$\begin{aligned}[U_L]_{ii} &= 1 - v^2 \frac{(\lambda_E^e)^2}{2M_E^2} \\[U_L]_{i2} &= v^2 \frac{M_E^2 \lambda_L^e \lambda_L^\mu y_e y_\mu - M_E M_L \bar{\lambda} \lambda_E^e \lambda_L^\mu y_\mu - M_E M_L \bar{\lambda} \lambda_L^e \lambda_E^\mu y_e + M_L^2 \lambda_E^e \lambda_E^\mu y_\mu^2}{M_E^2 M_L^2 (y_e^2 - y_\mu^2)} \\[U_L]_{i4} &= v^2 \frac{M_E^2 \lambda_L^e y_e - M_E M_L \bar{\lambda} \lambda_E^e - M_L^2 \lambda \lambda_E^e - M_L^2 \lambda_L^e y_e}{M_L^2 (M_E^2 - M_L^2)} \\[U_L]_{i5} &= v \frac{\lambda_E^e}{M_E} \\[U_L]_{2i} &= v^2 \frac{-M_E^2 \lambda_L^e \lambda_L^\mu y_e y_\mu + M_E M_L \bar{\lambda} \lambda_E^e \lambda_L^\mu y_\mu + M_E M_L \bar{\lambda} \lambda_L^e \lambda_E^\mu y_e - M_L^2 \lambda_E^e \lambda_E^\mu y_e^2}{M_E^2 M_L^2 (y_e^2 - y_\mu^2)} \\[U_L]_{22} &= 1 - v^2 \frac{(\lambda_E^\mu)^2}{2M_E^2} \\[U_L]_{24} &= v^2 \frac{M_E^2 \lambda_L^\mu y_\mu - M_E M_L \bar{\lambda} \lambda_E^\mu - M_L^2 \lambda \lambda_E^\mu - M_L^2 \lambda_L^\mu y_\mu}{M_L^2 (M_E^2 - M_L^2)} \\[U_L]_{25} &= v \frac{\lambda_E^\mu}{M_E} \\[U_L]_{4i} &= v^2 \frac{M_L \bar{\lambda} \lambda_E^e - M_E \lambda_L^e y_e}{M_E M_L^2} \\[U_L]_{42} &= v^2 \frac{M_L \bar{\lambda} \lambda_E^\mu - M_E \lambda_L^\mu y_\mu}{M_E M_L^2} \\[U_L]_{44} &= 1 - v^2 \frac{(M_E \lambda - M_L \bar{\lambda})^2}{2(M_E^2 - M_L^2)^2} \\[U_L]_{45} &= v \frac{M_E \lambda + M_L \bar{\lambda}}{M_E^2 - M_L^2} \\[U_L]_{5i} &= -v \frac{\lambda_E^e}{M_E} \\[U_L]_{52} &= -v \frac{\lambda_E^\mu}{M_E} \\[U_L]_{54} &= -v \frac{M_E \lambda + M_L \bar{\lambda}}{M_E^2 - M_L^2} \\[U_L]_{55} &= 1 - v^2 \frac{(M_E \lambda + M_L \bar{\lambda})^2}{2(M_E^2 - M_L^2)^2} - v^2 \frac{(\lambda_E^e)^2 + (\lambda_E^\mu)^2}{2M_E^2}\end{aligned}$$

A.2 Right SVD Matrix

$$\begin{aligned}
[U_R]_{11} &= 1 - v^2 \frac{(\lambda_L^e)^2}{2M_L^2} \\
[U_R]_{12} &= v^2 \frac{M_E^2 \lambda_L^e \lambda_L^\mu y_\mu^2 - M_E M_L \bar{\lambda} \lambda_E^e \lambda_L^\mu y_e - M_E M_L \bar{\lambda} \lambda_L^e \lambda_E^\mu y_\mu + M_L^2 \lambda_E^e \lambda_E^\mu y_e y_\mu}{M_E^2 M_L^2 (y_e^2 - y_\mu^2)} \\
[U_R]_{14} &= v \frac{\lambda_L^e}{M_L} \\
[U_R]_{15} &= v^2 \frac{M_E^2 \lambda \lambda_L^e + M_E^2 \lambda_E^e y_e + M_E M_L \bar{\lambda} \lambda_L^e - M_L^2 \lambda_E^e y_e}{M_E^2 (M_E^2 - M_L^2)} \\
[U_R]_{21} &= v^2 \frac{-M_E^2 \lambda_L^e \lambda_L^\mu y_e^2 + M_E M_L \bar{\lambda} \lambda_E^e \lambda_L^\mu y_e + M_E M_L \bar{\lambda} \lambda_L^e \lambda_E^\mu y_\mu - M_L^2 \lambda_E^e \lambda_E^\mu y_e y_\mu}{M_E^2 M_L^2 (y_e^2 - y_\mu^2)} \\
[U_R]_{22} &= 1 - v^2 \frac{(\lambda_L^\mu)^2}{2M_L^2} \\
[U_R]_{24} &= v \frac{\lambda_L^\mu}{M_L} \\
[U_R]_{25} &= v^2 \frac{M_E^2 \lambda \lambda_L^\mu + M_E^2 \lambda_E^\mu y_\mu + M_E M_L \bar{\lambda} \lambda_L^\mu - M_L^2 \lambda_E^\mu y_\mu}{M_E^2 (M_E^2 - M_L^2)} \\
[U_R]_{41} &= -v \frac{\lambda_L^e}{M_L} \\
[U_R]_{42} &= -v \frac{\lambda_L^\mu}{M_L} \\
[U_R]_{44} &= 1 - v^2 \frac{(M_L \lambda + M_E \bar{\lambda})^2}{2(M_E^2 - M_L^2)^2} - v^2 \frac{(\lambda_L^e)^2 + (\lambda_L^\mu)^2}{2M_L^2} \\
[U_R]_{45} &= v \frac{M_E \bar{\lambda} + M_L \lambda}{M_E^2 - M_L^2} \\
[U_R]_{51} &= v^2 \frac{M_E \bar{\lambda} \lambda_L^e - M_L \lambda_E^e y_e}{M_E^2 M_L} \\
[U_R]_{52} &= v^2 \frac{M_E \bar{\lambda} \lambda_L^\mu - M_L \lambda_E^\mu y_\mu}{M_E^2 M_L} \\
[U_R]_{54} &= v \frac{-M_E \bar{\lambda} - M_L \lambda}{M_E^2 - M_L^2} \\
[U_R]_{55} &= 1 - v^2 \frac{(M_E \bar{\lambda} - M_L \lambda)^2}{2(M_E^2 - M_L^2)^2}
\end{aligned}$$

