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On the trilinear Higgs coupling in the Minimal R-symmetric Supersymmetric Standard Model

Jonas Scheibler

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Betreuende Hochschullehrer

Prof. Dr. Dominik Stöckinger

Prof. Dr. Georg Weiglein

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Abstract

Since the discovery of a Standard Model-like Higgs in 2012, measurements of its properties have played a key role in the challenging search for Physics Beyond the Standard Model (BSM). To confront BSM theories with new experimental results, electroweak precision calculations are of vital importance. This phenomenological study investigates the Higgs sector of the Minimal R-symmetric Supersymmetric Standard Model (MRSSM). A strong focus is placed on the trilinear self-coupling of the lightest CP-even neutral Higgs. This coupling is investigated at tree level and one loop order, with an emphasis on the distinctive features of the model and its parameter space.

Zusammenfassung

Seit der Entdeckung des Standardmodell-ähnlichen Higgs im Jahr 2012 spielen Messungen dessen Eigenschaften eine entscheidende Rolle in der herausfordernden Suche nach Physik jenseits des Standardmodells (BSM). Um BSM-Theorien neue experimentelle Ergebnissen gegenüberzustellen, sind elektroschwache Präzisionsberechnungen von entscheidender Bedeutung. Diese phänomenologische Arbeit untersucht den Higgs-Sektor des Minimalen R-symmetrischen Supersymmetrischen Standardmodells (MRSSM). Im Vordergrund liegt die Analyse der trilinearen Selbstkopplung des leichtesten CP-geraden neutralen Higgs. Diese Kopplung wird auf führender Ordnung und in der ersten Ordnung in Störungstheorie untersucht, wobei ein besonderer Schwerpunkt auf den charakteristischen Merkmalen des Modells und seines Parameterraums liegt.

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Abbreviations

1PI	One Particle Irreducible
BSM	Beyond the Standard Model
EFT	Effective Field Theory
EWSB	Electroweak Symmetry Breaking
EWSM	Electroweak Standard Model
GUT	Grand Unified Theory
LHC	Large Hadron Collider
MRSSM	Minimal Supersymmetric R-Symmetric Standard Model
MSSM	Minimal Supersymmetric Standard Model
NMSSM	Next-To-Minimal Supersymmetric Standard Model
QCD	Quantum Chromo Dynamics
QFT	Quantum Field Theory
SM	Standard Model of particle physics
SSB	Spontaneous Symmetry Breaking
SUSY	Supersymmetry
vev	vacuum expectation value

1. Introduction

Despite direct evidence for Beyond the Standard Model (BSM) physics, the search for a suitable, more encompassing theory has proven notoriously difficult. Investigating the properties of the SM-like Higgs state, which has been discovered at the Large Hadron Collider (LHC) in 2012 provides a powerful probe in the search for new physics. Measuring the self couplings of the Higgs provides an insight in the shape of the Higgs potential possible signs of new physics. The trilinear Higgs coupling λ_{hhh} is crucial in this process and currently in the focus of experimental investigations. Since the trilinear coupling is not an observable, the experimental investigations focus on the study of interference effects in di-Higgs production, adjusted by the modifier

$$\kappa_\lambda = \frac{\lambda_{hhh}}{\lambda_{hhh}^{(0)SM}}. \tag{1.1}$$

Current bounds from ATLAS searches [1] on κ_λ are $-1.2 < \kappa_\lambda < 7.2$ at the 95% confidence level. From a theory perspective, calculating κ_λ and subsequently di-Higgs production in BSM models is necessary in order to compare to experiment. Out of many approaches to BSM physics, Supersymmetry (SUSY) provides a compelling framework with a rich collection of realization in various models. The Minimal Supersymmetric R-Symmetric Standard Model (MRSSM) is an extension of the Standard Model of particle physics (SM) that goes beyond the minimal implementation of SUSY. It incorporates a higher degree of symmetry than the Minimal Supersymmetric Standard Model (MSSM), namely R-symmetry. R-symmetry introduces a larger field content and unique terms in the superpotential as well as the soft breaking potential. These terms offer solutions to both theoretical and phenomenological issues of the MSSM. Through the incorporation of SUSY and R-symmetry, the Higgs sector of the MRSSM is a unique and non-trivial extension of the Higgs sector of the SM. Investigating the Higgs self couplings in the MRSSM is therefore highly motivated.

2. The Standard Model Higgs sector at tree level

To this day, the SM remains the most successful and comprehensive theory of fundamental interactions. Its predictions have been confirmed by a wide range of experiments with remarkable precision. The SM is formulated within the framework of Quantum Field Theory (QFT) and, more specifically, as a gauge theory based on the gauge group

$$\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y. \tag{2.1}$$

Here, $\mathrm{SU}(3)_C$ governs the interactions of color-charged particles and defines the theory of Quantum Chromo Dynamics (QCD), while the electroweak sector is described by the gauge group $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$, commonly referred to as the Electroweak Standard Model (EWSM). This work focuses primarily on the electroweak sector of supersymmetric theories and in particular, on the structure and phenomenology of their Higgs sector. It is therefore worthwhile to review the Higgs sector of the SM because it forms the basis of any discussion about extended frameworks.

The Higgs sector of the SM is defined by its scalar potential, which reads at tree level

$$V^{(0)}(\Phi) = -\mu^2|\Phi|^2 + \lambda|\Phi|^4 \quad \mu^2, \lambda > 0 \tag{2.2}$$

The theory is formulated in terms of the $\mathrm{SU}(2)_L$ doublet Φ. A key feature of the EWSM is the incorporation of Spontaneous Symmetry Breaking (SSB)

$$SU(2)_L \times U(1)_Y \xrightarrow{SSB} U(1)_Q. \tag{2.3}$$

In order to describe SSB effectively, the Higgs doublet Φ is parametrized as

$$\Phi = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}\left(v + h + iG^0\right) \end{pmatrix}, \tag{2.4}$$

where G^0 and G^+ describe the unphysical Goldstone bosons and the Higgs field h describes small fluctutions around the electroweak minimum, characterized by the vacuum expectation value (vev) v. The Higgs potential (2.2) after SSB reads

$$V^{(0)} \supset \underbrace{(\lambda v^3 - \mu^2 v)}_{\stackrel{!}{=}0 \Leftrightarrow \lambda v^2 = \mu^2} h + \underbrace{(\tfrac{3}{2}\lambda v^2 - \tfrac{\mu^2}{2})}_{m_h^2 = 2\lambda v^2} h^2 + \lambda v h^3 + \tfrac{\lambda}{4} h^4, \tag{2.5}$$

where the tree level tadpole condition

$$t^{(0)} = \left.\frac{\partial^3 V^{(0)}}{\partial h^3}\right|_{h=0} \stackrel{!}{=} 0, \tag{2.6}$$

ensures minimization and fixes μ^2. In the tree level minimum the mass of the Higgs m_h^2 is given as $m_h^2 = 2\lambda v^2$. The trilinear Higgs self coupling at tree level $\lambda_{hhh}^{(0)}$ is defined as the third derivative of the potential in the minimum. It relates to the mass by

$$\lambda_{hhh}^{(0)} = \left.\frac{\partial^3 V^{(0)}}{\partial h^3}\right|_{h=0} = \frac{3m_h^2}{v}. \tag{2.7}$$

In order to study BSM effects $\kappa_\lambda^{\text{BSM}}$ is defined as the trilinear Higgs coupling of the BSM theory normalized by the SM value

$$\kappa_\lambda^{\text{BSM}} = \frac{\lambda_{hhh}^{\text{BSM}}}{\lambda_{hhh}^{(0)SM}}. \tag{2.8}$$

The quartic Higgs coupling can be related to the mass as well

$$\lambda_{hhh}^{(0)} = \left.\frac{\partial^4 V^{(0)}}{\partial h^4}\right|_{h=0} = \frac{3m_h^2}{v^2}. \tag{2.9}$$

The relations (2.7) and (2.9) are investigated in the context of the decoupling limit in the MSSM and MRSSM in chapter 4 and chapter 7 at tree level and in chapter 9 at the one loop level.

3. Supersymmetry

The following chapter provides an overview alongside selected detailed aspects of Supersymmetry. Derivations are presented not for mathematical rigor, but to establish a clear and consistent framework for the discussions throughout this thesis. Given the abundance of pedagogical literature on this subject, the reader is referred to the following comprehensive resources, which form the foundation for much of the material presented in this chapter: [27], [19], [28], [4].

3.1. Spinor Notation and Structure

In most descriptions of QFT's involving fermions, Dirac Spinors are utilized. In the framework of SUSY it is however very convenient to turn to a description using Weyl Spinors which can be regarded as more fundamental objects. In order to spell this out, one may consider the Lorentz group $\mathrm{SO}(3,1)$ which can be decomposed into two $\mathrm{SU}(2)$ subgroups [29] [28]

$$\mathrm{SO}(3,1) \cong \mathrm{SU}(2) \otimes \mathrm{SU}(2). \tag{3.1}$$

In order to describe relativistic spin-$\frac{1}{2}$ fields, one turns to the $\left(\frac{1}{2},0\right)$ representation of (3.1). The representation objects of this 2-dimensional complex representation are by convention called left handed Weyl Spinors ψ_α where $\alpha = 1,2$. Representation objects of $\left(0,\frac{1}{2}\right)$ are called right handed Weyl Spinors $\overline{\psi}^{\dot{\alpha}}$ where $\dot{\alpha} = 1,2$. Both representations are related by complex conjugation

$$\overline{\psi}^{\dot{\alpha}} = (\psi^\alpha)^\dagger \,. \tag{3.2}$$

Raising and lowering of indices is done by using the two dimensional, antisymmetric ϵ-tensor

$$\psi_\alpha = \epsilon_{\alpha\beta}\psi^\beta, \qquad \overline{\psi}_{\dot{\alpha}} = \epsilon_{\dot{\alpha}\dot{\beta}}\bar{\psi}^{\dot{\beta}}. \tag{3.3}$$

The direct sum of $\left(\frac{1}{2},0\right)$ and $\left(0,\frac{1}{2}\right)$ is a four dimensional complex representation. The new representation objects are called Dirac Spinors Ψ and are constructed by two Weyl Spinors of each representation ψ_α and $\overline{\chi}^{\dot{\alpha}}$

$$\Psi = \begin{pmatrix} \psi_\alpha \\ \overline{\chi}^{\dot{\alpha}} \end{pmatrix}. \tag{3.4}$$

γ-matrices are essential objects when discussing four dimensional Spinors. In this discussion the Weyl-basis basis is used

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \overline{\sigma}^\mu & 0 \end{pmatrix}, \quad \gamma_5 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}. \tag{3.5}$$

Where

$$\sigma^0 = \overline{\sigma}^0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \qquad \sigma^1 = -\overline{\sigma}^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},$$
$$\sigma^2 = -\overline{\sigma}^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \sigma^3 = -\overline{\sigma}^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{3.6}$$

Projection operators P_L and P_R are defined in terms of γ_5 as

$$P_L = \frac{1-\gamma_5}{2} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \qquad P_R = \frac{1+\gamma_5}{2} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \tag{3.7}$$

where the matrix representation holds in the Weyl basis. Applying these projectors upon Ψ reveals the connection to the Weyl Spinors

$$\Psi_L = P_L \Psi = \begin{pmatrix} \psi_\alpha \\ 0 \end{pmatrix},$$
$$\Psi_R = P_R \Psi = \begin{pmatrix} 0 \\ \overline{\chi}^{\dot{\alpha}} \end{pmatrix}. \tag{3.8}$$

Barred and charge conjugated Spinors are defined in the usual way as

$$\overline{\Psi} = \Psi^\dagger \gamma^0, \qquad \Psi^C = i\gamma^0\gamma^2\overline{\Psi}^T. \tag{3.9}$$

Applied to (3.4) explicitly, this results in

$$\overline{\Psi} = \begin{pmatrix} \chi^\alpha & \overline{\psi}_{\dot{\alpha}} \end{pmatrix}, \quad \Psi^C = \begin{pmatrix} \chi_\alpha \\ \overline{\psi}^{\dot{\alpha}} \end{pmatrix}. \tag{3.10}$$

If a Spinor Ψ fulfills $\Psi = \Psi^C$, then $\psi_\alpha = \chi_\alpha$ in (3.4). These Spinors are called Majorana Spinors Ψ_{M} and describe particles that are their own antiparticles. So called Majorana mass-terms enter Lagrangians in the following way

$$\mathcal{L} \supset -\frac{1}{2} M \overline{\Psi}_{\mathrm{M}} \Psi_{\mathrm{M}}. \tag{3.11}$$

These mass-terms can be expressed using Weyl Spinors ψ_α

$$\mathcal{L} \supset -\frac{1}{2} M \left(\psi\psi + \overline{\psi}\,\overline{\psi} \right). \tag{3.12}$$

where the Spinor indices in (3.12) are fully contracted. At this point it is worthwhile to discuss the number of degrees of freedom n_f of the introduced types of Spinors. As stated above a Weyl Spinor ψ_α has two complex entries. After the process of canonical quantization, the number of degrees of freedom is reduced since the Lagrangian poses a second class constraint [26] [28]. Therefore the degrees of freedom are reduced to two. The same argument holds for the Dirac Spinor Ψ, reducing its number of degrees of freedom to 4. Because of the defining property a Majorana Spinor has two real degrees of freedom.

Spinor Type	n_f	Comment
Weyl Spinor ψ_α , $\overline{\psi}^{\dot{\alpha}}$	2	Second class constraint
Dirac Spinor Ψ	4	Second-class constraint
Majorana Spinor Ψ_{M}	2	$\Psi_{\mathrm{M}}^C = \Psi_{\mathrm{M}}$

Table 3.1.: Degrees of freedom for different types of Spinors.

3.2. The Supersymmetry Algebra

Relativistic Quantum Field Theory is compatible with Special Relativity and therefore incorporates the relativistic symmetry of spacetime. The Poincaré group is a 10-parameter Lie Group that encapsulates Lorentz transformations and translations and is characterized by its Algebra

$$\begin{aligned}
[P^\mu, P^\nu] &= 0, \\
[P^\mu, J^{\rho\sigma}] &= i\left(g^{\mu\rho}P^\sigma - g^{\mu\sigma}P^\rho\right), \\
[J^{\mu\nu}, J^{\rho\sigma}] &= i\left(g^{\nu\rho}J^{\mu\sigma} - g^{\nu\sigma}J^{\mu\rho} + g^{\mu\sigma}J^{\nu\rho} - g^{\mu\rho}J^{\nu\sigma}\right).
\end{aligned} \tag{3.13}$$

Quantum Field Theories can exhibit further continuous symmetries (e.g. gauge symmetries). The generators T of these symmetries are always Lorentz scalars.

$$[P^\mu, T] = [J^{\mu\nu}, T] = 0. \tag{3.14}$$

The Coleman Mandula theorem [7] answers the question whether these relations can become nontrivial. It states that the symmetry group of any interacting QFT factorises as

$$\text{Poincaré} \times \text{Internal}. \tag{3.15}$$

Loosening the underlying assumptions to the Coleman-Mandula theorem allows for evasions like Conformal Invariance or Supersymmetry.
It was worked out by Haag, Łopuszański and Sohnius [14] that the the Poincaré Algebra could be expanded by introducing fermionic generators Q_α.

$$\begin{aligned}
\{Q_\alpha, Q_\beta\} &= \{\overline{Q}_{\dot\alpha}, \overline{Q}_{\dot\beta}\} = 0, \\
\{Q_\alpha, \overline{Q}_{\dot\alpha}\} &= 2\sigma^\mu_{\alpha\dot\alpha}P_\mu, \\
[P^\mu, Q_\alpha] &= \left[P^\mu, \overline{Q}_{\dot\alpha}\right] = 0, \\
[J^{\mu\nu}, Q_\alpha] &= -\frac{1}{2}\left(\sigma^{\mu\nu}\right)_\alpha^{\ \beta} Q_\beta.
\end{aligned} \tag{3.16}$$

This evades the Coleman Mandula Theorem by diverting from a Lie Algebra to a $\mathbb{Z}_2$ graded Lie Algebra by the inclusion of anticommutation relations.

3.3. Superspace and Superfields

The parameter space of the group described by (3.16) is called Superspace. It is the product of the Minkoswki-space and two spinor-spaces described by Grassmann-valued spinors θ_α and $\overline{\theta}^{\dot\alpha}$. A superfield is a function on superspace $\Phi = \Phi\left(x^\mu, \theta, \overline{\theta}\right)$. An important class of superfields are the so called chiral superfields that are defined by

$$\overline{D}_{\dot\alpha}\Phi = 0 \tag{3.17}$$

with

$$\overline{D}_{\dot\alpha} = -\overline{\partial}_{\dot\alpha} + i(\theta\sigma^\mu)_{\dot\alpha}\partial_\mu. \tag{3.18}$$

An expansion of Φ in θ and $\overline{\theta}$ goes up to second order only because of anti-commutativity

$$\Phi(x,\theta,\overline{\theta}) = A(x) + \sqrt{2}\,\theta\psi(x) + \theta\theta F(x) - i\,\theta\sigma^\mu\overline{\theta}\,\partial_\mu A(x) - \frac{i}{\sqrt{2}}\,\theta\theta\overline{\theta}\,\overline{\sigma}^\mu\partial_\mu\psi(x) - \frac{1}{4}\,\theta\theta\,\overline{\theta\theta}\,\Box A(x), \tag{3.19}$$

where ψ is a left handed Weyl Spinor and A and F are complex scalars. The collection of ψ, A and F is called a chiral multiplet. F is an auxiliary field that can be eliminated by its

equations of motion, in other words F does not have its own dynamics since its equation of motion is purely algebraic. After imposing all equations of motion, the number of fermionic and bosonic degrees of freedom match, see (3.1).
Anti-Chiral superfields are constructed in a similar way by imposing the condition

$$D_\alpha \overline{\Phi} = 0 \tag{3.20}$$

with

$$D_\alpha = \partial_\alpha - i(\sigma^\mu \overline{\theta})_\alpha \partial_\mu. \tag{3.21}$$

Performing the same expansion as in (3.19) leads to

$$\overline{\Phi}(x,\theta,\overline{\theta}) = A^\dagger(x) + \sqrt{2}\,\overline{\theta}\overline{\psi}(x) + \overline{\theta}\overline{\theta}F^\dagger(x) + i\,\theta\sigma^\mu\overline{\theta}\,\partial_\mu A^\dagger(x) - \frac{i}{\sqrt{2}}\,\overline{\theta}\overline{\theta}\theta\,\sigma^\mu\partial_\mu\overline{\psi}(x) - \frac{1}{4}\,\theta\theta\,\overline{\theta}\overline{\theta}\,\Box A^\dagger(x), \tag{3.22}$$

where $\overline{\psi}$ is a right handed Weyl spinor and $A^\dagger$ and $F^\dagger$ are complex scalars. These fields form an anti-chiral multiplet.
Another essential type of superfield is the vector superfield V which is obtained by the reality condition $V = V^\dagger$. The expansion of V in Wess-Zumino Gauge [27] reads

$$V(x,\theta,\overline{\theta}) = \theta\sigma^\mu\overline{\theta}v_\mu(x) + i\theta\theta\overline{\theta}\overline{\lambda}(x) - i\overline{\theta}\overline{\theta}\theta\lambda(x) + \frac{1}{2}\theta\theta\overline{\theta}\overline{\theta}D(x), \tag{3.23}$$

where v_μ is a real vectorfield, λ and $\overline{\lambda}$ are left- and right-handed spinors and D is a real scalar field. Note that the fermionic and bosonic degrees of freedom match since D does not have its own dynamics and can therefore be eliminated by its equation of motion.