A.3 Lepton Masses

$$\begin{aligned}
m_1 &= vy_e - v^3 \frac{M_E^2 (\lambda_L^e)^2 y_e - 2M_E M_L \bar{\lambda} \lambda_E^e \lambda_L^e + M_L^2 (\lambda_E^e)^2 y_e}{2M_E^2 M_L^2} \\
m_2 &= vy_\mu - v^3 \frac{M_E^2 (\lambda_L^\mu)^2 y_\mu - 2M_E M_L \bar{\lambda} \lambda_E^\mu \lambda_L^\mu + M_L^2 (\lambda_E^\mu)^2 y_\mu}{2M_E^2 M_L^2} \\
m_{d_4} &= M_L + v^2 \frac{M_E^2 \left((\lambda_L^e)^2 + (\lambda_L^\mu)^2 \right) - 2M_E M_L \bar{\lambda} \lambda - M_L^2 \left(\bar{\lambda}^2 + \lambda^2 + (\lambda_L^e)^2 + (\lambda_L^\mu)^2 \right)}{2M_L (M_E^2 - M_L^2)} \\
m_5 &= M_E + v^2 \frac{M_E^2 \left(\bar{\lambda}^2 + \lambda^2 + (\lambda_E^e)^2 + (\lambda_E^\mu)^2 \right) + 2M_E M_L \bar{\lambda} \lambda - M_L^2 \left((\lambda_E^e)^2 + (\lambda_E^\mu)^2 \right)}{2M_E (M_E^2 - M_L^2)}
\end{aligned}$$

B Comparisons of branching ratio approaches

B.1 Leptoquark comparison

Firstly the Leptoquark sample NP result within MuonBridge, which was also discussed in Section 5.2 of [Haxton et al. \[26\]](#), was numerically compared to an equivalent result gotten through the Formula given in Equation (14) of [Kitano et al. \[20\]](#). The Leptoquark couplings y_i were all set to 10^{-1} and the VLL masses were set to 1 TeV . The calculation showed a difference ratio

$$2 \frac{\text{BR}(\text{MuonBridge}) - \text{BR}(\text{Kitano et al.})}{\text{BR}(\text{MuonBridge}) + \text{BR}(\text{Kitano et al.})} = 57\%$$

which is within the allowed margins.

B.2 VLL comparison

Secondly, branching ratio results of the VLL model were compared in the same way as for the Leptoquarks. Here the effective couplings were set to $\beta_{L,E}^\mu = 0.03$, $\beta_{L,E}^e = \frac{m_e}{m_u} \beta_{L,E}^\mu$ and the masses were set to the current limits. The resulting difference ratio is

$$2 \frac{\text{BR}(\text{MuonBridge}) - \text{BR}(\text{Kitano et al.})}{\text{BR}(\text{MuonBridge}) + \text{BR}(\text{Kitano et al.})} = 27\%$$

and therefore also in the acceptable range.

B.3 Model comparison

Lastly, the Leptoquark and VLL model were compared with respect to the order of magnitude of the calculated branching ratio in MuonBridge. For this both elementary couplings, y_i for the Leptoquark and λ_i for the VLL, as well as the NP masses were set to the same value of 10^{-1} and 1 TeV respectively. This gave a difference ratio

$$2 \frac{\text{BR}(\text{VLL}) - \text{BR}(\text{LQ})}{\text{BR}(\text{VLL}) + \text{BR}(\text{LQ})} = 8\%,$$

showing that also different model results within MuonBridge are self consistent.

C Renormalization Group Equations

The one-loop Renormalization Group Equations (RGEs) for the Vector-like Lepton Model are listed. The beta functions for the yukawa and gauge couplings are given by

$$(32\pi^2) \beta(y_i) = y_i \left[\frac{1}{2} + 3y_i^2 + 2(\lambda_{L,i})^2 + (\lambda_{E,i})^2 \right] - 4\lambda_{E,i}\lambda_{L,i}\bar{\lambda} \left(\mu + \frac{1}{\mu} \right) \quad (\text{C.1a})$$

$$(32\pi^2) \beta(\lambda_{L,i}) = \lambda_{L,i} \left[\frac{1}{2} + 3(\lambda_{L,i})^2 + 4y_i^2 + \lambda \left(\lambda - \frac{8\bar{\lambda}}{\mu} \right) \right] + 2\lambda_{E,i}y_i \left[\lambda + \bar{\lambda}2\mu \right] \\ + \sum_{k \neq i} \frac{\lambda_{L,i}}{y_i^2 - y_k^2} \left[\lambda_{L,i}\lambda_{L,k} (5y_i^2 - y_k^2) + 2\lambda_{E,i}\lambda_{E,k}y_iy_k \right. \\ \left. - 4\bar{\lambda} (\lambda_{L,i}\lambda_{E,k}y_k + \lambda_{E,i}\lambda_{L,k}y_i) \left(\mu + \frac{2}{\mu} \right) \right] \quad (\text{C.1b})$$

$$(32\pi^2) \beta(\lambda_{E,i}) = \lambda_{E,i} \left[\frac{1}{2} + 3(\lambda_{E,i})^2 + 5y_i^2 + 2\lambda (\lambda - 2\bar{\lambda}\mu) \right] + 4\lambda_{L,i}y_i \left[\lambda + \frac{2\bar{\lambda}}{\mu} \right] \\ + \sum_{k \neq i} \frac{\lambda_k^E}{y_i^2 - y_k^2} \left[\lambda_{E,i}\lambda_k^E (4y_i^2 - 2y_k^2) + 4\lambda_{L,i}\lambda_k^L y_iy_k \right. \\ \left. - 4\bar{\lambda} (\lambda_{L,i}\lambda_k^E y_i + \lambda_{E,i}\lambda_k^L y_k) \left(\mu + \frac{2}{\mu} \right) \right] \quad (\text{C.1c})$$