3.4. Supersymmetric model building

Gauge theories are essential in the context of describing nature with the apparatus of Quantum Field Theory. This section describes how to obtain Lagrangians from the superfield formalism and how to construct supersymmetric gauge theories. The derivations mainly follow [27].
Assume a theory including a chiral superfield Φ and a vector superfield $V = T^a V^a$ where T^a denote the generators of the gauge group. Gauge invariance is reflected by invariance under the following transformations of the superfields

$$\begin{aligned} \Phi &\to e^{-i2g\Lambda}\Phi, \\ \bar{\Phi} &\to \bar{\Phi}e^{i2g\bar{\Lambda}}, \\ e^{2gV} &\to e^{-i2g\bar{\Lambda}}e^{2gV}e^{i2g\Lambda}, \end{aligned} \tag{3.24}$$

where $\Lambda = \Lambda^a T^a$ and the Λ^a are chiral superfields. The superfield formalism allows to formulate theories that respect supersymmetry in a clear way. In order to obtrain supersymmetric Lagrangians that depend only on spacetime x^μ the θ dependence is integrated out. Since θ is fermionic the Berezin integral must be utilized. It can be shown that

$$\begin{aligned} &\int d^4\theta\,(\text{any product of superfields})\,, \\ &\int d^2\theta\,(\text{any product of chiral superields})\,|_{\overline{\theta}=0}\,, \end{aligned} \tag{3.25}$$

is supersymmetric up to total derivatives. Essential for gauge theories are kinetic terms for the matter fields and the minimal coupling to gauge fields. These terms are collected in

$$\int d^4\theta\overline{\Phi}e^{2gV}\Phi. \tag{3.26}$$

The expansion of the exponential e^{2gV} vanishes after the second order in V in Wess-Zumino gauge because of the anticommutativity of the θ and $\overline{\theta}$ in (3.23). (3.26) can be expanded as

$$\begin{aligned}\int d^4\theta\bar{\Phi}e^{2gV}\Phi =& F^\dagger F + \left(\partial^\mu A^\dagger - igA^\dagger v^\mu\right)\left(\partial_\mu A + igAv_\mu\right) + \bar{\psi}\bar{\sigma}^\mu\left(i\partial_\mu - gv_\mu\right)\psi \\ &- \sqrt{2}\left(i\overline{\psi\lambda}A - A^\dagger i\lambda\psi\right) + gA^\dagger DA,\end{aligned} \tag{3.27}$$

where $\bar{\psi}\bar{\sigma}^\mu\left(i\partial_\mu - gv_\mu\right)\psi$ gives rise to the minimally coupled Weyl equation. The covariant derivative is defined as usual by

$$D_\mu = \partial_\mu + igv_\mu. \tag{3.28}$$

Another key observation is that there exists no derivative term for F, which means that as stated in section 3.3, F is an auxiliary field which can be eliminated by its equations of motion. In order to acquire also kinetic terms for the gauge fields, the chiral field strength is defined as

$$W_\alpha = -\frac{1}{4}\overline{DD}\left(e^{-2gV}D_\alpha e^{2gV}\right). \tag{3.29}$$

It can be worked out [27], that

$$\begin{aligned}\int d^2\theta W^\alpha W_\alpha =& 4ig^2\lambda\sigma^\mu\left(\partial_\mu\overline{\lambda} + ig\left[v_\mu, \overline{\lambda}\right]\right) \\ &- 4ig^2\left(\partial^\mu\overline{\lambda} + ig\left[v^\mu, \overline{\lambda}\right]\right)\overline{\sigma}_\mu\lambda \\ &+ 4g^2DD + 2\left(\left[D_\mu, D_\nu\right]\right)^2\end{aligned} \tag{3.30}$$

up to a total derivative. $F^{\mu\nu}$ is defined as usual as

$$F_{\mu\nu} = F^a_{\mu\nu}T^a = \frac{1}{ig}\left[D_\mu, D_\nu\right] = \partial_\mu v_\nu - \partial_\nu v_\mu + ig\left[v_\mu, v_\nu\right]. \tag{3.31}$$

Taking the trace of (3.30) provides a gauge invariant and supersymmetric kinetic term for the gauge fields and gauginos and an interaction between them

$$\begin{aligned}&\int d^4\theta\frac{1}{16g^2\kappa}\mathrm{Tr}\left[W^\alpha W_\alpha\delta\left(\overline{\theta}\right) + h.c.\right] \\ &= \int d^4\theta\frac{1}{16g^2}\left[W^{a\alpha}W^a_\alpha\delta^2(\bar{\theta}) + h.c.\right] \\ &= \frac{1}{2}D^aD^a - \frac{1}{4}(F^a_{\mu\nu})^2 + \frac{i}{2}\bar{\lambda}^a\bar{\sigma}^\mu(D_\mu\lambda)^a + \frac{i}{2}\lambda^a\sigma^\mu(D_\mu\bar{\lambda})^a.\end{aligned} \tag{3.32}$$

κ in(3.32) is the normalization used in $\mathrm{Tr}\left(T^aT^b\right) = \kappa\delta^{ab}$.

3.5. The Superpotential

So far kinetic terms for matter and gauge fields have been introduced alongside their (minimal) interactions. The Superpotential W introduces further supersymmetric interactions by virtue of (3.25), although further constraints need to be respected. The requirement of gauge invariance and renormalizability only admit for [19],[27]

$$W(\Phi) = c_i\Phi_i + \frac{m_{ij}}{2}\Phi_i\Phi_j + \frac{g_{ijk}}{3!}\Phi_i\Phi_j\Phi_k, \tag{3.33}$$

where the couplings m_{ij} and g_{ijk} are totally symmetric. After specifying the superpotential, F and D^α terms can be eliminated by their equations of motion. All terms including D^α are

$$\mathcal{L}_D = \frac{1}{2}D^\alpha D^\alpha + A^\dagger gT^aD^aA. \tag{3.34}$$

Utilizing the Euler-Lagrange equations leads to

$$D^\alpha = -A^\dagger g T^a A \Rightarrow \mathcal{L}_D = -\frac{1}{2}\left(A^\dagger g T^a A\right)^2 . \tag{3.35}$$

The same strategy can be applied to F terms

$$\begin{aligned}\mathcal{L}_F &= F_i^\dagger F_i + \left(c_i F_i + m_{ij} F_i A_j + \frac{g_{ijk}}{2} F_i A_j A_k + h.c.\right)\\ &= F_i^\dagger F_i + \left(\frac{\partial W(A)}{\partial A_i} F_i + h.c.\right),\end{aligned} \tag{3.36}$$

which leads to

$$F_i^\dagger = -\frac{\partial W(A)}{\partial A_i} \Rightarrow \mathcal{L}_F = -\left|\frac{\partial W(A)}{\partial A_i}\right|^2 . \tag{3.37}$$

A renormalizable supersymmetric gauge theory is specified by the gauge transformation properties of all superfields and by its superpotential W. This provides the masses and the interactions of all particles. The general Lagrangian is given by

$$\mathcal{L} = \int d^4\theta \left(\bar{\Phi} e^{2gV} \Phi + \frac{1}{16g^2}\left(W^{a\alpha} W^a_\alpha \delta^2(\bar{\theta}) + h.c.\right) + \left(W(\Phi)\delta^2(\bar{\theta}) + h.c.\right)\right). \tag{3.38}$$

3.6. R-Symmetry

Equation (3.14) states that the generators of all internal symmetries must commute with the Poincaré generators. This holds true for the SUSY generators except for a global U(1) symmetry which is called R symmetry [19]. The generators Q_α and $\overline{Q}_{\dot{\alpha}}$ transform in the following way

$$Q_\alpha \rightarrow e^{-i\lambda} Q_\alpha \quad , \quad \overline{Q}_{\dot{\alpha}} \rightarrow e^{i\lambda} \overline{Q}_{\dot{\alpha}}, \tag{3.39}$$

which implies that they have R charges -1 and $+1$ respectively. They have nontrivial commutation relations with the symmetry generator R

$$[R, Q_\alpha] = -Q_\alpha \quad \text{and} \quad [R, \overline{Q}_{\dot{\alpha}}] = \overline{Q}_{\dot{\alpha}}. \tag{3.40}$$

The coordinates of superspace θ and $\overline{\theta}$ have respective charges $+1$ and -1. An R-charge r_s for an entire superfield $S(x^\mu, \theta, \overline{\theta})$ is specified by its transformation

$$S(x^\mu, \theta, \overline{\theta}) \rightarrow e^{ir_s\lambda} S(x^\mu, e^{i\lambda}\theta, e^{-i\lambda}\overline{\theta}). \tag{3.41}$$

A chiral superfield Φ transforms with an R-charge r_Φ and because of (3.19) its components transform with

$$R[A] = r_\Phi, \quad R[\psi] = r_\Phi - 1, \quad \text{and} \quad R[F] = r_\Phi - 2 \tag{3.42}$$

Since a vector superfield is real its R-charge is zero. This translates to the following R-charges of its components

$$R[v_\mu] = 0, \quad R[\lambda] = 1, \quad \text{and} \quad R[D] = 0. \tag{3.43}$$

Two immediate consequences of (3.43) are that the inclusion of Majorana mass terms for gauginos as in (3.12) breaks R-symmetry and that kinetic terms (3.26) preserve R-symmetry .

Because θ carries R-charge $+1$, $d^2\theta$ must carry R-charge -2. Combined with (3.38) this means that the Superpotential W of an R-symmetric theory is required to have R-charge +2. The construction of the Minimal R-Symmetric Supersymmetric Standard Model is discussed in chapter 5.

3.7. Supersymmetry breaking

A fully supersymmetric theory predicts the same masses for partners in a multiplet. When looking at SUSY from a phenomenological perspective, this disagreement with experimental observation needs to be accounted for by breaking the supersymmetry. By explicitly adding non-supersymmetric terms to the Lagrangian, the desired aspects of a SUSY theory are kept while making the model phenomenologically viable. Soft terms are not supersymmetric and have couplings with positive mass dimension. This ensures a separation of the electroweak scale and a larger SUSY scale. A general Lagrangian $\mathcal{L}_{\text{soft}}$ is given by the Girardello-Grisaru terms that do not introduce quadratic divergences [19]

$$\mathcal{L}_{\text{soft}} = -\left(\frac{1}{2}M_a\lambda^a\lambda_a + \frac{1}{6}a^{ijk}\phi_i\phi_j\phi_k + \frac{1}{2}b^{ij}\phi_i\phi_j + t^i\phi_i + h.c.\right) - \left(m^2\right)^i_j \phi^{j*}\phi_i. \tag{3.44}$$

The first term describes Majorana mass terms for the gauginos. The other terms describe a triple scalar interaction with the coupling a^{ijk}, scalar masses b^{ij} and $\left(m^2\right)^i_j$ and a tadole term with the coupling t^i. There is another class of possible soft terms that needs to be further investigated

$$\mathcal{L}_{\text{soft candidates}} = -\left(\frac{1}{2}c_i^{jk}\phi^{*i}\phi_j\phi_k + M^a_{\text{Dirac}}\lambda^a\psi_a + \frac{1}{2}m^{ij}\psi_i\psi_j + h.c\right). \tag{3.45}$$

The first term in (3.45) is problematic since it can give rise to quadratic divergences. The second term is particularly interesting in the context of this thesis since it presents a way to introduce mass terms for the gauginos without breaking R-Symmetry as dicussed in section 3.6. The inclusion of a Dirac-mass term for the gauginos requires a chiral superfield that transforms in the adjoint representation. The last term in (3.45) introduces masses for the fermions of a chiral multiplett but such a term can be absorbed by redefinitions [19].
Soft SUSY breaking terms can be generated by spontaneous symmetry breaking in the sense that the full theory is supersymmetric but this symmetry is not shared by its ground state. The dynamical breaking of SUSY can be moderated by a Spurion that acquires a vev in its F or D term. The breaking arising from an F-term vev known as O'Raifeartaigh breaking while the D-term breaking is called Fayet-Iliopoulos breaking.
One possibility for generating a D term induced breaking is by considering the operator

$$\int d^2\theta \frac{\sqrt{2}\mathcal{W}^\alpha}{M} W^a_\alpha \Phi^a, \tag{3.46}$$

where $\mathcal{W}^\alpha = \theta^\alpha D$ describes a spurion field strength that acquires a vev $\langle D\rangle$ for the D term. In this way a Dirac mass parameter M^D can be generated dynamically

$$\int d^2\theta \frac{\sqrt{2}\mathcal{W}^\alpha}{M} W^a_\alpha \Phi^a \overset{\text{vev}}{=} -M^D\left(\lambda^a\psi^a - \sqrt{2}D^a\phi^a\right), \tag{3.47}$$

where $M^D = \frac{\langle D\rangle}{M}$. For a detailed derivation see [13]. This generation of a Dirac mass term has unique properties and is featured in the MRSSM.

4. The Minimal Supersymetric Standard Model

This chapter provides an introduction to the Minimal Supersymmetric Standard Model and highlights several elements that will be revisited throughout the thesis, with a particular emphasis on R-symmetry and its profound consequences. For simplicity only one generation of matter is introduced. Most derivations follow [27] and [19].

4.1. Lagrangian and field content of the MSSM

The Minimal Supersymmetric Standard Model (MSSM) is obtained by extending the particle content of the Standard Model into the minimal amount of supersymmetric multipletts needed in order to ensure a SUSY theory while necessarily adding a second Higgs doublett. In order to make it a realistic theory, the supersymmetric Lagrangian needs to be supplemented by soft breaking terms

$$\mathcal{L}_{\text{MSSM}} = \mathcal{L}_{\text{SUSY}} + \mathcal{L}_{\text{soft}} \tag{4.1}$$

Since the MSSM inherits the gauge group of the SM: $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$, the model is specified by providing the gauge quantum numbers of the fields alongside its superpotential and the breaking terms. Utilizing (3.38), the supersymmetric Lagrangian $\mathcal{L}_{\text{SUSY}}$ is

$$\begin{aligned}
\mathcal{L}_{\text{SUSY}} = \int d^4\theta \, \Big(& \overline{Q}\, e^{2gV+2g'V'+2g_sV_s} Q + \overline{U}\, e^{2gV+2g'V'-2g_sV_s^T} U + \overline{D}\, e^{2gV+2g'V'-2g_sV_s^T} D \\
& + \overline{L}\, e^{2gV+2g'V'} L + \overline{E}\, e^{2gV+2g'V'} E \\
& + \overline{H}_d\, e^{2gV+2g'V'} H_d + \overline{H}_u\, e^{2gV+2g'V'} H_u \Big) \\
+ \int d^2\theta & \left(\frac{1}{16g^2} W^{\alpha a} W^a_\alpha + \frac{1}{16g'^2} W'^{\alpha} W'_\alpha + \frac{1}{16g_s^2} W_s^{\alpha a} W^a_{s\alpha} \right) + h.c. \\
+ \int d^2\theta & \, W_{\text{MSSM}} + h.c.,
\end{aligned} \tag{4.2}$$

where $V_s = V_s^a \frac{\lambda^a}{2}$, $V = V^a \frac{\sigma^a}{2}$ and $V' = v'Y$.

Table 4.1 shows the contents of the superfields and transformation properties. The generators for the transformations in each representation are given in table 4.2.

Superfield	Components	$\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$
Φ	A, ψ	
V	λ, v_μ	
Q	$\tilde{q}_L = \begin{pmatrix} \tilde{u}_L \\ \tilde{d}_L \end{pmatrix}, \quad q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$	$(3, 2, 1/6)$
U	$\tilde{u}_R^\dagger$, u_R	$(3^*, 1, -2/3)$
D	$\tilde{d}_R^\dagger$, d_R	$(3^*, 1, 1/3)$
L	$\tilde{l}_L = \begin{pmatrix} \tilde{\nu}_L \\ \tilde{e}_L \end{pmatrix}, \quad l_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$	$(1, 2, -1/2)$
E	$\tilde{e}_R^\dagger$, e_R	$(1, 1, 1)$
H_d	$H_d = \begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix}, \quad \psi_{H_d}$	$(1, 2, -1/2)$
H_u	$H_u = \begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix}, \quad \psi_{H_u}$	$(1, 2, 1/2)$
V'	λ', B_μ	$(1, 1, 0)$
V^a	λ^a, W_μ^a	$(1, 3, 0)$
V_s^a	λ_s^a, G_μ^a	$(8, 1, 0)$

Table 4.1.: Superfields in the MSSM with their components and representations under gauge transformations.

Group	Label	Generator	Comment
$\mathrm{SU}(2)_L$	2	$T^a = \frac{\sigma^a}{2}$	Fundamental representation
	3	$\left(T^{\mathrm{ad}}\right)^a_{bc} = -i f_{abc} = -i\epsilon_{abc}$	Adjoint representation
$\mathrm{SU}(3)_C$	3	$T_s^a = \frac{\lambda^a}{2}$	Fundamental representation
	3^*	$T_s^a = -\frac{\lambda^{a*}}{2} = -\frac{\lambda^{aT}}{2}$	Anti-fundamental representation
	8	$\left(T_s^{\mathrm{ad}}\right)^a_{bc} = -i f_{abc}$	Adjoint representation

Table 4.2.: Representations of gauge group generators.

The superpotential of the MSSM is given as

$$W_{\mathrm{MSSM}} = y_d H_d Q D + y_u H_u Q U + y_e H_d L E - \mu H_d H_u, \tag{4.3}$$

where the first three terms correspond to supersymmetric versions of Yukawa interactions with H_u and H_d. Since the superpotential needs to be holomorphic there is no directly corresponding term to the Higgs mass term of the SM. The μ-term however takes this role as will be discussed in the next section. In order to completely specify the MSSM, the soft terms need to be stated. They are the following collection of Girardello-Grisaru terms

$$\begin{aligned}\mathcal{L}_{\text{soft}} =& - M_{\tilde{Q}}^2|\tilde{q}_L|^2 - M_{\tilde{U}}^2|\tilde{u}_R|^2 - M_{\tilde{D}}^2|\tilde{d}_R|^2 - M_{\tilde{L}}^2|\tilde{l}_L|^2 \\ & - M_{\tilde{E}}^2|\tilde{e}_R|^2 - M_{H_d}^2|H_d|^2 - M_{H_u}^2|H_u|^2 \\ & + \frac{1}{2}\left(M_1\lambda'\lambda' + M_2\lambda^a\lambda^a + M_3\lambda_s^a\lambda_s^a + h.c.\right) \\ & - \left(A_d\, y_d\, H_d\tilde{q}_L\tilde{d}_R^\dagger + A_u\, y_u\, H_u\tilde{q}_L\tilde{u}_R^\dagger + A_e\, y_e\, H_d\tilde{l}_L\tilde{e}_R^\dagger + h.c.\right) \\ & - \left(B_\mu\, H_d H_u + h.c.\right).\end{aligned} \tag{4.4}$$

The first two lines in (4.4) introduce mass terms for the squarks and sleptons while the third line consists of Majorana mass terms for the gauginos. The last two lines include trilinear scalar interactions and the bilinear B_μ-term.

4.2. The Higgs sector of the MSSM

Compared to the SM, supersymmetric models like the MSSM or MRSSM offer a rich and considerably more intricate Higgs sector. Considering only the supersymmetric part of (4.1), the number of parameters is less than in the SM because of the higher degree of symmetry. The absent coupling is the quartic coupling λ of the Higgs potential. In section 3.5 it is argued that the superpotential cannot include a term that consists of the product of four chiral superfields. Quartic scalar terms can only arise from F- and D-terms which implies that the mass of the lightest (SM-like) Higgs is predicted rather than a free parameter of the theory. In Table 4.1, the two Higgs doubletts H_u and H_d, have been introduced as the scalar field components of the chrial superfields, which share the same names. The Lagrangian parts only containing H_u and H_d can be divided into the parts arising from F-terms, D-terms and soft breaking terms [27]

$$\begin{aligned}\mathcal{L}^D_{H_u,H_d} &= -\frac{g^2}{2}\left(H_d^\dagger T^a H_d + H_u^\dagger T^a H_u\right)^2 - \frac{g'^2}{2}\left(Y_{H_d}H_d^\dagger H_d + Y_{H_u}H_u^\dagger H_u\right)^2, \\ \mathcal{L}^F_{H_u,H_d} &= -|\mu|^2\left(H_d^\dagger H_d + H_u^\dagger H_u\right), \\ \mathcal{L}^{\text{soft}}_{H_u,H_d} &= -\left(M_{H_d}^2 H_d^\dagger H_d + M_{H_u}^2 H_u^\dagger H_u + (B\mu H_d H_u + \text{h.c.})\right),\end{aligned} \tag{4.5}$$

where $Y_{H_d} = -\frac{1}{2}$ and $Y_{H_u} = \frac{1}{2}$. The Higgs potential V is defined as the negative of (4.5). Multiplications $H_d H_u$ are $\mathrm{SU}(2)_L$ invariant via $H_d H_u = \epsilon_{ij} H_d^i H_u^j$ and $|H_d|^2 = H_d^\dagger H_d = H_d^i H_d^i$. In order to describe Electroweak Symmetry Breaking (EWSB) effectively the following parametrization of the neutral doublett components H_d^0,H_u^0 is used

$$H_d = \begin{pmatrix} \frac{1}{\sqrt{2}}\left(v_d + \phi_d + i\sigma_d\right) \\ H_d^- \end{pmatrix}, \qquad H_u = \begin{pmatrix} H_u^+ \\ \frac{1}{\sqrt{2}}\left(v_u + \phi_u + i\sigma_u\right). \end{pmatrix}. \tag{4.6}$$

This corresponds to the convention[1] used in the `Mathematica` package `SARAH` [22], [23]. Tadpole equations correspond to the minimization condition of the Potential. They read

$$\begin{aligned}\left.\frac{\partial V}{\partial \phi_d}\right|_{\text{all fields }=0} &= -B_\mu\, v_u + \frac{1}{8}(g^2 + g'^2)\, v_d\left(v_d^2 - v_u^2\right) + v_d\left(M_{H_d}^2 + \mu^2\right), \\ \left.\frac{\partial V}{\partial \phi_u}\right|_{\text{all fields }=0} &= -B_\mu\, v_d + \frac{1}{8}(g^2 + g'^2)\, v_u\left(v_u^2 - v_d^2\right) + v_u\left(M_{H_u}^2 + \mu^2\right).\end{aligned} \tag{4.7}$$

[1]In [27] and [19] vacuum expectation values in the doubletts are defined without the normalization factor $\frac{1}{\sqrt{2}}$.

The two complex doubletts H_u and H_d have eight real degrees of freedom. Because $SU(2)_L \times U(1)_Y \xrightarrow{SSB} U(1)_e$ breaks three generators, three Goldstone bosons exist. These are denoted as $G^\pm$ and G^0. The mixing of gauge and mass eigenstates is summarized by [27]

$$\begin{aligned} \begin{pmatrix} H \\ h \end{pmatrix} &= \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \phi_d \\ \phi_u \end{pmatrix}, \\ \begin{pmatrix} G^0 \\ A^0 \end{pmatrix} &= \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} \sigma_d \\ \sigma_u \end{pmatrix}, \\ \begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix} &= \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} H_d^\pm \\ H_u^\pm \end{pmatrix}. \end{aligned} \tag{4.8}$$

The real parts of the neutral components of the doubletts ϕ_u and ϕ_d mix into the mass eigenstates h and H. These are the CP-even and neutral states where it is assumed that the lighter state h is the SM-like state. The other physical Higgs states are the charged Higgses $H^\pm$ and the CP odd Higgs A^0. The following definitions are useful in shortening calculations

$$\begin{aligned} \tan(\beta) &= \frac{v_u}{v_d}, \quad v^2 = v_u^2 + v_d^2, \\ m_Z^2 &= \tfrac{(g^2+g'^2)}{4} v^2, \quad m_A^2 = \frac{2B_\mu}{\sin(2\beta)}. \end{aligned} \tag{4.9}$$

4.3. The mass of the lightest Higgs in the MSSM

The mass matrix M_h^2 consisting of bilinears of ϕ_u and ϕ_d reads

$$M_h^2 = \begin{pmatrix} m_Z^2\cos^2\beta + m_A^2\sin^2\beta & -(m_A^2+m_Z^2)\cos\beta\sin\beta \\ -(m_A^2+m_Z^2)\cos\beta\sin\beta & m_A^2\cos^2\beta + m_Z^2\sin^2\beta \end{pmatrix}. \tag{4.10}$$

The rotation matrix U^T given in (4.8), that diagonalizes M_h^2 is a general orthogonal 2x2 matrix. The mixing angle α can be determined by considering

$$U^T M_h^2 U = D, \tag{4.11}$$

where D is the diagonal matrix of eigenvalues of M_h^2. The (1,2) (or (2,1)) component of (4.11) reads

$$\begin{aligned} &\frac{1}{2}\left(m_A^2\sin(2\alpha-2\beta) - m_Z^2\sin(2\alpha+2\beta)\right) = 0, \\ &\Rightarrow \tan(2\alpha) = \tan(2\beta)\left(\frac{m_Z^2+m_A^2}{m_A^2-m_Z^2}\right). \end{aligned} \tag{4.12}$$

A particularly interesting limit in the context of SUSY phenomenology is to consider $v/M_{\text{SUSY}} \to 0$. Keeping $v/M_{\text{SUSY}} = \epsilon$ finite defines a very powerful expansion parameter which will be used extensively throughout this thesis. When considering (4.12) in this limit $\left(\frac{m_Z^2}{m_A^2} = \epsilon\right)$, the mixing angle α tends to β

$$\begin{aligned} &\tan(2\alpha) = \tan(2\beta)\left(1 + \frac{2m_Z^2}{m_A^2} + \dots\right) \\ &\Rightarrow \alpha \overset{\epsilon\to 0}{=} \beta + n\frac{\pi}{2}, \quad n \in \mathbb{Z} \end{aligned} \tag{4.13}$$

The eigenvalues $m_{h,H}^2$ of M_h^2 are the masses of the physical particles H^0 and h^0 [27]

$$m_{h,H}^2 = \frac{1}{2}\left(m_A^2 + m_Z^2 \mp \sqrt{\left(m_A^2+m_Z^2\right)^2 - 4m_A^2 m_Z^2\cos^2(2\beta)}\right) \tag{4.14}$$

Expanding in $\frac{m_Z}{m_A}$ delivers at leading order the famous MSSM predicition for m_h^2

$$\begin{aligned} m_h^2 &= m_Z^2 \cos(2\beta)^2 + \dots \\ m_H^2 &= m_A^2 + m_Z^2 \sin(2\beta)^2 + \dots \end{aligned} \tag{4.15}$$

The same result can be obtained by perturbatively diagonalizing M_h^2 in orders of ϵ. This procedure is applied to M_h^2 in section A.1.

4.4. The trilinear Higgs coupling

In order to determine the trilinear coupling of the lightest CP even Higgs state h at tree level, all trilinear ϕ_d and ϕ_u terms must be obtained from the potential V

$$V|_{\phi_i\phi_j\phi_k} = \frac{\sqrt{2}\left(g^2 + g'^2\right)}{8} \left(v_u\phi_u^3 + v_d\phi_d^3 - v_u\phi_u\phi_d^2 - v_d\phi_u^2\phi^d\right). \tag{4.16}$$

Using (4.8) the relation between the gauge eigenstates ϕ_d, ϕ_u and h can be expressed as

$$\begin{aligned} \phi_u &\sim \cos(\alpha)\, h, \\ \phi_d &\sim -\sin(\alpha)\, h. \end{aligned} \tag{4.17}$$

The trilinear coupling of the physical state h is then given as

$$\begin{aligned} \lambda_{hhh} = \frac{\partial^3 V}{\partial h^3} &= 3\frac{\sqrt{2}\left(g^2 + g'^2\right)}{4} \underbrace{\left(v_u \cos^3(\alpha) - v_d \sin^3 - v_u \cos(\alpha)\sin^2(\alpha) + v_d \sin(\alpha)\cos^2(\alpha)\right)}_{\cos(2\alpha)(v_u\cos(\alpha)+v_d\sin(\alpha))} \\ &= \frac{3 m_Z g}{2\cos(\theta_W)} \cos(2\alpha)\sin(\alpha+\beta)\,, \end{aligned} \tag{4.18}$$

where $\cos(\theta_W) = \frac{g}{\sqrt{g^2+g'^2}}$ is the weak mixing angle. This confirms the literature result [15]. In the decoupling limit, where $\epsilon \to 0$ and therefore $\alpha \to \beta - \frac{\pi}{2}$, the trilinear tends to

$$\lambda_{hhh} \to \frac{3 m_Z g}{2\cos(\theta_W)} \cos^2(2\beta) = \frac{3m_h^2}{v}, \tag{4.19}$$

which is the well known SM relation between the mass and the trilinear.

4.5. The quartic Higgs coupling

The quartic Higgs coupling is defined by the collection of all quartic terms in the potential

$$V|_{\phi_i\phi_j\phi_k\phi_l} = \frac{1}{32}\left(g'^2 + g^2\right)\left(\phi_d^2 - \phi_u^2\right)^2. \tag{4.20}$$

In mass eigenstates it reads

$$\begin{aligned} \lambda_{hhhh} = \frac{\partial^4 V}{\partial h^4} &= \frac{3}{4}\left(g'^2 + g^2\right)\left(\cos^2\alpha - \sin^2\alpha\right)^2 \\ &= \frac{3}{4}\left(g'^2 + g^2\right)\cos^2(2\alpha) \\ &= \frac{3g^2}{4\cos^2(\theta_W)}\cos^2(2\alpha) \end{aligned} \tag{4.21}$$

Which is also confirmed by [15]. In the decoupling limit $\alpha \to \beta - \frac{\pi}{2}$ the quartic coupling also tends towards the SM relation between the mass and the quartic coupling

$$\lambda_{hhhh} \to \frac{3g^2}{4\cos^2(\theta_W)}\cos^2(2\beta) = \frac{3m_h^2}{v^2}. \tag{4.22}$$

5. The Minimal R-symmetric Supersymmetric Standard Model

The pervious chapter introduced the Minimal Supersymmetric extension of the Standard Model, the MSSM. Even though the MSSM is a rather large model and extremely rich in its phenomenology, extensions that go beyond the minimal implementation of SUSY have been developed to adress some of the shortcomings of the MSSM. As an example, the "μ-term" in the superpotential of the MSSM (4.3) includes the dimensionful μ-parameter. Explaining why μ is naturally of the order of the EW scale rather than the Grand Unified Theory (GUT)- or Planck scale is called the μ-problem. The Next-To-Minimal Supersymmetric Standard Model (NMSSM) adresses this by introducing an additional singlet chiral superfield S that generates the equivalent of the μ-term dynamically by its vev $-\mu H_d H_u \to -\lambda S H_d H_u$, where λ is a dimensionless coupling.
Another problematic feature of the MSSM arises from dangerous lepton- and baryon number violating terms which could in principle be added to the superpotential of the MSSM. In order to prevent this problem, the concept of R-parity is introduced [19].
It is important to mention that flavor physics observables strongly depend on certain model parameters of the MSSM, e.g. BR $\left(B_s \to \mu^+\mu^- \propto \tan^6\beta\right)$ [9]. The absence of the soft trilinear squark mixing terms ("A-terms") in (4.4) relaxes these constraints significantly [18].

5.1. Lagrangian and field content of the MRSSM

The Minimal R-symmetric Supersymmetric Standard Model (MRSSM) addresses all of the above by enforcing R-symmetry upon the Lagrangian. As described in section 3.6, the superpotential is therefore required to have an R-charge of +2 while soft breaking terms need to have an R-charge of 0. All SM fields are assigned an R-charge of 0. Table 5.1 includes the R-charges for the whole field content of the MRSSM. The problematic terms mentioned above, a μ-term $-\mu H_d H_u$, Majorana mass-terms for the gauginos and the A-terms have non vanishing R-charges and are therefore not allowed. Since phenomenology requires mass terms for gauginos, they can be replaced as Dirac mass terms by adding

$$\mathcal{L}_{\text{soft,Dirac}} = M_B^D\left(\tilde{B}\tilde{S} - \sqrt{2}D_B S\right) + M_W^D\left(\tilde{W}^a\tilde{T}^a - \sqrt{2}D_W^a T^a\right) + M_g^D\left(\tilde{g}^a\tilde{O}^a - \sqrt{2}D_g^a O^a\right) + h.c. \tag{5.1}$$

to the soft breaking Lagrangian. These terms are theoretically more appealing as they can be generated via a D-type Spurion as described in section 3.7. However this mechanism of generating Dirac masses for the gauginos requires the existence of fermions in the respective adjoint representation. Therefore the particle content of the theory is enlarged by the S,

T and O fields. The gauge superfields together with the additional chiral superfields in the adjoint representation are N=2 supersymmetric [10].
The μ-term as it exists in the MSSM also violates R-symmetry. By introducing the R-Higgs fields which carry an R-charge of +2, the μ-term in the superpotential can be replaced by

$$-\mu H_d H_u \to \mu_d R_d H_d + \mu_u R_u H_u. \tag{5.2}$$

These terms can be dynamically generated by the Giudice-Masiero mechanism where they arise from the vev of an F-type spurion which automatically explains why their size is not at the Planck scale.
Enforcing R-symmetry allows to generate terms that are theoretically very well motivated and simultaneously explains the absence of the less desirable terms of the MSSM. These new terms give rise to a rich phenomenology especially in the Higgs sector.

Field	Superfield	Boson	Fermion	R-Charges
Gauge Vector	g, W, B	g, W, B	$\tilde{g}, \tilde{W}, \tilde{B}$	$(0, 0, +1)$
Matter	l, e q, d, u	$\tilde{l}, \tilde{e}_R^*$ $\tilde{q}, \tilde{d}_R^*, \tilde{u}_R^*$	l, e_R^* q, d_R^*, u_R^*	$(+1, +1, 0)$
Higgs	$H_{d,u}$	$H_{d,u}$	$\tilde{H}_{d,u}$	$(0, 0, -1)$
R-Higgs	$R_{d,u}$	$R_{d,u}$	$\tilde{R}_{d,u}$	$(+2, +2, +1)$
Adjoint Chiral	O, T, S	O, T, S	$\tilde{O}, \tilde{T}, \tilde{S}$	$(0, 0, -1)$

Table 5.1.: R-charge assignments for the supermultiplets of the MRSSM. Notation for Fermions and Bosons is taken from [12].

The gauge transformation properties of the new fields are summarized in Table 5.2.

Superfield	Components	$\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$
R_u	$\begin{pmatrix} R_u^0 \\ R_u^- \end{pmatrix}, \begin{pmatrix} \widetilde{R}_u^0 \\ \widetilde{R}_u^- \end{pmatrix}$	$(1, 2, -\frac{1}{2})$
R_d	$\begin{pmatrix} R_d^+ \\ R_d^0 \end{pmatrix}, \begin{pmatrix} \widetilde{R}_d^+ \\ \widetilde{R}_d^0 \end{pmatrix}$	$(1, 2, +\frac{1}{2})$
S	$S, \quad \widetilde{S}$	$(1, 1, 0)$
T	$T, \quad \widetilde{T}$	$(1, 3, 0)$
O	$O, \quad \widetilde{O}$	$(8, 1, 0)$

Table 5.2.: Additional chiral superfields in the MRSSM and their representations under the gauge group.