$$(32\pi^2) \beta(\lambda) = \lambda \left[\frac{1}{2} + 3\lambda^2 + \sum_i (4(\lambda_{E,i})^2 + 5(\lambda_{L,i})^2) \right] \\ + \sum_i \left[6y_i\lambda_{E,i}\lambda_{L,i} + 4\bar{\lambda}\mu(\lambda_{E,i})^2 + \frac{2(\lambda_{L,i})^2}{\mu} \right] \quad (\text{C.1d})$$

$$(32\pi^2) \beta(\bar{\lambda}) = \bar{\lambda} \left[\frac{1}{2} + 3\bar{\lambda}^2 \right] \quad (\text{C.1e})$$

$$(32\pi^2) \beta(g_Y) = \frac{53}{3} g_Y^3 \quad (\text{C.1f})$$

$$(32\pi^2) \beta(g_W) = -5g_W^3 \quad (\text{C.1g})$$

$$(32\pi^2) \beta(g_S) = -14g_S^3 \quad (\text{C.1h})$$

with the abbreviations

$$\mathbb{A} = 2\bar{\lambda}^2 + 2\lambda^2 + 2 \sum_j \left((\lambda_j^E)^2 + (\lambda_j^L)^2 + y_j^2 + 3(y_j^u)^2 + 3(y_j^d)^2 \right) - \frac{9}{2}g_W^2 - \frac{15}{2}g_Y^2, \quad (\text{C.2})$$

$$\mu = \frac{M_L}{M_E}. \quad (\text{C.3})$$

The RGEs for the VLL masses are given by

$$(32\pi^2)\beta(m_L) = 4\bar{\lambda}\lambda m_E + \left(\lambda^2 + \bar{\lambda}^2 + \sum_j (\lambda_j^L)^2 - 9g_W^2 - 3g_Y^2 \right) m_L \quad (\text{C.4a})$$

$$(32\pi^2)\beta(m_E) = 8\bar{\lambda}\lambda m_L + 2 \left(\lambda^2 + \bar{\lambda}^2 + \sum_j (\lambda_j^E)^2 - 6g_Y^2 \right) m_E. \quad (\text{C.4b})$$

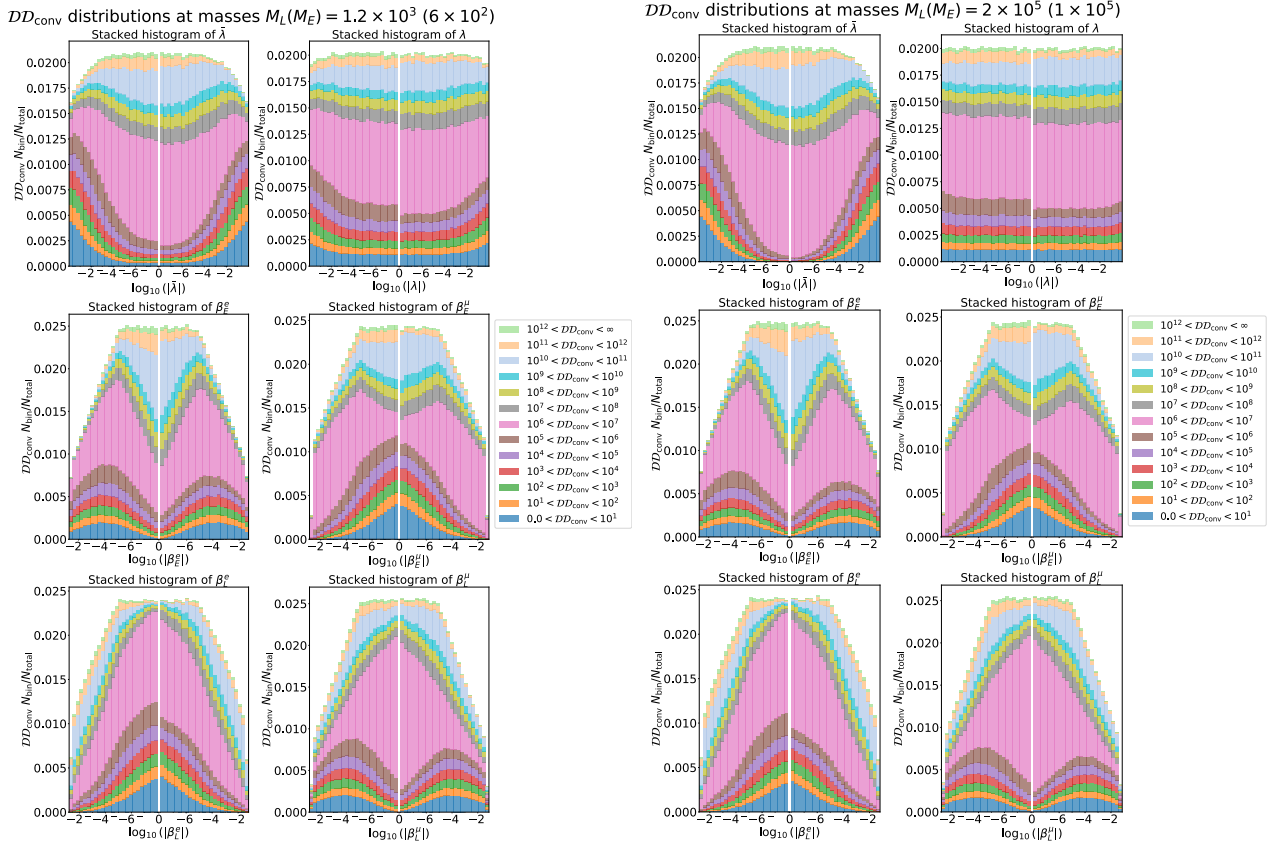
The beta function of the quartic Higgs coupling is given in Equation (4.63).