The $\mathrm{SU}(2)_L$ triplet T is defined as

$$T = \begin{pmatrix} T^0/\sqrt{2} & T^+ \\ T^- & -T^0/\sqrt{2} \end{pmatrix}, \quad \text{where } (T^+)^\dagger \neq T^-, \quad T' = \left\{ \frac{\mathrm{Sp}\, \sigma^j T}{\sqrt{2}} \right\}_{j=1}^{3}. \tag{5.3}$$

The Superpotential of the MRSSM is

$$\begin{aligned} W =& \mu_d\, R_d H_d + \mu_u R_u H_u + \Lambda_d\, R_d T H_d + \Lambda_u\, R_u T H_u + \lambda_d\, S R_d H_d + \lambda_u\, S R_u H_u \\ & - Y_d\, dq H_d - Y_e\, el H_d + Y_u\, uq H_u \end{aligned} \tag{5.4}$$

where μ_u and μ_d have mass dimension one and the couplings $\lambda_u, \lambda_d, \Lambda_u$ and Λ_d are dimensionless. The soft part of the MRSSM Lagrangian includes the Dirac mass terms $\mathcal{L}_{\text{soft,Dirac}}$, mass terms for the new fields S, T, O and R_u, R_d alongside MSSM like mass terms for squarks and sleptons and Higgs soft masses and a bilinear B_μ-term [12]

$$\begin{aligned} \mathcal{L}_{\text{soft}} =& \left[M_B^D \left(\tilde{B}\tilde{S} - \sqrt{2} D_B S \right) + M_W^D \left(\tilde{W}^a \tilde{T}^a - \sqrt{2} D_W^a T^a \right) + M_g^D \left(\tilde{g}^a \tilde{O}^a - \sqrt{2} D_g^a O^a \right) + h.c. \right] \\ & + \left[B_\mu \left(H_d^0 H_u^0 - H_d^- H_u^+ \right) + h.c. \right] \\ & - m_{H_d}^2 \left(|H_d^0|^2 + |H_d^-|^2 \right) - m_{H_u}^2 \left(|H_u^0|^2 + |H_u^+|^2 \right) \\ & - m_{R_d}^2 \left(|R_d^0|^2 + |R_d^+|^2 \right) - m_{R_u}^2 \left(|R_u^0|^2 + |R_u^-|^2 \right) \\ & - m_S^2 |S|^2 - m_T^2 |T^0|^2 - m_T^2 |T^-|^2 - m_T^2 |T^+|^2 - m_O^2 |O|^2 \\ & - \tilde{d}_{L,i}^* \, (m_q^2)_{ij} \, \tilde{d}_{L,j} - \tilde{d}_{R,i}^* \, (m_d^2)_{ij} \, \tilde{d}_{R,j} - \tilde{u}_{L,i}^* \, (m_q^2)_{ij} \, \tilde{u}_{L,j} - \tilde{u}_{R,i}^* \, (m_u^2)_{ij} \, \tilde{u}_{R,j} \\ & - \tilde{e}_{L,i}^* \, (m_l^2)_{ij} \, \tilde{e}_{L,j} - \tilde{e}_{R,i}^* \, (m_e^2)_{ij} \, \tilde{e}_{R,j} - \tilde{\nu}_{L,i}^* \, (m_l^2)_{ij} \, \tilde{\nu}_{L,j}. \end{aligned} \tag{5.5}$$

In total, the MRSSM Lagrangian is given as

$$\mathcal{L}_{\text{MRSSM}} = \mathcal{L}_{\text{SUSY}} + \mathcal{L}_{\text{soft}}, \tag{5.6}$$

where $\mathcal{L}_{\text{SUSY}}$ is obtained in the same way as for the MSSM by incorporating the new fields in the matter kinetic terms according to their gauge transformation properties shown in Table 5.2 and plugging the superpotential W (5.4) into (3.38).

5.2. Higgs sector of the MRSSM

The MRSSM has a larger Higgs sector than the MSSM. Since $SU(2)_L \times U(1)_Y \xrightarrow{SSB} U(1)_e$ is the same breaking in the MRSSM as in the MSSM and the SM, states with the same electric charge, spin and color mix. This means that the neutral, CP even Higgses do not only consist of ϕ_u and ϕ_d, but S and T^0 need to be taken into account as well. The following parametrization is chosen for the neutral components

$$\begin{aligned} H_d^0 &= \tfrac{1}{\sqrt{2}}(v_d + \phi_d + i\sigma_d), \quad H_u^0 = \tfrac{1}{\sqrt{2}}(v_u + \phi_u + i\sigma_u), \\ S &= \tfrac{1}{\sqrt{2}}(v_s + \phi_s + i\sigma_s), \qquad T^0 = \tfrac{1}{\sqrt{2}}(v_t + \phi_t + i\sigma_t). \end{aligned} \tag{5.7}$$

As in the case of the MSSM (4.5), the contributions to the Higgs sector are made up of F-, D- and soft-terms

$$\begin{aligned} \mathcal{L}_{\text{Higgs}}^D =& -\frac{1}{2} \left(H_d^\dagger g T^a H_d + H_u^\dagger g T^a H_u + T'^{\dagger b} g T_{\text{adj}}^{abc} T'^c + \sqrt{2} M_W^D \left(T'^a + T'^{\dagger a} \right) \right)^2 \\ & - \frac{1}{2} \left(H_d^\dagger g' Y_{H_d} H_d + H_u^\dagger g' Y_{H_u} H_u + \sqrt{2} M_B^D \left(S + S^\dagger \right) \right)^2 \\ \mathcal{L}_{\text{Higgs}}^F =& -|-\mu_u H_u - \Lambda_u T H_u - \lambda_u S H_u|^2 \\ & + u \to d \\ \mathcal{L}_{\text{Higgs}}^{SB} =& -m_S^2 |S|^2 - m_T^2 \left(|T^0|^2 + |T^+|^2 + |T^-|^2 \right) + \left[B_\mu \left(H_d^0 H_u^0 - H_d^- H_u^+ \right) + h.c. \right]. \end{aligned} \tag{5.8}$$

The Higgs potential is defined by as the negative of these terms, while the electroweak part which includes only the neutral fields V^{EW} is given in [12] as

$$\begin{aligned}
V^{EW} =& (m_{H_d}^2 + \mu_d^2) \left|H_d^0\right|^2 + (m_{H_u}^2 + \mu_u^2) \left|H_u^0\right|^2 - B_\mu \left(H_d^0 H_u^0 + \text{h.c.}\right) \\
&+ (m_{R_d}^2 + \mu_d^2) \left|R_d^0\right|^2 + (m_{R_u}^2 + \mu_u^2) \left|R_u^0\right|^2 + m_T^2 \left|T^0\right|^2 + m_S^2 \left|S\right|^2 \\
&+ \left(2M_W^D \,\Re(T^0)\right)^2 - \left(2M_B^D \,\Re(S)\right)^2 \\
&+ \frac{1}{8}(g'^2 + g^2) \left(\left|H_d^0\right|^2 - \left|H_u^0\right|^2 - \left|R_d^0\right|^2 + \left|R_u^0\right|^2\right)^2 \\
&- \left[\left(g' M_B^D - \sqrt{2}\lambda_d \mu_d\right) \sqrt{2}\Re(S) - \left(g M_W^D + \Lambda_d \mu_d\right) \sqrt{2}\Re(T^0) - \left|\lambda_d S + \tfrac{1}{\sqrt{2}}\Lambda_d T^0\right|^2\right] \left|H_d^0\right|^2 \\
&+ \left[\left(g' M_B^D - \sqrt{2}\lambda_u \mu_u\right) \sqrt{2}\Re(S) - \left(g M_W^D + \Lambda_u \mu_u\right) \sqrt{2}\Re(T^0) + \left|\lambda_u S - \tfrac{1}{\sqrt{2}}\Lambda_u T^0\right|^2\right] \left|H_u^0\right|^2 \\
&+ \left[\left(g' M_B^D + \sqrt{2}\lambda_d \mu_d\right) \sqrt{2}\Re(S) - \left(g M_W^D - \Lambda_d \mu_d\right) \sqrt{2}\Re(T^0) + \left|\lambda_d S + \tfrac{1}{\sqrt{2}}\Lambda_d T^0\right|^2\right] \left|R_d^0\right|^2 \\
&- \left[\left(g' M_B^D - \sqrt{2}\lambda_u \mu_u\right) \sqrt{2}\Re(S) - \left(g M_W^D - \Lambda_u \mu_u\right) \sqrt{2}\Re(T^0) - \left|\lambda_u S - \tfrac{1}{\sqrt{2}}\Lambda_u T^0\right|^2\right] \left|R_u^0\right|^2 \\
&+ \left(\lambda_d^2 + \tfrac{1}{2}\Lambda_d^2\right) \left|H_d^0 R_d^0\right|^2 + \left(\lambda_u^2 + \tfrac{1}{2}\Lambda_u^2\right) \left|H_u^0 R_u^0\right|^2 \\
&- \left(\lambda_d \lambda_u - \tfrac{1}{2}\Lambda_d \Lambda_u\right) \left(H_d^0 R_d^0 H_u^{0\dagger} R_u^{0\dagger} + \text{h.c.}\right)
\end{aligned} \tag{5.9}$$

Tadpole equations of the MRSSM are derivatives of the potential V^{EW} with respect to the neutral fields

$$\begin{aligned}
t_d &= \left.\frac{\partial V^{EW}}{\partial \phi_d}\right|_{\text{all fields} = 0} = v_d \left[\tfrac{1}{8}(g'^2 + g^2)(v_d^2 - v_u^2) - g' M_B^D v_S + g M_W^D v_T + m_{H_d}^2 + (\mu_d^{\text{eff},+})^2\right] - v_u B_\mu \,, \\
t_u &= \left.\frac{\partial V^{EW}}{\partial \phi_u}\right|_{\text{all fields} = 0} = v_u \left[\tfrac{1}{8}(g'^2 + g^2)(v_u^2 - v_d^2) + g' M_B^D v_S - g M_W^D v_T + m_{H_u}^2 + (\mu_u^{\text{eff},-})^2\right] - v_d B_\mu \,, \\
t_t &= \left.\frac{\partial V^{EW}}{\partial \phi_t}\right|_{\text{all fields} = 0} = \tfrac{1}{2} g M_W^D (v_d^2 - v_u^2) + \tfrac{1}{2}\Lambda_d v_d^2 \mu_d^{\text{eff},+} - \Lambda_u v_u^2 \mu_u^{\text{eff},-} + 4(M_W^D)^2 v_T + m_T^2 v_T \,, \\
t_s &= \left.\frac{\partial V^{EW}}{\partial \phi_s}\right|_{\text{all fields} = 0} = \tfrac{1}{2} g' M_B^D (v_u^2 - v_d^2) + \tfrac{1}{\sqrt{2}}\lambda_d v_d^2 \mu_d^{\text{eff},+} + \lambda_u v_u^2 \mu_u^{\text{eff},-} + 4(M_B^D)^2 v_S + m_S^2 v_S \,,
\end{aligned} \tag{5.10}$$

where the useful abbreviations $\mu_i^{eff,\pm,0}$ are defined as [12]

$$\mu_i^{\text{eff},\pm} = \mu_i + \frac{\lambda_i v_S}{\sqrt{2}} \pm \frac{\Lambda_i v_T}{2}, \qquad \mu_i^{\text{eff},0} = \mu_i + \frac{\lambda_i v_S}{\sqrt{2}}, \qquad i = u, d. \tag{5.11}$$

The minimization of V^{EW} at tree level is ensured by

$$0 = t_d = t_u = t_t = t_s \tag{5.12}$$

6. Intermezzo: Perturbative Techniques for Matrix Diagonalization

The transfer from gauge to mass eigenstates is crucial in order to make predictions about the masses and couplings of particles. The diagonalization of the mass matrix of the CP even neutral Higgses in the MSSM (4.10) admits a simple treatment mainly because it is a 2x2 matrix. In the MRSSM, the situation is more challenging for most mass matrices because of the increase in dimension and the number of parameters of the model. While exact diagonalization is sometimes still possible, the results are mostly intractable. This chapter introduces perturbative methods that systematically obtain eigenvalues and eigenvectors up to a desired accuracy while retaining transparency and analytic control.

6.1. Setup

MRSSM scalar mass matrices in the CP conserving case are real and symmetric matrices M^2. The masses of the physical particles are the respective eigenvalues of M^2. For Fermions, the same argumentation holds for $M^T M$. A real and symmetric matrix M^2 can be orthogonally diagonalized.

$$\exists\, U, D \quad \text{such that} \quad U^T M^2 U = D \tag{6.1}$$

Where $U^{-1} = U^T$ and $D = \text{diag}(m_1^2, \ldots, m_n^2)$. U is the matrix of the normalized eigenvectors, and m_i^2 are the eigenvalues of M^2. The analytical determination of the eigenvalues and eigenvectors of M^2 is cumbersome and yields expressions that are not easily interpreted. Perturbative techniques provide a more manageable approach. In the MRSSM, mass matrices can be systematically organized according to the parameter $\epsilon = v/M_{\text{SUSY}}$.

$$M^2 = \underbrace{M_0^2}_{\sim\epsilon^0} + \underbrace{M_1^2}_{\sim\epsilon} + \underbrace{M_2^2}_{\sim\epsilon^2} \tag{6.2}$$

Interesting regions of the parameter space allow for ϵ around the neighborhood of 0 and if the eigenvectors and eigenvlaues are analytic in ϵ, the following ansatz for U and D can be made

$$U = U_0(\mathbb{1} + \underbrace{\Delta_1}_{\sim\epsilon} + \underbrace{\Delta_2}_{\sim\epsilon^2} + \ldots) \quad \text{with} \quad U_0^{\mathrm{T}} U_0 = \mathbb{1} \tag{6.3}$$

$$D = D_0 + \underbrace{D_1}_{\sim\epsilon} + \underbrace{D_2}_{\sim\epsilon^2} + \ldots \tag{6.4}$$

Finding the eigenvalues and eigenvectors breaks down to solving the following matrix equations order by order in ϵ

$$\mathbb{1} = U^{\mathrm{T}} U \quad \Leftrightarrow \quad \mathbb{1} = \left(\mathbb{1} + \Delta_1^{\mathrm{T}} + ...\right) U_0^{\mathrm{T}} U_0 \left(\mathbb{1} + \Delta_1 + ...\right) \tag{6.5}$$

$$\begin{aligned} U^{\mathrm{T}} M^2 U = D \quad \Leftrightarrow \quad & \left(\mathbb{1} + \Delta_1^{\mathrm{T}} + ...\right) U_0^{\mathrm{T}} \left(M_0^2 + M_1^2 + M_2^2 + ...\right) U_0 \left(\mathbb{1} + \Delta_1 + ...\right) \\ & = D_0 + D_1 + D_2 + ... \end{aligned} \tag{6.6}$$

6.2. Distinct eigenvalue case

In the following, the leading order calculation as well as relevant formulae up to third order corrections will be discussed explicitly.

6.2.1. Leading order calculation

At leading order, (6.5) and (6.6) are solved at $\mathcal{O}(\epsilon^0)$. This breaks down to orthogonally diagonalizing M_0^2

$$U_0^{\mathrm{T}} M_0^2 U_0 = D_0 \quad \text{with} \quad U_0^{\mathrm{T}} U_0 = \mathbb{1} \tag{6.7}$$

6.2.2. First order correction

At $\mathcal{O}(\epsilon^1)$, equation (6.5) yields

$$\begin{aligned} 0 &= \Delta_1^{\mathrm{T}} + \Delta_1 \\ \Leftrightarrow (\Delta_1)_{ij} &= - (\Delta_1)_{ji} \ \wedge \ (\Delta_1)_{ii} = 0 \end{aligned} \tag{6.8}$$

Equation (6.6) at this order gives

$$U_0^{\mathrm{T}} M_0^2 U_0 \Delta_1 + U_0^{\mathrm{T}} M_1^2 U_0 + \Delta_1^{\mathrm{T}} U_0^{\mathrm{T}} M_0^2 U_0 = D_1 \tag{6.9}$$

Equations (6.8) and (6.9) provide enough information to uniquely determine Δ_1.

6.2.3. Second order correction

At $\mathcal{O}(\epsilon^2)$, equation (6.5) results in

$$0 = \Delta_2^{\mathrm{T}} + \Delta_1^{\mathrm{T}} \Delta_1 + \Delta_2 \tag{6.10}$$

Equation (6.6) leads to

$$\begin{aligned} & U_0^{\mathrm{T}} M_0^2 U_0 \Delta_2 + U_0^{\mathrm{T}} M_1^2 U_0 \Delta_1 + U_0^{\mathrm{T}} M_2^2 U_0 \\ & + \Delta_1^{\mathrm{T}} U_0^{\mathrm{T}} M_0^2 U_0 \Delta_1 + \Delta_1^{\mathrm{T}} U_0^{\mathrm{T}} M_1^2 U_0 + \Delta_2^{\mathrm{T}} U_0^{\mathrm{T}} M_0^2 U_0 = D_2 \end{aligned} \tag{6.11}$$

6.2.4. Third order correction

At $\mathcal{O}(\epsilon^3)$, equation (6.5) reads

$$0 = \Delta_3^{\mathrm{T}} + \Delta_1^{\mathrm{T}} \Delta_2 + \Delta_2^{\mathrm{T}} \Delta_1 + \Delta_3 \tag{6.12}$$

And equation (6.6) produces the following

$$\begin{aligned} & U_0^{\mathrm{T}} M_0^2 U_0 \Delta_3 + U_0^{\mathrm{T}} M_1^2 U_0 \Delta_2 + U_0^{\mathrm{T}} M_2^2 U_0 \Delta_1 + \Delta_1^{\mathrm{T}} U_0^{\mathrm{T}} M_0^2 U_0 \Delta_2 + \Delta_1 U_0^{\mathrm{T}} M_1^2 U_0 \Delta_1 \\ & + \Delta_1^{\mathrm{T}} U_0^{\mathrm{T}} M_2^2 U_0 + \Delta_2^{\mathrm{T}} U_0^{\mathrm{T}} M_0^2 U_0 \Delta_1 + \Delta_2^{\mathrm{T}} U_0^{\mathrm{T}} M_1^2 U_0 + \Delta_3^{\mathrm{T}} U_0^{\mathrm{T}} M_0^2 U_0 = D_3 \end{aligned} \tag{6.13}$$

6.3. Degenerate eigenvalue case

Since M^2 is real and symmetric, it has a full set of eigenvectors. If the spectrum of M^2 is degenerate, each eigenvalue has the same algebraic and geometric multiplicity. Because of (6.2), M_0^2 also inherits these properties.
In the following, we consider the case that M_0^2 has a degenerate eigenvalue m_0^2, with the geometric multiplicity m by assuming the following structure

$$M_0^2 = \begin{pmatrix} M_{01}^2 & \mathbf{0} \\ \mathbf{0} & M_{02}^2 \end{pmatrix} = \begin{pmatrix} m_0^2 \mathbb{1}_{m\times m} & \mathbf{0} \\ \mathbf{0} & M_{02}^2 \end{pmatrix} \tag{6.14}$$

6.3.1. Leading order calculation

In order to diagonalize M_0^2, U_0 has the following structure

$$U_0 = \begin{pmatrix} U_{0_1} & \mathbf{0} \\ \mathbf{0} & U_{0_2} \end{pmatrix} \quad \text{with} \quad U_{0_1}^{\mathrm{T}} U_{0_1} = \mathbb{1}_{m\times m}, \quad U_{0_2}^{\mathrm{T}} U_{0_2} = \mathbb{1} \tag{6.15}$$

A key difference to the non-degenerate case is that U_{0_1} has an the degree of freedom of an orthogonal $m \times m$-matrix V due to the diagonal structure of M_{01}^2.

$$U_{0_1} \to V U_{0_1} \tag{6.16}$$

$$U_{0_1}^{\mathrm{T}} M_{01}^2 U_{0_1} = m_0^2\, U_{0_1}^{\mathrm{T}} U_{0_1} = m_0^2\, (V U_{0_1})^{\mathrm{T}} V U_{0_1} = m_0^2 \mathbb{1}_{m\times m} \tag{6.17}$$

In other words, unlike in the non-degenerate case, U_{0_1} is not uniquely determined in the leading order calculation.

6.3.2. First order correction

At $\mathcal{O}(\epsilon^1)$, due to equation (6.8), Δ_1 and M_1^2 have the following block structure

$$\Delta_1 = \begin{pmatrix} (\Delta_1)_{11} & (\Delta_1)_{12} \\ -(\Delta_1)_{12} & (\Delta_1)_{22} \end{pmatrix} \qquad M_1^2 = \begin{pmatrix} (M_1^2)_{11} & (M_1^2)_{12} \\ (M_1^2)_{21} & (M_1^2)_{22} \end{pmatrix} \tag{6.18}$$

Consider now only the 11-block of equation (6.9) in order to analyze the degeneracy

$$\begin{aligned} &\underbrace{U_{0_1}^{\mathrm{T}} M_{01}^2 U_{0_1} (\Delta_1)_{11}}_{m_0^2 \mathbb{1}_{m\times m} (\Delta_1)_{11}} + U_{0_1}^{\mathrm{T}} (M_1^2)_{11} U_{0_1} + \underbrace{\left(U_{0_1}^{\mathrm{T}} M_{01}^2 U_{0_1} (\Delta_1)_{11}\right)^{\mathrm{T}}}_{m_0^2 \mathbb{1}_{m\times m} (\Delta_1)_{11}^{\mathrm{T}}} = (D_1)_{11} \\ \Leftrightarrow\ & m_0^2 \underbrace{\left((\Delta_1)_{11} + (\Delta_1)_{11}^{\mathrm{T}}\right)}_{\overset{(6.8)}{=} 0} + U_{0_1}^{\mathrm{T}} (M_1^2)_{11} U_{0_1} = (D_1)_{11} \\ \Leftrightarrow\ & U_{0_1}^{\mathrm{T}} (M_1^2)_{11} U_{0_1} = (D_1)_{11} \end{aligned} \tag{6.19}$$

At leading order, U_{0_1} remained undetermined due to the degeneracy of $(M_0^2)_{11}$. Equation (6.19) addresses this ambiguity by uniquely fixing U_{0_1}. Terms involving $(\Delta_1)_{11}$ neatly drop out due to the orthogonality condition (6.8) and are determined at $\mathcal{O}(\epsilon^2)$. This confirms the result from section 3.2 in [5]

6.3.3. Second order correction

The 11-block of equation (6.11) reads

$$\begin{aligned} &\underbrace{m_0^2 \left((\Delta_2)_{11} + (\Delta_2)_{11}^{\mathrm{T}}\right)}_{\overset{(6.10)}{=} -m_0^2 \left((\Delta_1)_{11}^{\mathrm{T}} (\Delta_1)_{11} + (\Delta_1)_{12}^{\mathrm{T}} (\Delta_1)_{12}\right)} + m_0^2 (\Delta_1)_{11}^{\mathrm{T}} (\Delta_1)_{11} + (\Delta_1)_{12}^{\mathrm{T}} (D_0)_{22} (\Delta_1)_{12} \\ &+ (D_1)_{11} (\Delta_1)_{11} + (\Delta_1)_{11}^{\mathrm{T}} (D_1)_{11} - U_{0_1}^{\mathrm{T}} (M_1^2)_{12} U_{0_2} (\Delta_1)_{12} - (\Delta_1)_{12}^{\mathrm{T}} U_{0_2}^{\mathrm{T}} (M_1^2)_{21} U_{0_1} \\ &+ U_{0_1}^{\mathrm{T}} (M_2^2)_{11} U_{0_1} \end{aligned} \tag{6.20}$$

Again, the second order contributions $(\Delta_2)_{11}$ drop out because of the orthogonality constraint (6.10) at $\mathcal{O}(\epsilon^2)$, enabling the determination of $(\Delta_1)_{11}$. Note that it is required for $(\Delta_1)_{11}$ to have distinct entries, otherwise the combination $(D_1)_{11}(\Delta_1)_{11} + (\Delta_1)_{11}^{\mathrm{T}}(D_1)_{11}$ drops out of (6.20).

6.3.4. Higher order corrections

At order $\mathcal{O}(\epsilon^n)$ the orthogonality constraint (6.5) produces in the 11-block

$$0 = (\Delta_n)_{11}^{\mathrm{T}} + (\Delta_n)_{11} + \dots \tag{6.21}$$

At order $\mathcal{O}(\epsilon^n)$ the 11-block of equation (6.6) only includes terms involving $(\Delta_n)_{11}$ of the form

$$U_{0_1}^{\mathrm{T}}(M_0^2)_{11}U_{0_1}(\Delta_n)_{11} + (\Delta_n)_{11}^{\mathrm{T}}U_{0_1}^{\mathrm{T}}(M_0^2)_{11}U_{0_1} = m_0^2\left((\Delta_n)_{11}^{\mathrm{T}} + (\Delta_n)_{11}\right) \tag{6.22}$$

The combination $(\Delta_n)_{11}^{\mathrm{T}} + (\Delta_n)_{11}$ in equation (6.22) can be replaced by using equation (6.21) with terms not involving $(\Delta_n)_{11}$, allowing for a determination at $\mathcal{O}(\epsilon^{n+1})$.

7. Tree level Higgs self couplings in the MRSSM

Since the MRSSM is a relatively large model, only a subset of its parameter space is considered in order to investigate the impact of specific parameters in a clear way. Throughout this thesis, only the CP-conserving case is considered, where all parameters are assumed to be real. Further assumptions are specifically in the following tree level discussion. These will be partly dropped in the one loop analysis.

7.1. The mass of the lightest Higgs

The main difference to the MSSM is that, as described in section 5.2, the CP even neutral states ϕ_d and ϕ_u are complemented by ϕ_s and ϕ_t. The following assumptions upon the MRSSM specific parameters are made throughout this chapter [12]

$$\lambda = \lambda_u = -\lambda_d \ , \ \Lambda = \Lambda_u = \Lambda_d \ , \ \mu_u = \mu_d = \mu \ , \ v_S \approx v_T \approx 0. \tag{7.1}$$

The relations between the $\lambda_{u,d}$ and $\Lambda_{u,d}$ are motivated by N=2 SUSY relations between the Higgs- and R Higgs-supermultiplets. The assumption $v_T \approx 0$ is justified by phenomenology since in the MRSSM the $\hat{\rho}$ parameter is shifted at tree level $\hat{\rho}_{\text{tree}} = 1 + \frac{4v_T^2}{v^2}$ [12]. $v_S \approx 0$ at tree level is ensured by large adjoint masses m_S^2, m_T^2. This will not be the case in scenarios with light scalars as discussed in [11] and [17].
The tree level mass matrix of the MRSSM respects the tree-level tadpole conditions (5.12). Assuming (7.1), it reads

$$M^2 = \begin{pmatrix} m_Z^2 c_\beta^2 + m_A^2 s_\beta^2 & -(m_Z^2 + m_A^2) s_\beta c_\beta & -v_d(\sqrt{2}\lambda\mu + g_1 M_B^D) & v_d(\Lambda\mu + g_2 M_W^D) \\ -(m_Z^2 + m_A^2) s_\beta c_\beta & m_Z^2 s_\beta^2 + m_A^2 c_\beta^2 & v_u(\sqrt{2}\lambda\mu + g_1 M_B^D) & -v_u(\Lambda\mu + g_2 M_W^D) \\ -v_d(\sqrt{2}\lambda\mu + g_1 M_B^D) & v_u(\sqrt{2}\lambda\mu + g_1 M_B^D) & 4(M_B^D)^2 + m_S^2 + \frac{\lambda^2 v^2}{2} & -\frac{\lambda\Lambda v^2}{2\sqrt{2}} \\ v_d(\Lambda\mu + g_2 M_W^D) & -v_u(\Lambda\mu + g_2 M_W^D) & -\frac{\lambda\Lambda v^2}{2\sqrt{2}} & 4(M_W^D)^2 + m_T^2 + \frac{\Lambda^2 v^2}{4} \end{pmatrix}, \tag{7.2}$$

where m_Z^2 and m_A^2 are as defined in (4.9). The upper left 2x2 component of (7.2) describes the mixing between ϕ_u and ϕ_d is the same as in the MSSM (4.10). In order to obtain eigenvalues and the rotation matrix U, perturbative diagonalization is applied as described in section A.2. The lightest eigenvalue of (7.2) is up to $\mathcal{O}(\epsilon^2)$

$$\left(m_h^0\right)^2 = m_Z^2 \cos^2(2\beta) - v^2 \left(\frac{\left(g_1 M_B^D + \sqrt{2}\lambda\mu\right)^2}{4(M_B^D)^2 + m_S^2} + \frac{\left(g_2 M_W^D + \Lambda\mu\right)^2}{4(M_W^D)^2 + m_T^2} \right) \cos^2(2\beta) \tag{7.3}$$

which is confirmed by eq. (2.20) in [12]. This important result contains the MSSM result in the first term (4.15) and an MRSSM specific correction which reduces the Higgs mass, shifting the focus even further on loop corrections in order to achieve the experimentally measured Higgs mass.
In Figure 7.1, the analytical result (7.2) is compared to the numerical determination of the eigenvalues of (7.2). The error is defined by the second lightest SUSY mass of BMP3, which is defined in Appendix B. The obtained Higgsmass (7.3) provides a suitable approximation of the tree level Higgs mass.

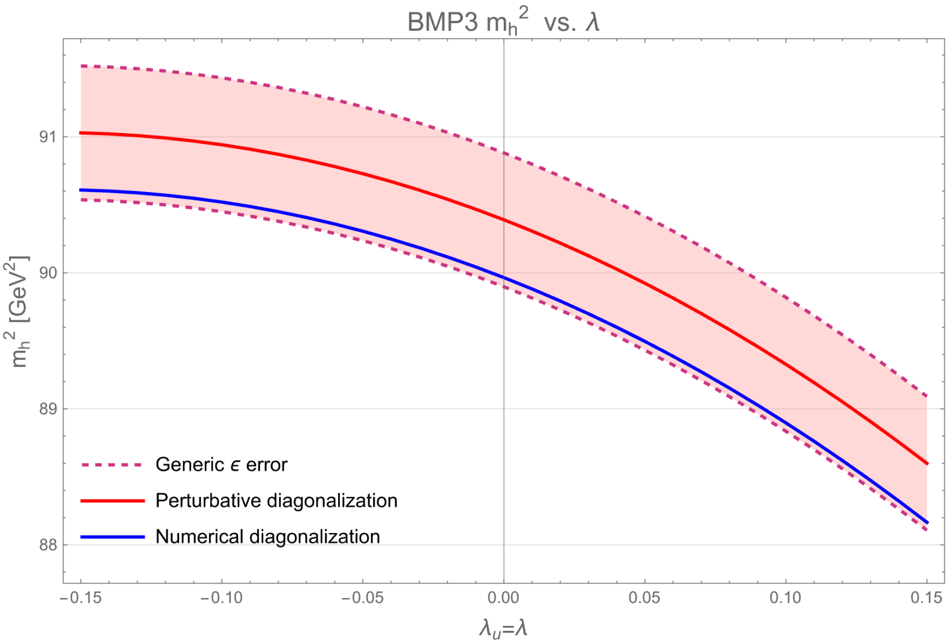


Figure 7.1.: Comparison between the analytical eigenvalue and numerical diagonalization.

7.2. The trilinear coupling of the lightest Higgs

The trilinear coupling in gauge eigenstates reads

$$\begin{aligned} V^{EW}\big|_{\phi_i\phi_j\phi_k} =& \frac{1}{8}(\phi_d-\phi_u)(\phi_d+\phi_u)\left(g'^2 v_d\phi_d + g^2 v_d\phi_d - 4g' M_B^D\phi_s + 4gM_W^D\phi_t - (g'^2+g^2)v_u\phi_u\right) \\ &+\frac{1}{4}\Big\{2\Lambda\mu\phi_t(\phi_d-\phi_u)(\phi_d+\phi_u) + \left(2\lambda^2\phi_s^2+\Lambda^2\phi_t^2\right)(v_d\phi_d+v_u\phi_u) \\ &\qquad - 2\sqrt{2}\lambda\phi_s\Big(\mu(\phi_d-\phi_u)(\phi_d+\phi_u) + \Lambda\phi_t(v_d\phi_d+v_u\phi_u)\Big)\Big\}, \end{aligned} \tag{7.4}$$

where the first line arises from D-term- and the last two lines from F-term-contributions. The D-terms include the MSSM result alongside contributions from (5.1). The relationship between the gauge- and mass-eigenstates is established via the rotation matrix U which has been obtained by diagonalizing M^2 in section A.2. The mixing at leading order in the ϵ-

expansion for the SM-like state is given by

$$\begin{aligned}\phi_d &\sim \cos(\beta)\, h + \mathcal{O}(\epsilon^2)\\ \phi_u &\sim \sin(\beta)\, h + \mathcal{O}(\epsilon^2)\\ \phi_s &\sim \frac{v\left(g' M_B^D + \sqrt{2}\lambda\mu\right)\cos(2\beta)}{4(M_B^D)^2 + m_S^2} h + \mathcal{O}(\epsilon^3)\\ \phi_t &\sim -\frac{v\left(g M_W^D + \Lambda\mu\right)\cos(2\beta)}{4(M_W^D)^2 + m_T^2} h + \mathcal{O}(\epsilon^3)\end{aligned} \tag{7.5}$$

The tree level trilinear coupling λ^0_{hhh} of the lightest Higgs in the MRSSM is

$$\lambda^0_{hhh} = \frac{\partial^3 V^{EW}}{\partial h^3} = \frac{3}{4} v \left(g'^2 + g^2 - 4\left(\frac{(g' M_B^D + \sqrt{2}\lambda\mu)^2}{4(M_B^D)^2 + m_S^2} + \frac{(g M_W^D + \Lambda\mu)^2}{m_T^2 + 4(M_W^D)^2} \right) \right) \cos^2(2\beta) + \mathcal{O}(\epsilon^2) \tag{7.6}$$

This result is in first order in the perturbative diagonalization and directly corresponds to the decoupling limit. The relation to the mass (7.3) in the decoupling limit is the same as in the SM

$$\lambda^0_{hhh} = \frac{3\left(m_h^0\right)^2}{v}. \tag{7.7}$$

In Figure 7.2, κ_λ is plotted w.r.t. the parameter $\lambda_u = \lambda$ using (7.6). The MSSM limit is defined by (4.19). The plot showcases the impact of the strictly negative corrections unique to the MRSSM and emphasizes the importance of the one loop corrections.

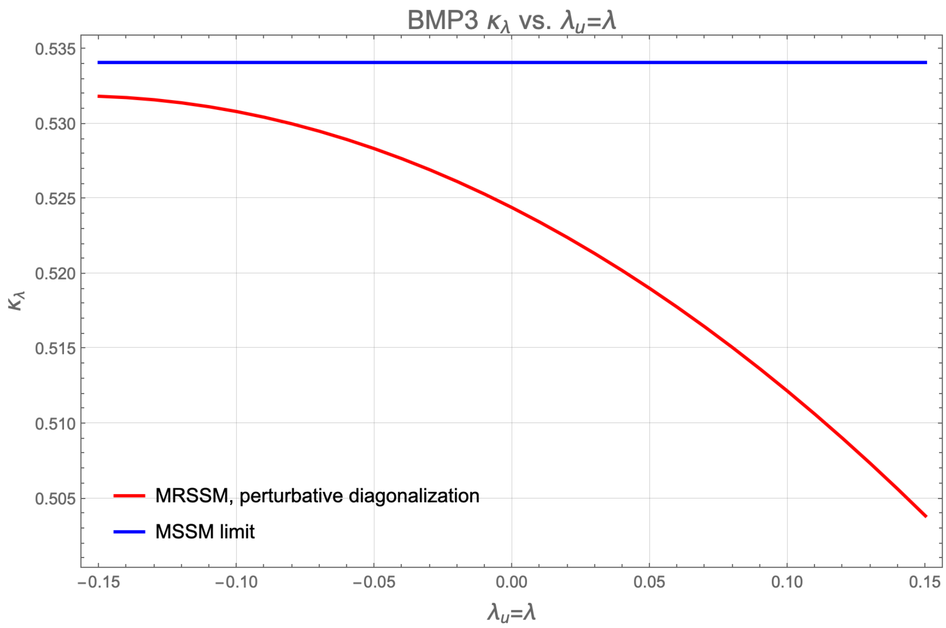


Figure 7.2.: Comparison between κ_λ in the MRSSM and MSSM for BMP3.