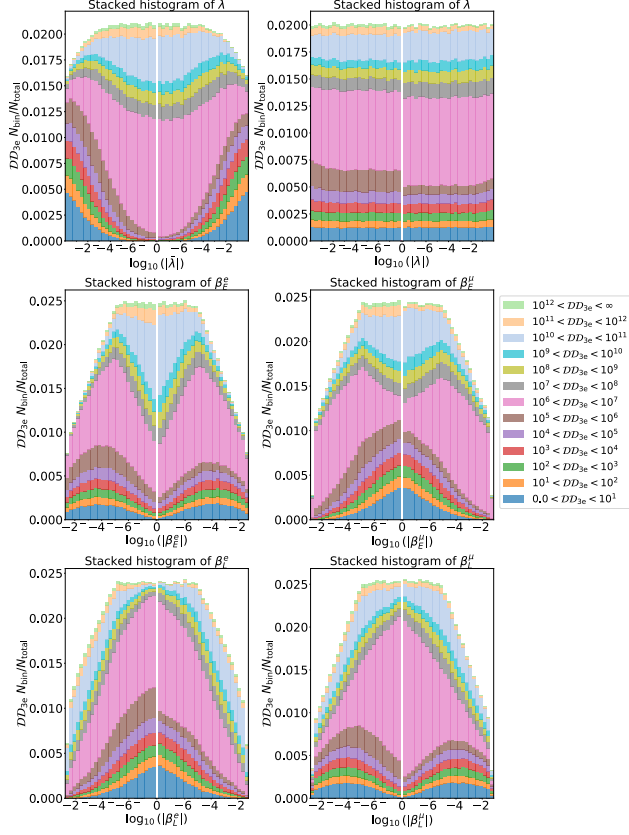
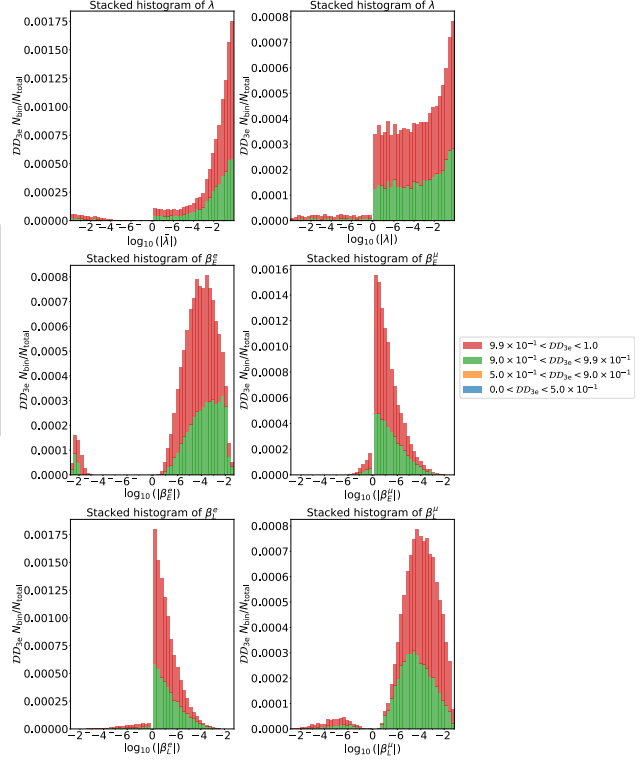
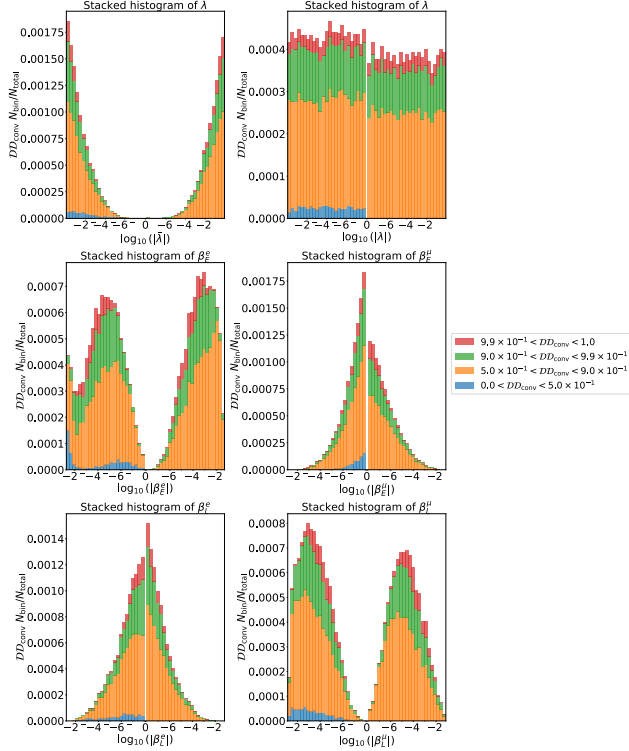
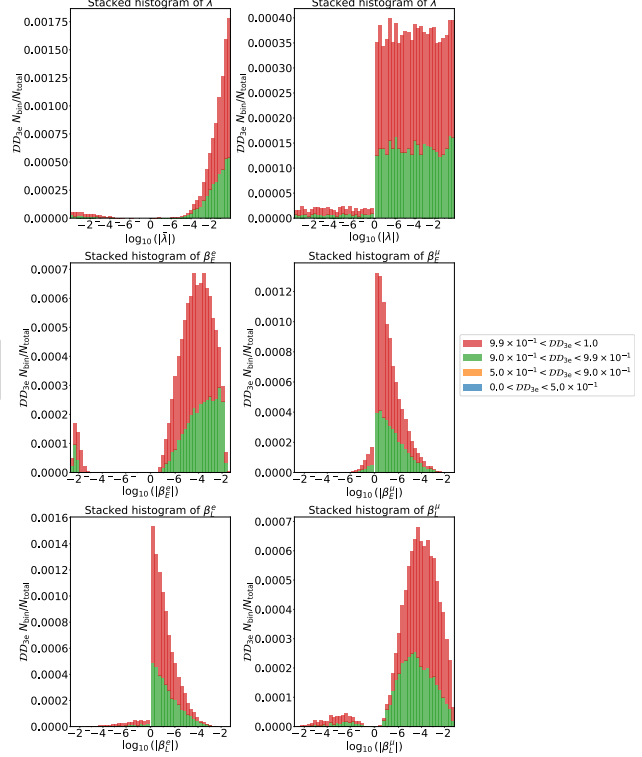
D Miscellaneous Histograms

In this chapter the omitted histograms for VLL random scans are shown. The plot titles indicate the observable and masses, the legend shows the observed ranges. The superscript “–” on the tickmarks indicates the negative sign of the plotted axis tick label. The first section shows histograms without applying the vacuum stability constraint, while the second section shows histograms with the vacuum stability constraint applied. The sections are divided subsections containing VLL-parameter histograms and Δa_μ histograms respectively.

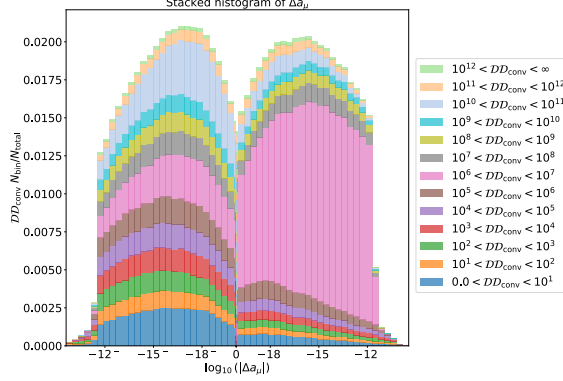
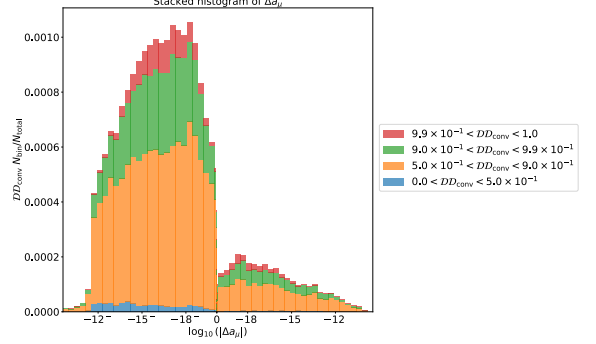
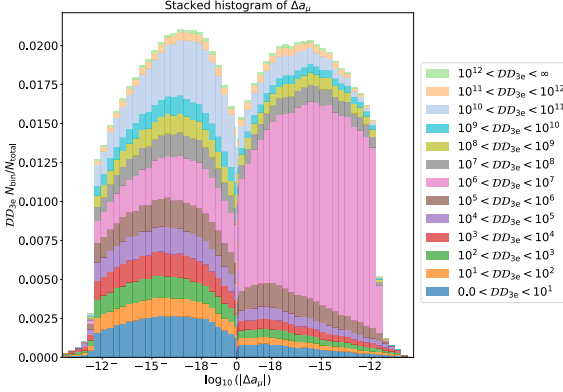
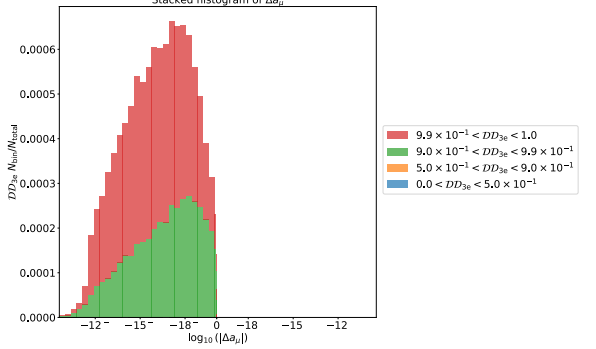
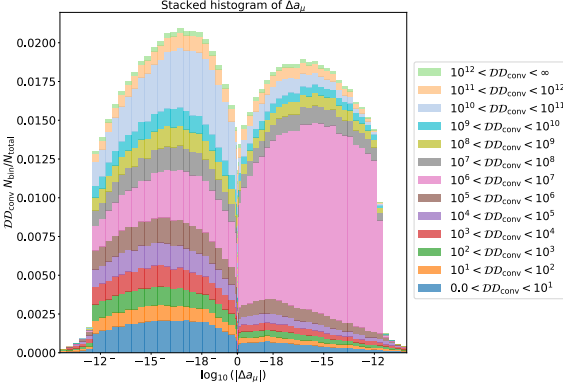
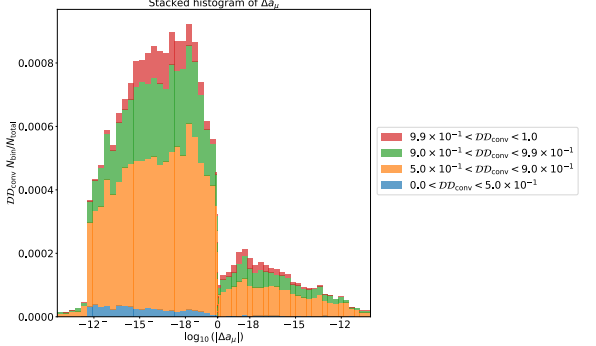
D.1 Histograms without Vacuum Stability Constraint