7.3. The quartic coupling of the lightest Higgs

The quartic coupling is obtained in the same fashion as the trilinear coupling. In gauge eigenstates it reads

$$\begin{aligned}V^{EW}\big|_{\phi_i\phi_j\phi_k\phi_l} =& \frac{1}{32}\left(g'^2 + g^2\right)\left(\phi_d^2 - \phi_u^2\right)^2\\ &+ \frac{1}{8}\left(2\lambda^2\phi_s^2 + \Lambda\phi_t\left(\Lambda\phi_t - 2\sqrt{2}\lambda\phi_s\right)\right)\left(\phi_d^2 + \phi_u^2\right),\end{aligned} \tag{7.8}$$

Where the first line arises from D-term- and the second line from F-term-contributions. In mass eigenstates, the quartic coupling of the lightest Higgs reads

$$\lambda^{(0)}_{hhhh} = \frac{\partial^4 V^{EW}}{\partial h^4} = \frac{3}{4}\left(g'^2 + g^2\right)\cos^2(2\beta) + \mathcal{O}(\epsilon^2) \tag{7.9}$$

This expression corresponds to the decoupling limit but it does not fulfil the expected SM relation

$$\lambda^0_{hhhh} = \frac{3\left(m_h^0\right)^2}{v^2}. \tag{7.10}$$

In order to achieve the correct result, the four point h function needs to be properly matched at tree level. Matching is a very powerful and popular procedure in the context of Effective Field Theory (EFT) and can be applied if a theory has heavy and light degrees of freedom, as the MRSSM does. The key steps are outlined here, a full derivation can be found in [24]. Consider the process $hh \to hh$ via one of the heavy Higgses $X = H, H_s, H_t$ at tree level and assume the external momenta p_1, p_2, p_3, p_4 are small in comparison to their mass. Since the

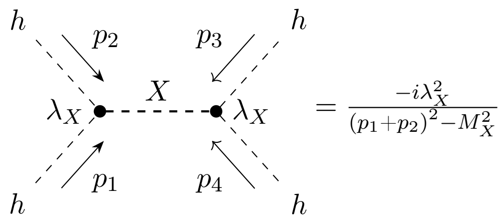


Figure 7.3.: $hh \to hh$ via exchange of a heavy scalar X.

process happens at low energy, the propagator of the heavy particle X can be Taylor-expanded as

$$\frac{i}{(p_1+p_2)^2 - M_X^2} = \frac{-i}{M_X^2}\left[1 + \frac{(p_1+p_2)^2}{M_X^2} + \frac{(p_1+p_2)^4}{M_X^4} + \ldots\right]. \tag{7.11}$$

Calculating the diagram in Figure 7.3 while approximating the propagator of the heavy interal particle by (7.11) generates a polynomial expression in the momenta p_i. Since polynomial in momentum space correspond to local interactions, these terms can be neatly interpreted as vertices in an effective theory without the heavy particles. The procedure of calculating these interactions and mapping them upon an effective theory up to a desired order is called matching.

Since the considered process $hh \to hh$ is described by a dimension 4 operator h^4, it is possible that in the decoupling limit where the propagator mass M_X tends to infinity the contribution of the diagram tends to a constant. This is the case if the dimensionful trilinear coupling $\lambda_X \sim M_{\text{SUSY}}$. The couplings between the lightest state h and the heavy states H_s and H_t is obtained by the perturbative diag and reads in leading order

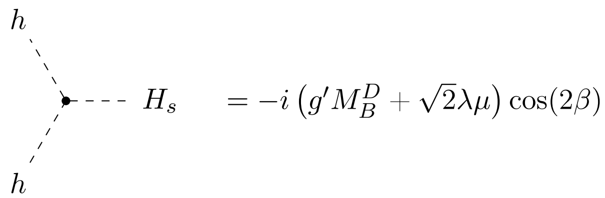
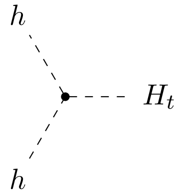


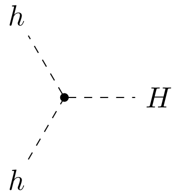
Figure 7.4.: Trilinear coupling $h^2 H_s$.



$$= i\left(g\, M_W^D + \Lambda\mu\right)\cos(2\beta)$$

Figure 7.5.: Trilinear coupling $h^2 H_t$.

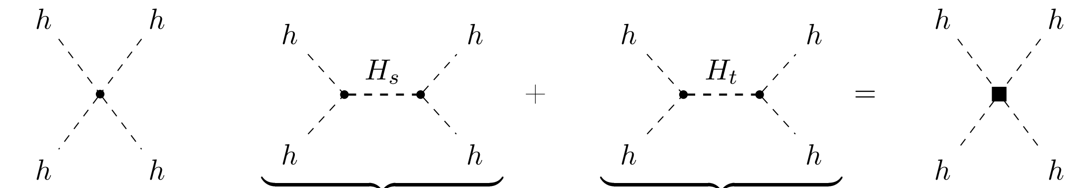
In section 4.5, the decoupling behaviour of the MSSM quartic Higgs has been investigated. In the MSSM case, there are no contributions from an intermediate H in the decoupling limit since the quartic coupling arises strictly from D-terms which are $\sim v$. This behaviour is replicated by the MRSSM.



$$\sim v$$

Figure 7.6.: Trilinear coupling $h^2 H$.

The matching in the decoupling limit can be expressed diagramatically by



$$\underbrace{\tfrac{3}{4}(g'^2+g^2)\cos^2(2\beta)} \quad \underbrace{-\frac{3\left(g' M_B^D+\sqrt{2}\lambda\mu\right)^2}{4(M_B^D)^2+m_S^2}\cos^2(2\beta)} + \underbrace{-\frac{3\left(g\, M_W^D+\Lambda\mu\right)^2}{m_T^2+4(M_W^D)^2}\cos^2(2\beta)} = \underbrace{\frac{3\left(m_h^0\right)^2}{v^2}} \tag{7.12}$$

where the proper decoupling relation for the quartic coupling arises (7.10). The factor 3 for the H_s and H_t diagrams arises from the inclusion of s, t and u channel diagrams in (7.12).

8. Effective potential in the MRSSM

8.1. Formal setup and introduction of the effective potential

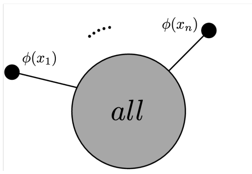
8.1.1. Generating Functionals

The following chapter introduces key concepts and notions connected to the effective potential. Derivations will be based on [25] and [21] but do not strive for completeness.
Green functions are core objects in QFT and therefore many powerful tools have been developed in order to study their properties. The generating functional $Z[J]$ is specified by the Lagrangian $\mathcal{L}$ and introduces classcial sources J for all quantum fields of the theory.

$$Z[J] = \int \mathcal{D}\phi e^{i\int(\mathcal{L}+J\phi)} = \langle\Omega|\mathcal{T}e^{i\int J\Phi}|\Omega\rangle \tag{8.1}$$

Where Φ denotes the field operator in the Heisenberg picture and $|\Omega\rangle$ describes the vacuum of the interacting theory. Taking derivatives of $Z[J]$ wrt. J allows to generate n-point Green functions.

$$\frac{\frac{\delta^n Z[J]}{\delta iJ(x_1)\ldots\delta iJ(x_n)}|_{J=0}}{Z[0]} = \tilde{G}^{(n)}(x_1,\ldots,x_n) = \tag{8.2}$$



Dividing by $Z[0]$ cancels out vacuum diagrams. Expanding $Z[J]$ in powers of J leads to a representation in terms of the Green functions.

$$\begin{aligned} Z[J] &= \int \mathcal{D}\phi\, e^{i\int(\mathcal{L}+J\phi)} = \sum_{n=0}^{\infty}\frac{i^n}{n!}\int \mathcal{D}\phi\, e^{i\int\mathcal{L}}\int d^4x_1\ldots d^4x_n\, J(x_1)\phi(x_1)\ldots J(x_n)\phi(x_n) \\ &= \sum_{n=0}^{\infty}\frac{i^n}{n!}\int d^4x_1\ldots d^4x_n\, J(x_1)\ldots J(x_n)\, G^{(n)}(x_1,\ldots,x_n) \end{aligned} \tag{8.3}$$

A particularly interesting subset of all Green functions are connected graphs $G_c^{(n)}(x_1,\ldots,x_n)$.The generating functional $W[J]$ which generates $G_c^{(n)}(x_1,\ldots,x_n)$ is connected to $Z[J]$ by the exponential function.

$$Z[J] = e^{iW[J]} \Leftrightarrow W[J] = -i\log(Z[J]) \tag{8.4}$$

Again the expansion of $W[J]$ reveals the representation in terms of the $G_c^{(n)}(x_1,\ldots,x_n)$

$$iW[J] = \sum_{n=0}^{\infty}\frac{i^n}{n!}\int d^4x_1\ldots d^4x_n J(x_1)\ldots J(x_n)\, G_c^{(n)}(x_1,\ldots,x_n) \tag{8.5}$$

Using $W[J]$, field expectation values can be expressed as

$$\begin{aligned} \phi_{vac} &= \langle \phi \rangle = \langle \Omega | \Phi | \Omega \rangle = \left. \frac{\delta W}{\delta J} \right|_{J=0} \\ \phi_{cl} &= \langle \phi \rangle_J = \frac{\delta W}{\delta J} \end{aligned} \tag{8.6}$$

8.1.2. The Effective Action

The effective action $\Gamma(\phi_{cl})$ is introduced as the Legendre transformation of $W[J]$ via a mapping between ϕ_{cl} and the sources J

$$\Gamma(\phi_{cl}) = W[J] - \left. \int J \phi_{cl} \right|_{\phi_{cl} = \frac{\delta W}{\delta J}} \tag{8.7}$$

On the level of Green functions the effective action $\Gamma(\phi_{cl})$ generates One Particle Irreducible (1PI) graphs $\Gamma^{(n)}(x_1, \ldots, x_n)$. Analogous to (8.3) and (8.5) $\Gamma(\phi_{cl})$ may also be expanded.

$$\Gamma(\phi_{cl}) = \sum_{n=0}^{\infty} \frac{1}{n!} \int d^4x_1 \ldots d^4x_n \phi_{cl}(x_1) \ldots \phi_{cl}(x_n) \Gamma^{(n)}(x_1, \ldots x_n) \tag{8.8}$$

$\Gamma(\phi_{cl})$ can be interpreted as a generalization of the action S incorporating quantum effects. Therefore $\Gamma(\phi_{cl})$ evaluated at constant values of ϕ_{cl} generalizes the notion of a potential. In momentum space, $\Gamma^{(n)}(x_1, \ldots, x_n)$ transforms as follows

$$\Gamma^{(n)}(x_1, \ldots, x_n) = \int \frac{d^4p_1}{(2\pi)^4} \cdots \frac{d^4p_n}{(2\pi)^4} e^{-ix_1p_1} \ldots e^{-ix_np_n} \delta^{(4)}(p_1 + \cdots + p_n) \Gamma^{(n)}(p_1, \ldots, p_n) \tag{8.9}$$

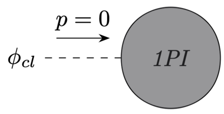
Where the $\delta^{(4)}(p_1 + \cdots + p_n)$ corresponds to momentum conservation. Evaluating (8.8) at constant field values ϕ_{cl}^c which correspond to vanishing field momentum, the effective potential $V_{\text{eff}}(\phi_{cl}^c)$ is defined as in [21] as

$$V_{\text{eff}}(\phi_{cl}^c) = -\sum_{n=0}^{\infty} \frac{1}{n!} (\phi_{cl}^c)^n \Gamma^{(n)}(p_i = 0) \tag{8.10}$$

8.1.3. Computing V_{eff}

(8.10) states that obtaining V_{eff} is a matter of computing 1PI Green functions with vanishing momentum. Utilizing derivatives of $\Gamma(\phi_{cl})$ reduces the computation to a subset of 1PI graphs [25].

$$\left. \frac{\delta \Gamma}{\delta \phi_{cl}} \right|_{\phi_{cl} = \phi_{cl}^c} = -\frac{\partial V_{\text{eff}}}{\partial \phi_{cl}} = (-i) \tag{8.11}$$



In practice, the 1PI tadpole graphs are determined by going to a shifted theory with arbitrary but constant ϕ_{cl}^c. V_{eff} is then obtained up to a constant by integrating (8.11). Performing the sum over all one loop diagrams, the effective potential is given by the Coleman-Weinberg potential [8] [16] as

$$V_{\text{eff}}^{1l} = \frac{1}{64\pi^2} \sum_n (-1)^{2s_n} x_n \left(m_n^2\right)^2 \left[\log\left(\frac{m_n^2}{Q^2} \right) - c_n \right] \tag{8.12}$$

Where the sum is performed over the possible particles with masses m_n^2, spin s_n and x_n degrees of freedom (e.g. color, particle/antiparticle, charge). Q is the renormalization scale and c_n must be choosen according to regularization and renormalization scheme. In SUSY theories the $\overline{\textbf{DR}}$ scheme is a convenient choice that leads to $c_n = \frac{3}{2}$ [20].

8.2. The one loop effective potential in the MRSSM

The one loop effective potential of the MRSSM can be conveniently calculated using the Coleman Weinberg potential (8.12). In this work, an emphasis is put on the contributions that involve distinguishing features of the MRSSM. All calculations are carried out in Landau gauge.

- **CP-even neutral scalars:** The fields (ϕ_d, ϕ_u, ϕ_s, ϕ_t) mix into four physical states via the 4×4 mass matrix $M^2_{H^0}$.
- **CP-odd neutral scalars:** The fields (σ_d, σ_u, σ_s, σ_t) mix into three physical states and one massless Goldstone boson G^0 (at tree level) via the 4×4 matrix M^2_A.
- **Charged Higgses:** The fields $\left(\left(H_d^-\right)^\dagger, H_u^+, \left(T^-\right)^\dagger, T^+\right)$ and their hermitian conjugates mix into three physical states and one massless Goldstone boson $G^\pm$ (at tree level) via the mass matrix $M^2_{H^\pm}$.
- **Neutral R-Higgses:** The fields $\left(R_d^0, R_u^0\right)$ mix via the 2×2 mass matrix M^2_R.
- **Charged R-Higgses:** The fields R_d^+, R_u^- do not mix due to R-symmetry.
- **Charginos:** The eight charged fermions in the weak basis separate into two submatrices that make up χ^+ and ρ^- Dirac-charginos. The fields $\left(\tilde{T}^-, \tilde{H}_d^-\right)$ and $\left(\tilde{W}^+, \tilde{R}_d^+\right)$ constitute the χ^+ charginos with the 2×2 mass matrix M_{χ^+}. The fields $\left(\tilde{W}^-, \tilde{R}_u^-\right)$ and $\left(\tilde{T}^+, \tilde{H}_u^+\right)$ constitute the ρ^- charginos with the 2×2 mass matrix M_{ρ^-}.
- **Neutralinos:** The eight neutral fermion fields $\left(\tilde{B}, \tilde{W}^0, \tilde{R}_d^0, \tilde{R}_u^0\right)$ and $\left(\tilde{S}, \tilde{T}^0, \tilde{H}_d^0, \tilde{H}_u^0\right)$ constitute physical Dirac-neutralinos that mix in a 4×4 mass matrix M_χ.
- **top-quark & stop:** Because of the size of the top yukawa coupling, contributions from the top-quark and stop are included. They are especially important in the MSSM. The impact of the absence of the squark mixing terms on the mass is investigated in chapter 9.

8.3. Setup of the analytical calculation

Obtaining an analytical expression for (8.12) requires calculating the eigenvalues of scalar mass matrices m_S^2 or squared fermion mass eigenvalues $m_F^T m_F$ for the contributions listed above. The method of perturbative diagonalization is applied in order to allow for analytical control and maintains transparency in the structure of the results.
Apart from the top-quark, which mass is directly given by the vev, the masses m^2 of all contributions listed in section 8.2 are of the form

$$m^2 = m^2_{\text{SUSY}} + \sum_{i=1}^{n} c_i^2 \left(\frac{v_j}{m_{\text{SUSY}}}\right)^i , \tag{8.13}$$

where c_i can be any dimensionful parameter of the theory and is determined by perturbative diagonalization. n is the order up to which the perturbative diagonalization has been carried out. (8.13) allows to expand $\log\left(\frac{m^2}{Q^2}\right)$ for $v_j/m_{\text{SUSY}} \to 0$. Therefore V^{CW} is a polynomial in v_j. Expanding V^{CW} beyond v_j^4 introduces terms that are supressed by factors of m_{SUSY} because of the dimension of the potential, e.g. v_j^6/m^2_{SUSY}. Including dimension-6 terms

is particularly interesting since they directly change the relationship between the trilinear coupling and the mass. It is important to note that the inclusion of the top-quark changes the potential non-trivially, since its mass is $m_t = y_t v/\sqrt{2}$ and therefore $\log\left(\frac{m_t^2}{Q^2}\right)$ cannot be expanded into a polynomial in v.
In order to demonstrate this, consider a toy model containing one scalar field h with the associated vev v and the following potential

$$V_{\text{toy model}} = \mu^2(h+v)^2 + \lambda(h+v)^4 + \frac{c_6(h+v)^6}{m^2_{\text{SUSY}}} - \frac{3y_t^4(h+v)^4}{64\pi^2}\log\left(\frac{y_t^2(h+v)^2}{2Q^2}\right) \tag{8.14}$$

Where $V_{\text{toy model}}$ can be interpreted as a potential containing tree level and one loop corrections, such that $\mu^2 = \left(\mu^{(0)}\right)^2 + \left(\mu^{(1)}\right)^2$ and $\lambda = \lambda^{(0)} + \lambda^{(1)}$. The last term arises from the one loop correction from the top quark. After imposing tadpole equations and calculating the mass m_h^2 and the trilinear λ_{hhh}, their relationship deviates from $\lambda_{hhh} = \frac{3m_h^2}{v}$. The mismatch is

$$\frac{3m_h^2}{v} - \lambda_{hhh} = \frac{3vy_t^4}{4\pi^2} - \frac{48\,c_6\,v^3}{\Lambda^2} \tag{8.15}$$

This relation states that non trivial effects in the trilinear and subsequently κ_λ can be achieved by studying c_6. This relation can be carried over to the MRSSM.
After obtaining the Coleman Weinberg potential V^{CW}, the loop corrected quantities can be derived from the corrected potential V,

$$V = V^{(0)} + V^{CW}. \tag{8.16}$$

In order to take derivatives of V w.r.t. the fields ϕ_j, the vacuum expectation values can be expanded as $v_j \to v_j + \phi_j$. Tadpole equations are needed in order to ensure the evaluation at the shifted minimum. They are obtained by the first derivatives of V

$$\begin{aligned}
T_d &= \frac{\partial V}{\partial \phi_d}\bigg|_{\text{all fields}\,=\,0} = t_d^{(0)} + t_d^{(1)},\\
T_u &= \frac{\partial V}{\partial \phi_u}\bigg|_{\text{all fields}\,=\,0} = t_u^{(0)} + t_u^{(1)},\\
T_t &= \frac{\partial V}{\partial \phi_t}\bigg|_{\text{all fields}\,=\,0} = t_t^{(0)} + t_t^{(1)},\\
T_s &= \frac{\partial V}{\partial \phi_s}\bigg|_{\text{all fields}\,=\,0} = t_s^{(0)} + t_s^{(1)},
\end{aligned} \tag{8.17}$$

where the $t_i^{(0)}$ are the tree level tadpole equations as in (5.10). The one loop minimum is ensured by

$$0 = T_d = T_u = T_t = T_s. \tag{8.18}$$

In this work, the tadpole equations of the MRSSM at one loop have been solved perturbatively in terms of the loop factor κ. This procedure is described in subsection 8.3.1 for a simplified model including two scalar fields.
The calculation of the mass and higher order Higgs self couplings is obtained by calculating derivatives of the corrected potential V in the one loop minimum. The one loop corrected mass matrix reads

$$\left(M_{H^0}^{(1l)}\right)^2_{ij} = \frac{\partial V^2}{\partial\phi_i\partial\phi_j}\bigg|_{\text{all fields}\,=\,0;\ \text{1lmin}}. \tag{8.19}$$

In order to obtain corrected self couplings of physical Higgs states, the one loop corrected mass matrix $\left(M_{H^0}^{(1l)}\right)^2$ needs to be diagonalized by an orthogonal matrix U, containing the mixings between the gauge and mass eigenstates.