D.1.1 VLL-Parameter Histograms

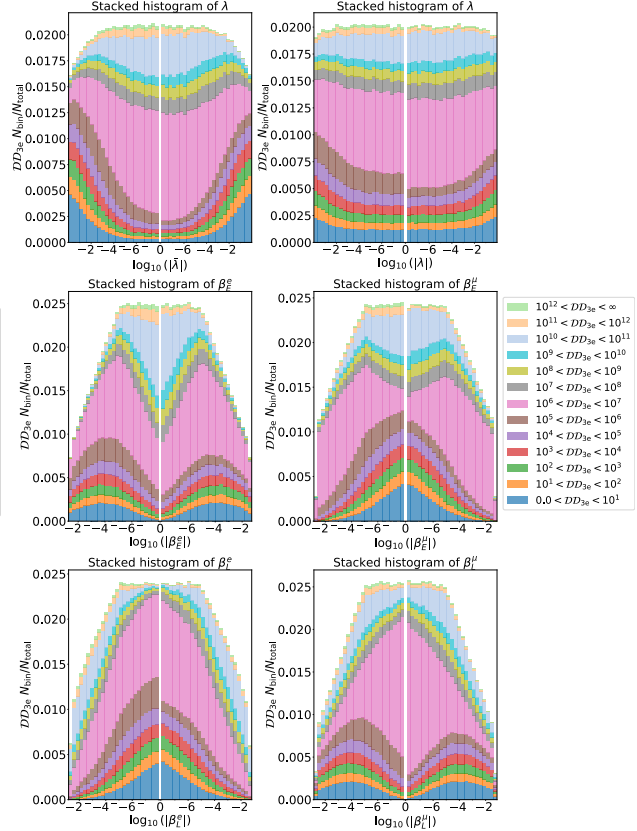


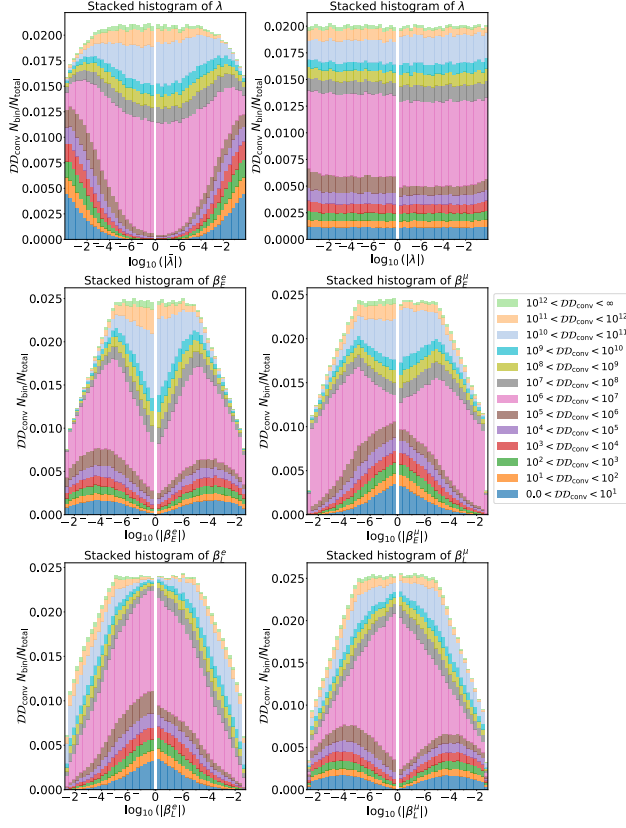
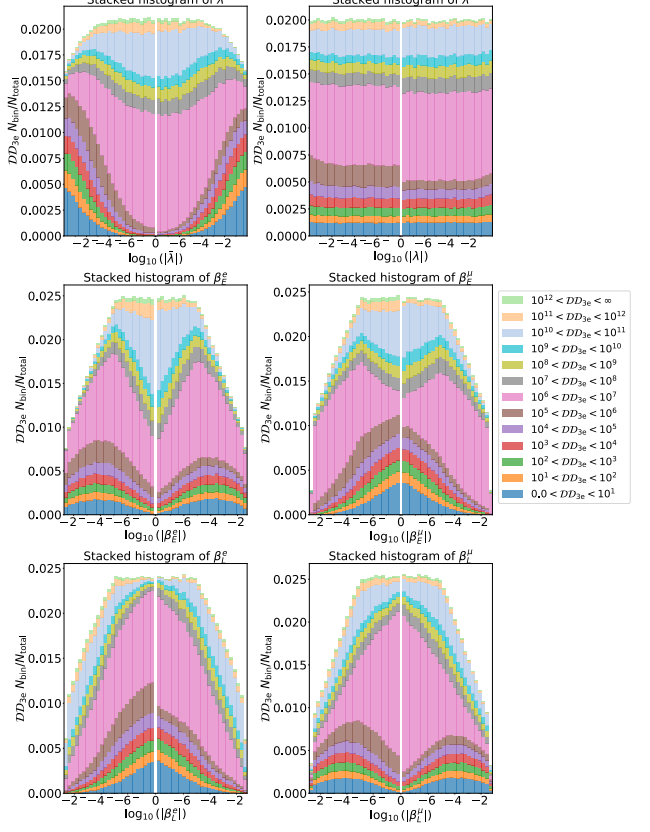
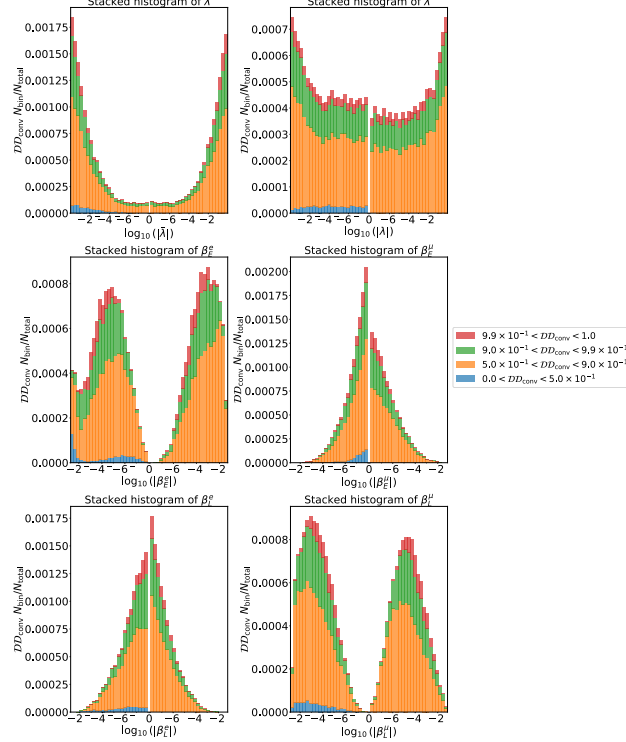
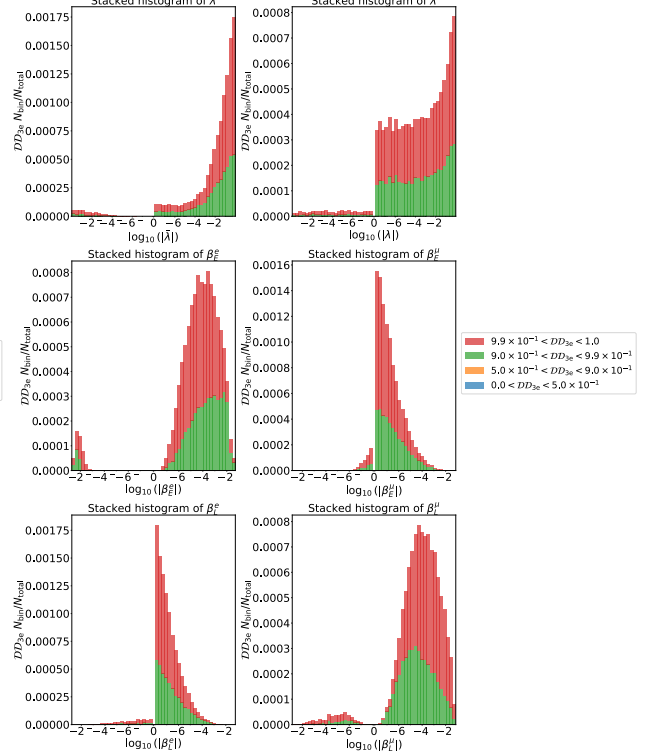
DD_{3e} distributions at masses $M_L(M_E) = 2 \times 10^5 (1 \times 10^5)$  DD_{3e} distributions at masses $M_L(M_E) = 1.2 \times 10^3 (6 \times 10^2)$  DD_{conv} distributions at masses $M_L(M_E) = 2 \times 10^5 (1 \times 10^5)$  DD_{3e} distributions at masses $M_L(M_E) = 2 \times 10^5 (1 \times 10^5)$ 