8.3.1. Perturbative treatment of Tadpole equations

Up to this point the expansion in orders of $\epsilon = v/M_{\text{SUSY}}$ has been employed to obtain perturbative results on the level of eigenvalues and rotation matrices. Going beyond tree level calculations introduces another expansion parameter κ which appears in terms of loop orders. As an example, all quantities which are obtained from the Coleman-Weinberg potential (8.12) are (at least) suppressed by $\frac{1}{64\pi^2}$. Consider the following setup of a theory with two scalar fields ϕ, φ and a potential V composed of a tree level (zero order) part $V^{(0)}$ and a one loop part $V^{(1)}$ where the loop factor κ has been explicitly factored out.

$$V = V^{(0)} + \kappa\, V^{(1)} \tag{8.20}$$

Tadpole equations that depend on the fields and model specific parameters $(\lambda_1, \ldots, \lambda_n)$ directly inherit the property of (8.20)

$$\begin{aligned} T_\phi(\lambda_1, \ldots, \lambda_n) &= \frac{\partial V}{\partial \phi} = T^{(0)}_\phi(\lambda_1, \ldots, \lambda_n) + \kappa\, T^{(1)}_\phi(\lambda_1, \ldots, \lambda_n) \\ T_\varphi(\lambda_1, \ldots, \lambda_n) &= \frac{\partial V}{\partial \varphi} = T^{(0)}_\varphi(\lambda_1, \ldots, \lambda_n) + \kappa\, T^{(1)}_\varphi(\lambda_1, \ldots, \lambda_n) \end{aligned} \tag{8.21}$$

At the minimum of the potential the tadppole equations vanish

$$\left.\frac{\partial V}{\partial \phi}\right|_{\phi=v_\phi,\, \varphi=v_\varphi} = \left.\frac{\partial V}{\partial \varphi}\right|_{\phi=v_\phi,\, \varphi=v_\varphi} = 0 \quad \Leftrightarrow \quad T_\phi(\lambda_1, \ldots, \lambda_n) = T_\varphi(\lambda_1, \ldots, \lambda_n) = 0 \tag{8.22}$$

(8.21) then reduce to algebraic equations that relate the parameters to each other. Assume that the tadpole equations at the minumum are solved for λ_i and λ_j. (8.21) justifies the following Ansatz

$$\begin{aligned} \lambda_i &= \lambda_i^{(0)} + \kappa\, \lambda_i^{(1)} + \mathcal{O}(\kappa^2) \\ \lambda_j &= \lambda_j^{(0)} + \kappa\, \lambda_j^{(1)} + \mathcal{O}(\kappa^2) \end{aligned} \tag{8.23}$$

An order by order solution in κ is obtained by plugging (8.23) into (8.21) and performing a Taylor expansion. The resulting system of equations needs to vanish order by order in κ and therefore allows to determine the solutions of $\lambda_{i,j}$.

$$\begin{aligned} 0 =& T^{(0)}_\phi(\lambda_1, \ldots, \lambda_i^{(0)}, \lambda_j^{(0)}, \ldots, \lambda_n) + \kappa \left[T^{(1)}_\phi(\lambda_1, \ldots, \lambda_i^{(0)}, \lambda_j^{(0)}, \ldots, \lambda_n) + \sum_{l=i,j} \left.\frac{\partial T^{(0)}_\phi}{\partial \lambda_l}\right|_{\lambda_l = \lambda_l^{(0)}} \lambda_l^{(1)} \right] \\ 0 =& T^{(0)}_\varphi(\lambda_1, \ldots, \lambda_i^{(0)}, \lambda_j^{(0)}, \ldots, \lambda_n) + \kappa \left[T^{(1)}_\varphi(\lambda_1, \ldots, \lambda_i^{(0)}, \lambda_j^{(0)}, \ldots, \lambda_n) + \sum_{l=i,j} \left.\frac{\partial T^{(0)}_\varphi}{\partial \lambda_l}\right|_{\lambda_l = \lambda_l^{(0)}} \lambda_l^{(1)} \right] \end{aligned} \tag{8.24}$$

8.4. Numerical implementation

The analytical setup described in section 8.3 is realized under several sets of of assumptions in chapter 9. To validate the analytical implementations, the effective potential calculation has also been carried out within a numerical framework in `Mathematica` that relies on minimal assumptions. By specifying numerical values of the input parameters while keeping the fields ϕ_d, ϕ_u, $\phi_s\, \phi_t$ as variables, derivatives of the potential are calculated by utilizing the routine `ND` of the package `NumericalCalculus`.

9. Higgs self couplings in the MRSSM at one loop

In chapter 7 the tree-level self couplings of the lightest, CP even neutral Higgs h have been discussed. When considering the mass of h it is especially important to consider loop effects since the tree level prediction of the mass in the MRSSM (7.3) is even lower than in the MSSM (4.15).

9.1. Simplest Setup: Dimension 4 and restricted parameter space

The first implementation of one loop corrections from the effective potential is based on the following assumptions

(i) The parameter space restricitons of chapter 7

$$\lambda = \lambda_u = -\lambda_d \ , \ \Lambda = \Lambda_u = \Lambda_d \ , \ \mu_u = \mu_d = \mu \ , \ v_S \approx v_T \approx 0. \tag{9.1}$$

(ii) The mass hierarchy

$$m_{S,T} \gg M^D_{B,W} \gg \mu. \tag{9.2}$$

(iii) The large $\tan\beta$ limit, such that the SM-like state is predominantly composed of the ϕ_u gauge eigenstate.

(iv) The gaugeless limit $g, g' \to 0$ in V^{CW}.

9.1.1. The Mass of the lightest Higgs state

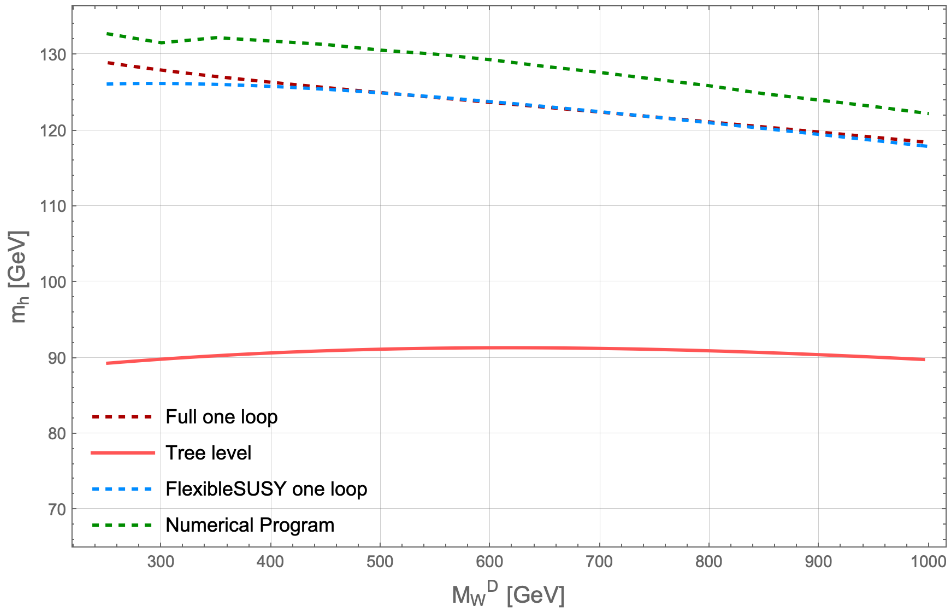
The mass and self couplings of the lightest CP-even Higgs are obtained by the method as outlined in section 8.3. The explicit form of the mass matrices with their associated eigenvalues obtained via perturbative diagonalization can be found in section A.3. The one loop correction to the mass of the ϕ_u-state can be split into terms involving λ and Λ denoted as $\Delta m^2_{h,\lambda,\Lambda}$ and terms involving the top-Yukawa coupling y_t denoted as $\Delta m^2_{h,y_t}$. They read

$$\begin{aligned}\Delta m^2_{h,\lambda,\Lambda} = \frac{2v^2}{16\pi^2}\Bigg[& \left(\frac{\Lambda^2\lambda^2}{2} + \frac{4\lambda^4 + 4\lambda^2\Lambda^2 + 5\Lambda^4}{8}\right)\log\frac{m^2_{R_u}}{Q^2} \\ & + \left(\frac{\lambda^4}{2} - \frac{\lambda^2\Lambda^2}{2}\frac{m_S^2}{m_T^2 - m_S^2}\right)\log\frac{m_S^2}{Q^2} \\ & + \left(\frac{5}{8}\Lambda^4 + \frac{\lambda^2\Lambda^2}{2}\frac{m_T^2}{m_T^2 - m_S^2}\right)\log\frac{m_T^2}{Q^2} \\ & - \left(\frac{5}{4}\Lambda^4 - \lambda^2\Lambda^2\frac{(M_W^D)^2}{(M_B^D)^2 - (M_W^D)^2}\right)\log\frac{(M_W^D)^2}{Q^2} \\ & - \left(\lambda^4 + \lambda^2\Lambda^2\frac{(M_B^D)^2}{(M_B^D)^2 - (M_W^D)^2}\right)\log\frac{(M_B^D)^2}{Q^2}\Bigg]\end{aligned} \tag{9.3}$$

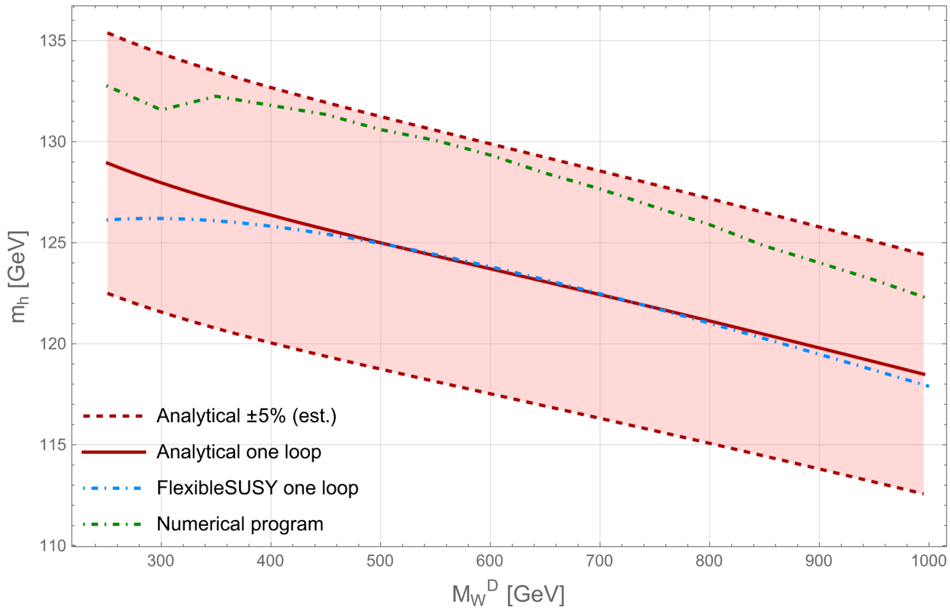
and

$$\Delta m^2_{h,y_t} = \frac{6v^2y_t^4}{16\pi^2}\log\left(\frac{m_{Q,33}\,m_{U,33}}{m_t^2}\right) \tag{9.4}$$

which is in agreement with [12] and [6]. Figure 9.1a shows the impact of the one loop correction upon the mass in BMP3 (see Appendix B). As argued in [12], the MRSSM is able to comfortably accomodate for a 125 GeV Higgsmass without requiring stop masses above 1 TeV. In order to cross-chek the the tree level- and one loop-analytical approximation, also the same plot is produced using the numerical program and FlexibleSUSY [3] [2]. Figure 9.1b shows in detail the predictions of the different programs. The predictions of the numerical program and FlexibleSUSY are within a ±5% range of the analytical prediction.



(a) Lightest CP even Higgsmass at tree level and one loop order



(b) Comparision of the analytical approximation with the numerical program and FlexibleSUSY.

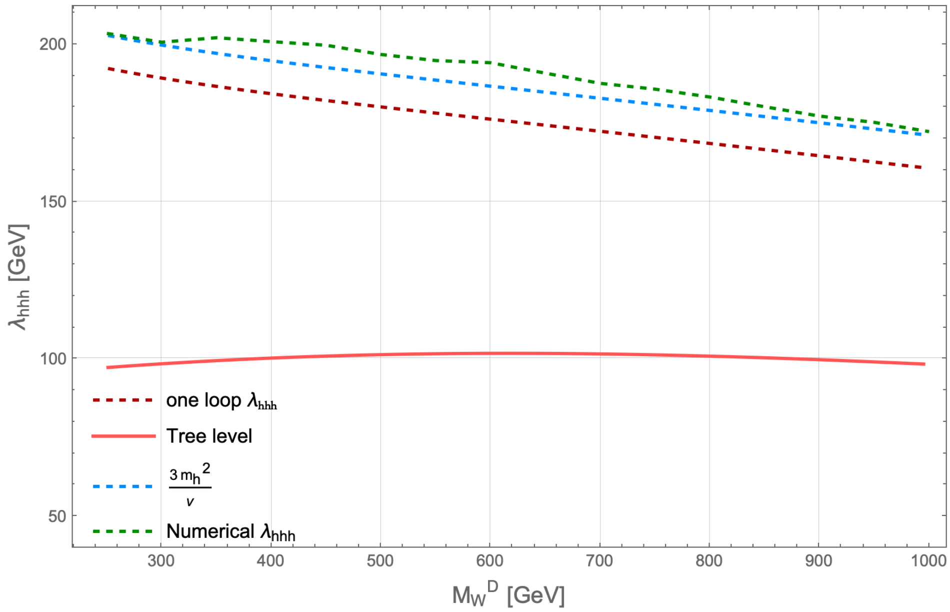
Figure 9.1.: Prediction of the Higgsmass and cross-checks in BMP3.

9.1.2. Trilinear Higgs coupling at one Loop order

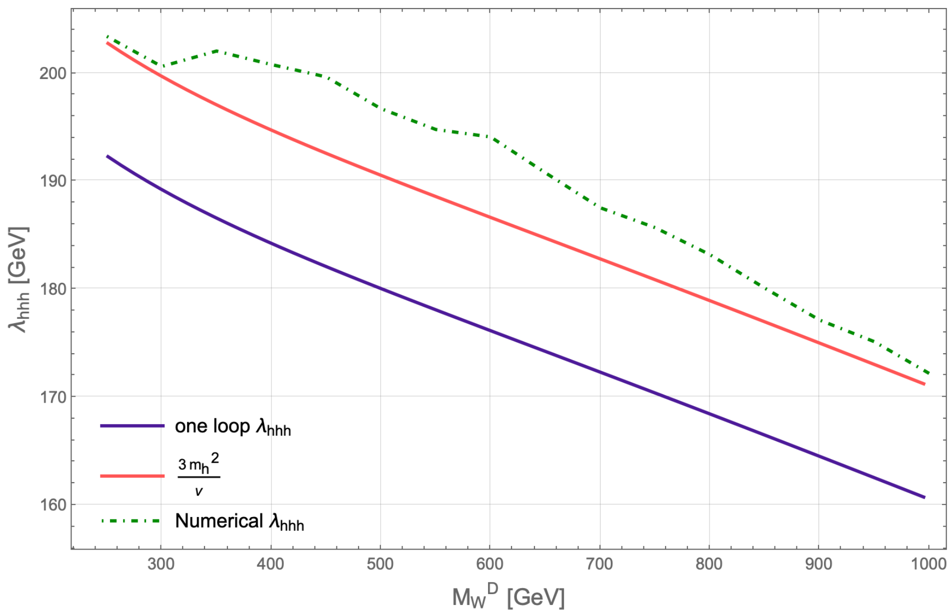
Since the effective potential is calculated up to fourth order, no dim6 correction is included. Therefore in the analytical setup, the trilinear is

$$\lambda^{(1)}_{hhh} = \frac{3\Delta m_h^2}{v} - \frac{3vy_t^4}{4\pi^2}, \tag{9.5}$$

where $\Delta m_h^2 = \Delta m^2_{h,\lambda,\Lambda} + \Delta m^2_{h,y_t}$. This is in agreement with (8.15), where $c_6 = 0$ and provides a valuable cross-check for the analytical calculation.



(a) Tree level and one loop predictions for the trilinear coupling λ_{hhh}.



(b) Comparison of the different one loop predictions.

Figure 9.2.: Predictions of the trilinear coupling at one-loop order for BMP3.

Figure 9.2a shows the impact of the one loop correction on the trilinear coupling. The correction is of similar size as the mass correction. Alongside the analytical tree-level, one-loop and $\frac{3m_h^2}{v}$ predictions, the numerical prediction is plotted. Figure 9.2b shows the detailed comparison of the one loop predictions. Overall good agreement is reached between the predictions. It is important to note that Figure 9.2b clearly confirms (8.15), since the curve for $\frac{3m_h^2}{v}$ is at a constant offset above the prediction of λ_{hhh}. The question, wether the impact of dim-6 effects can elevate λ_{hhh} above $\frac{3m_h^2}{v}$ is investigated in the next section.