D.1.2 Δa_μ Histograms

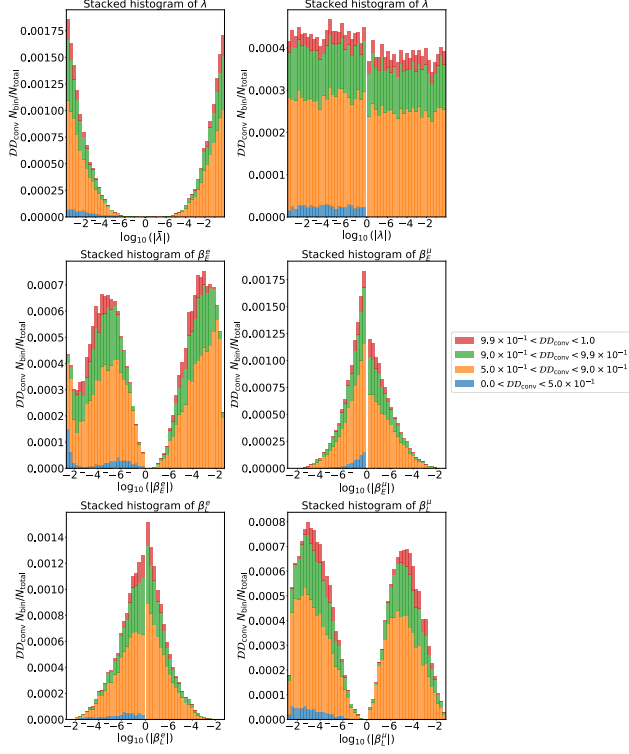
Distributions of DD_{conv} with masses $M_L(M_E) = 1.2 \times 10^3 (6 \times 10^2)$ Distributions of DD_{conv} with masses $M_L(M_E) = 1.2 \times 10^3 (6 \times 10^2)$ Distributions of DD_{3e} with masses $M_L(M_E) = 1.2 \times 10^3 (6 \times 10^2)$ Distributions of DD_{3e} with masses $M_L(M_E) = 1.2 \times 10^3 (6 \times 10^2)$ Distributions of DD_{conv} with masses $M_L(M_E) = 2 \times 10^5 (1 \times 10^5)$ Distributions of DD_{conv} with masses $M_L(M_E) = 2 \times 10^5 (1 \times 10^5)$ 

Stacked histogram of Δa_μ

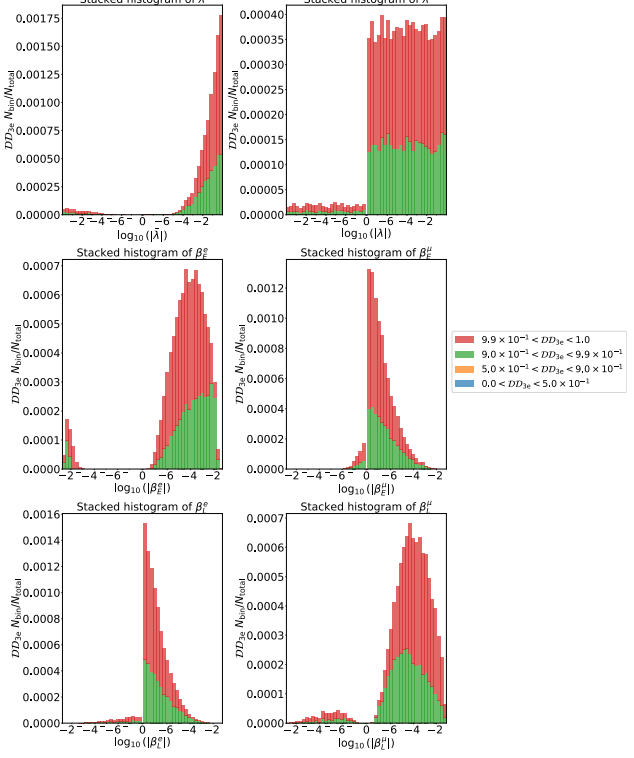


DD_{conv} distributions at masses $M_L(M_E) = 2 \times 10^5$ (1×10^5) DD_{3e} distributions at masses $M_L(M_E) = 2 \times 10^5$ (1×10^5) DD_{conv} distributions at masses $M_L(M_E) = 1.2 \times 10^3$ (6×10^2) DD_{3e} distributions at masses $M_L(M_E) = 1.2 \times 10^3$ (6×10^2)

DD_{conv} distributions at masses $M_L(M_E) = 2 \times 10^5 (1 \times 10^5)$

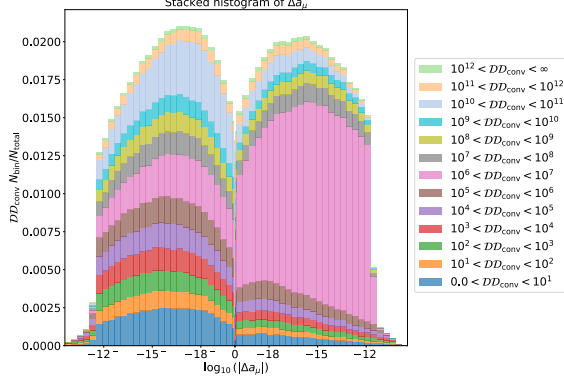


DD_{3e} distributions at masses $M_L(M_E) = 2 \times 10^5 (1 \times 10^5)$

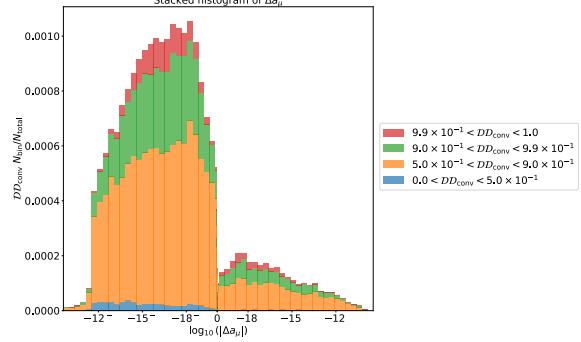


D.2.2 Δa_μ Histograms

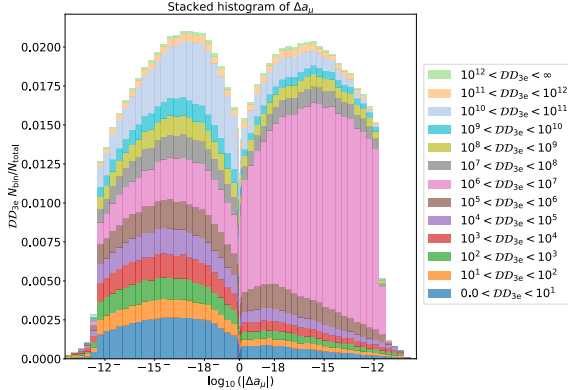
Distributions of DD_{conv} with masses $M_L(M_E) = 1.2 \times 10^3 (6 \times 10^2)$



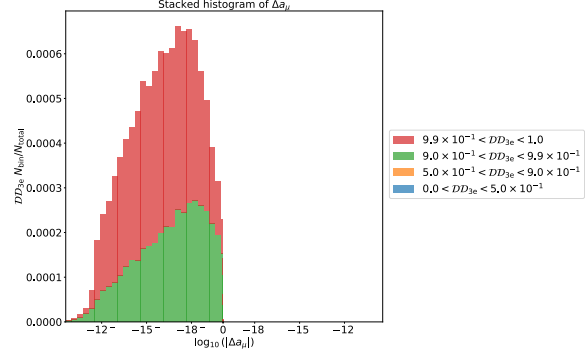
Distributions of DD_{conv} with masses $M_L(M_E) = 1.2 \times 10^3 (6 \times 10^2)$

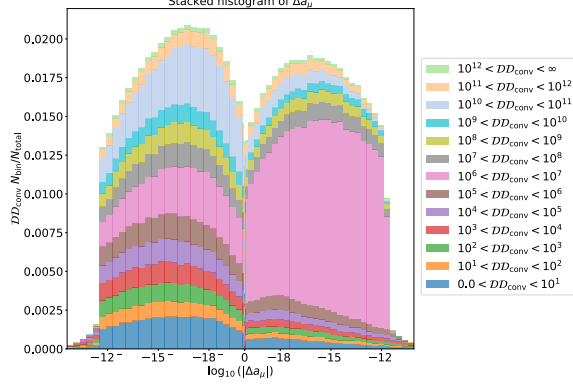
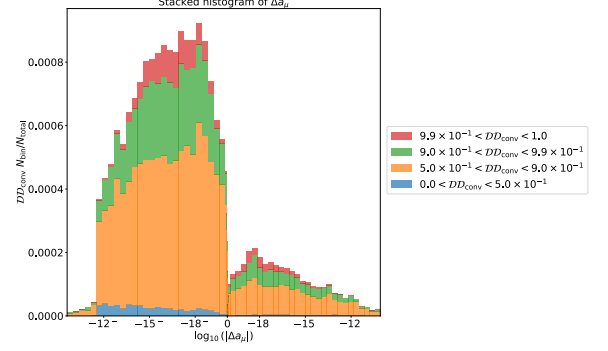
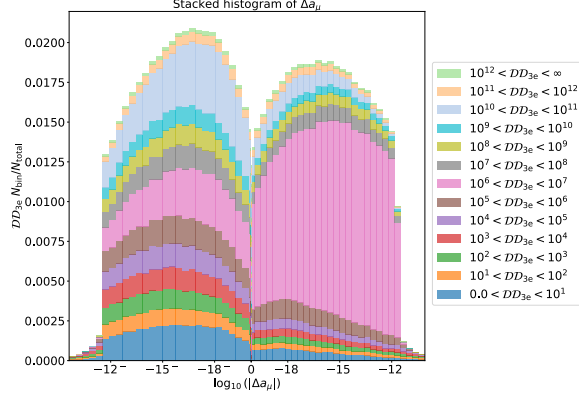
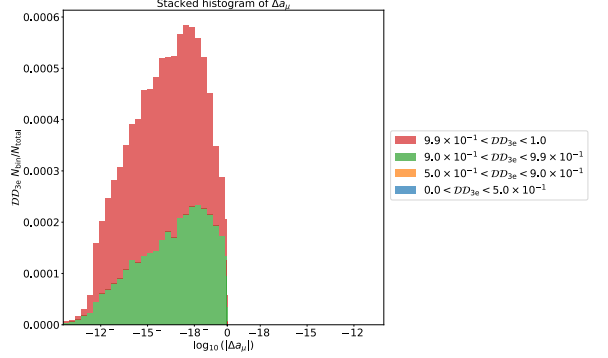


Distributions of DD_{3e} with masses $M_L(M_E) = 1.2 \times 10^3 (6 \times 10^2)$



Distributions of DD_{3e} with masses $M_L(M_E) = 1.2 \times 10^3 (6 \times 10^2)$



Distributions of DD_{conv} with masses $M_L(M_E) = 2 \times 10^5(1 \times 10^5)$ Distributions of DD_{conv} with masses $M_L(M_E) = 2 \times 10^5(1 \times 10^5)$ Distributions of DD_{3e} with masses $M_L(M_E) = 2 \times 10^5(1 \times 10^5)$ Distributions of DD_{3e} with masses $M_L(M_E) = 2 \times 10^5(1 \times 10^5)$ 

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