9.2. Extended Setup: Dimension 6 impact analysis

Equation 8.15 provides an intriguing way to non-trivially influence the prediction of λ_{hhh} and subsequently κ_λ. In order to achieve an analytical prediction that includes dim-6 effects, the Coleman Weinberg-potential has been calculated up to v^6. Therefore the perturbative diagonalization framework was implemented to include genuine $(v/m_{\text{SUSY}})^6$ effects. The assumptions of section 9.1 for the one loop corrections have been loosened into the following list, in order to allow for a less constrained investigation of the parameter space.

(i) The large $\tan\beta$ limit, such that the SM-like state is predominantly composed of the ϕ_u gauge eigenstate.

(ii) The gaugeless limit $g, g' \to 0$ in V^{CW}.

Figure 9.3 shows the prediction of the Higgs mass including dim-6 terms and only including dim-4 terms alongside the prediction of the numerical program. The inclusion of dim-6 terms allow for an effect of several GeV.

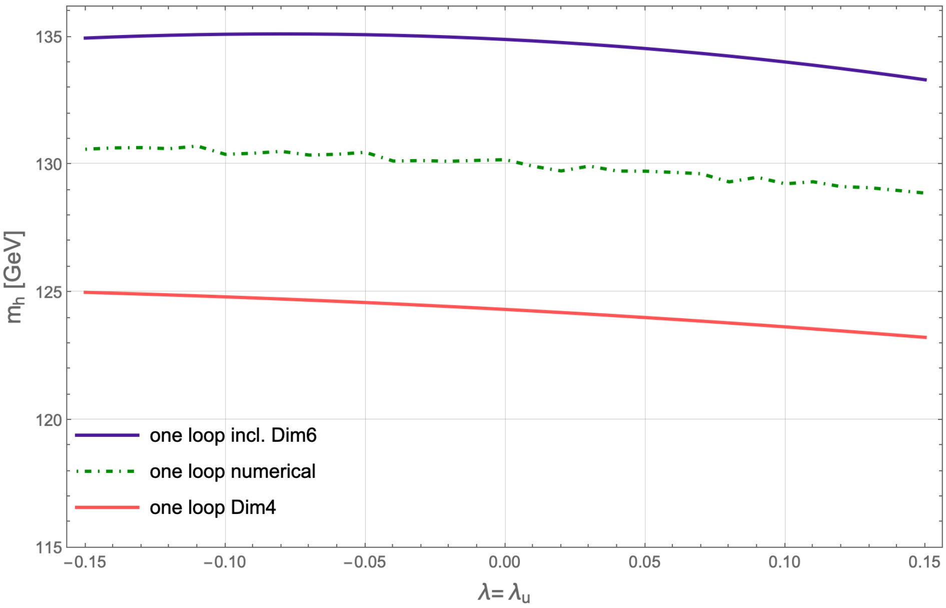
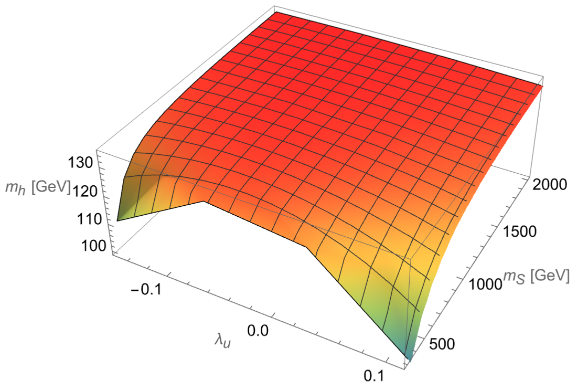
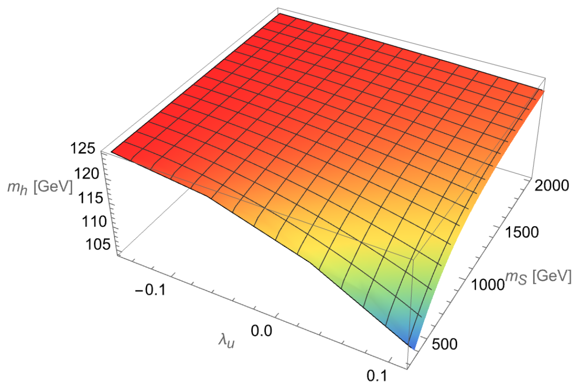


Figure 9.3.: Comparison between the dim-6, dim-4 and numerical prediction of the Higgs mass.

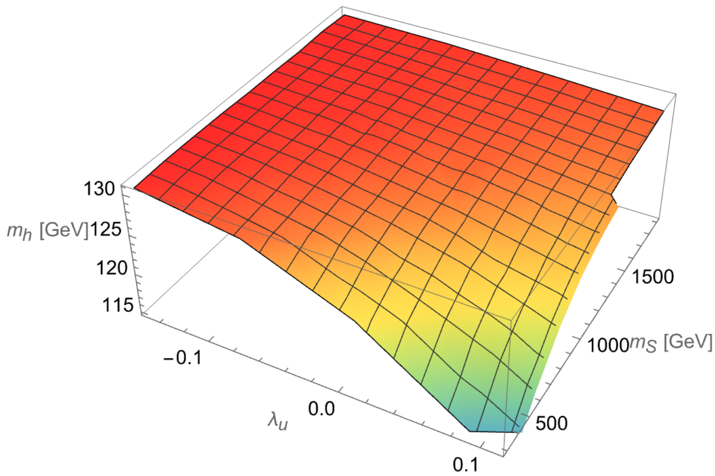
Figure 9.4 shows the comparison of the Higgs-mass predictions of the dim-6 analytical program with the numerical program and FlexibleSUSY. Though overall agreement is established between all predictions, both FlexibleSUSY and the numerical program make a highly non-symmetric prediction regarding λ_u. This lies beyond the scope of the present work and is left for future investigation.



(a) One-loop including dim-6.



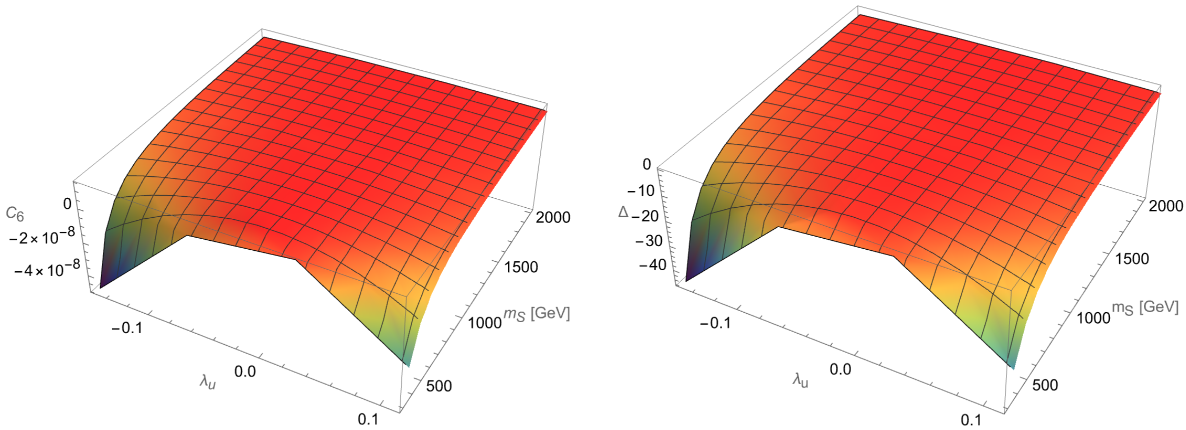
(b) One loop FlexibleSUSY prediction.



(c) One-loop numerical prediction.

Figure 9.4.: Comparison of Higgs mass predictions for BMP3 depending on λ_u and m_S

An interesting aspect in investigating the effect of the dim-6 contributions is by looking at the decoupling behaviour. For $m_{\text{SUSY}} \to \infty$, the effects described by c_6 need to vanish. This implies that for finite and if allowed, rather small values of parameters $\sim m_{\text{SUSY}}$ the effect is enhanced. Considering the same parameter scan as in Figure 9.4,the same scan can be performed on c_6, which is extracted from the analytical program. Figure 9.5 shows c_6 alongside $\Delta = \lambda_{hhh} - \frac{3m_h^2}{v}$, confirming the relation (8.15).

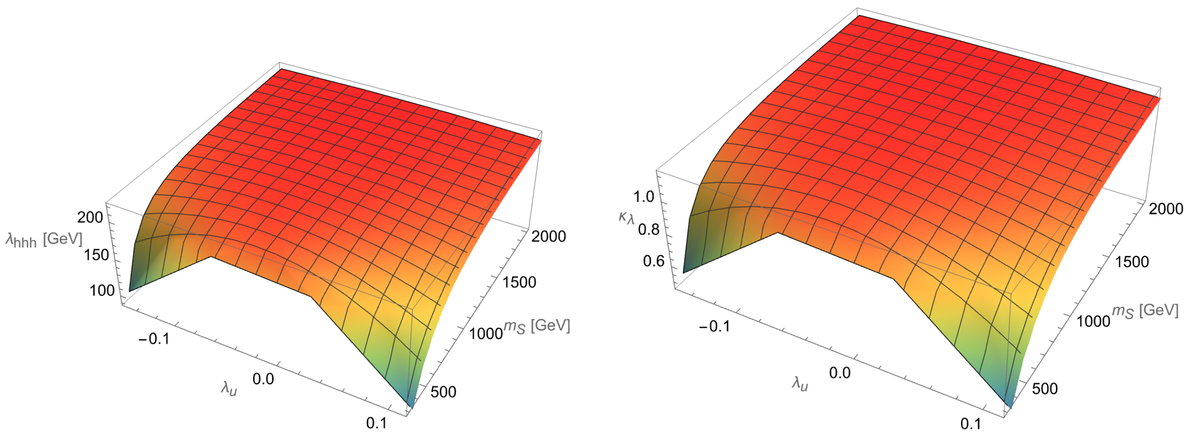


(a) c_6 scan.

(b) $\Delta = \lambda_{hhh} - \frac{3m_h^2}{v}$ scan.

Figure 9.5.: Comparison of c_6 and Δ for BMP3.

The trilinear coupling λ_{hhh} is plotted alongside κ_λ in Figure 9.6 for the same parameter scan as the plots above. A particularly interesting parameter region is at low values of m_S enhanced by large values for the coupling λ_u. This coincides with the region where c_6 is largest, highlighting the impact of the dim-6 contributions.



(a) λ_{hhh} scan.

(b) κ_λ scan.

Figure 9.6.: Comparison of Higgs mass predictions for BMP3 depending on λ_u and m_S

9.3. Light Scalar Scenarios

Scenarios with a light scalar in the MRSSM have been considered in [11] and [17]. The dim-6 analytical approach allows for calculating the masses of the lightest (m_{h_1}) and next to lightest (SM-like) Higgs (m_{h_2}) as well as their trilinear couplings. Table 9.1 shows the masses and the trilinear coupling of the SM-like state for BMP7 and BMP8 from [17]. These results agree reasonably with [17]. Further investigation of the light scalar scenario requires an alternative setup of the perturbative diagonalization since $mS, M_D^B < v$.

	BMP7	BMP8
m_{h_1}	90.83	96.77
m_{h_2}	134.25	132.85
λ_{hhh}	188.13	182.46

Table 9.1.: Higgs masses and trilinear coupling for BMP7 und BMP8 of [17].

10. Conclusions and Outlook

In this thesis, the trilinear Higgs coupling in the Minimal R-symmetric Supersymmetric Standard Model has been investigated in at tree level as well as the one loop level. The MRSSM is a theoretically appealing and phenomenologically highly motivated extension of the Standard Model that offers a structurally very unique Higgs sector.
A central tool in this analysis has been the method of perturbative diagonalization, which enables great analytical control in the determination of eigenvalues and eigenvectors. In chapter 6, this method was systematically developed beyond leading order to account for block degeneracies, thereby extending its applicability in scenarios with degenerate submatrices. This allows for a very versatile and wide applicability of the method on diagonalizable (mass) matrices as used in chapters 4, 7 and 9.
In chapter 7, the Higgs self-couplings were derived in both the gauge- and mass-eigenstates. The resulting expressions were found to be consistent with MSSM limits and existing results in the literature, thereby serving as a robust validation of the approach.
Furthermore, analytical one-loop corrections to the Higgs self-couplings and its mass were calculated in chapter 9 at both dimension-4 and dimension-6 levels using the Coleman-Weinberg potential. These results were confronted with a numerical implementations and cross-checked against the FlexibleSUSY, yielding good agreement and providing a non-trivial confirmation of the analytical framework.

While the results presented in this work provide an analytical and numerical foundation for understanding Higgs self-coupling, they represent only a first step in exploring the full potential of the parameter space of the MRSSM. In particular, the impact of the paremeters unique to the MRSSM have not been fully investigated.
In summary, this thesis not only provides new insights into the Higgs sector of the MRSSM but also develops powerful tools that can be used in investigations that extend beyond the presented analysis.

A. Perturbative diagonalization in action

A.1. Tree Level Higgs Mass matrix in the MSSM

The mass Matrix M_h^2 reads

$$M_h^2 = \begin{pmatrix} m_Z^2 \cos^2\beta + m_A^2 \sin^2\beta & -(m_A^2 + m_Z^2)\cos\beta\sin\beta \\ -(m_A^2 + m_Z^2)\cos\beta\sin\beta & m_A^2 \cos^2\beta + m_Z^2 \sin^2\beta \end{pmatrix}. \tag{A.1}$$

M_h^2 can be split in the following way

$$\frac{M_h^2}{m_{\text{SUSY}}^2} = \underbrace{\frac{\left(M_h^{(0)}\right)^2}{m_{\text{SUSY}}^2}}_{\sim\epsilon^0} + \underbrace{\frac{\left(M_h^{(1)}\right)^2}{m_{\text{SUSY}}^2}}_{\sim\epsilon} + \underbrace{\frac{\left(M_h^{(2)}\right)^2}{m_{\text{SUSY}}^2}}_{\sim\epsilon^2}, \tag{A.2}$$

where

$$\begin{aligned}
\left(M_h^{(0)}\right)^2 &= \begin{pmatrix} m_A^2 \sin^2\beta & -m_A^2 \cos\beta\sin\beta \\ -m_A^2 \cos\beta\sin\beta & m_A^2 \cos^2\beta \end{pmatrix}, \\
\left(M_h^{(1)}\right)^2 &= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \\
\left(M_h^{(2)}\right)^2 &= \begin{pmatrix} m_Z^2 \cos^2\beta & -m_Z^2 \cos\beta\sin\beta \\ -m_Z^2 \cos\beta\sin\beta & m_Z^2 \sin^2\beta \end{pmatrix}.
\end{aligned} \tag{A.3}$$

U and D from (4.11) are analytic in ϵ. They read up to order ϵ^2

$$\begin{aligned}
D^{(0)} &= \begin{pmatrix} 0 & 0 \\ 0 & m_A^2 \end{pmatrix}, \\
D^{(1)} &= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \\
D^{(2)} &= \begin{pmatrix} m_Z^2 \cos(2\beta)^2 & 0 \\ 0 & m_Z^2 \sin(2\beta)^2 \end{pmatrix}
\end{aligned} \tag{A.4}$$

and

$$
\begin{aligned}
U^{(0)} &= \begin{pmatrix} \cos(\beta) & -\sin(\beta) \\ \sin(\beta) & \cos(\beta) \end{pmatrix}, \\
U^{(1)} &= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \\
U^{(2)} &= \begin{pmatrix} -\frac{m_Z^2 \sin(\beta)\sin(4\beta)}{2m_A^2} & -\frac{m_Z^2 \cos(\beta)\sin(4\beta)}{2m_A^2} \\ \frac{m_Z^2 \cos(\beta)\sin(4\beta)}{2m_A^2} & -\frac{m_Z^2 \sin(\beta)\sin(4\beta)}{2m_A^2} \end{pmatrix},
\end{aligned}
\tag{A.5}
$$

which confirms (4.13) and (4.15) neatly.

A.2. Tree Level Higgs Mass matrix in the MRSSM

The higgs mass matrix of the MRSSM M^2 describes the mixing between the gauge eigenstates $(\phi_d, \phi_u, \phi_s, \phi_t)$,

$$
M^2 = \begin{pmatrix} m_Z^2 c_\beta^2 + m_A^2 s_\beta^2 & -(m_Z^2 + m_A^2) s_\beta c_\beta & -v_d(\sqrt{2}\lambda\mu + g_1 M_B^D) & v_d(\Lambda\mu + g_2 M_W^D) \\ -(m_Z^2 + m_A^2) s_\beta c_\beta & m_Z^2 s_\beta^2 + m_A^2 c_\beta^2 & v_u(\sqrt{2}\lambda\mu + g_1 M_B^D) & -v_u(\Lambda\mu + g_2 M_W^D) \\ -v_d(\sqrt{2}\lambda\mu + g_1 M_B^D) & v_u(\sqrt{2}\lambda\mu + g_1 M_B^D) & 4(M_B^D)^2 + m_S^2 + \frac{\lambda^2 v^2}{2} & -\frac{\lambda\Lambda v^2}{2\sqrt{2}} \\ v_d(\Lambda\mu + g_2 M_W^D) & -v_u(\Lambda\mu + g_2 M_W^D) & -\frac{\lambda\Lambda v^2}{2\sqrt{2}} & 4(M_W^D)^2 + m_T^2 + \frac{\Lambda^2 v^2}{4} \end{pmatrix}.
\tag{A.6}
$$

Since M^2 is real and symmetric, it can be orthogonally diagonalized via the perscription described in chapter 6. The relationship between the gauge and mass eigenstates is described by the matrix U

$$
\begin{pmatrix} \phi_d \\ \phi_u \\ \phi_s \\ \phi_t \end{pmatrix} = U \begin{pmatrix} h \\ H \\ H_s \\ H_t \end{pmatrix}.
\tag{A.7}
$$

As in the MSSM, M^2 contains corrections up to ϵ^2, where $\epsilon = v/M_{SUSY}$. SUSY masses are $m_S, m_T, m_A, M_W^D, M_B^D, \mu$.

$$
\frac{M^2}{m_{\text{SUSY}}^2} = \underbrace{\frac{\left(M^{(0)}\right)^2}{m_{\text{SUSY}}^2}}_{\sim\epsilon^0} + \underbrace{\frac{\left(M^{(1)}\right)^2}{m_{\text{SUSY}}^2}}_{\sim\epsilon} + \underbrace{\frac{\left(M^{(2)}\right)^2}{m_{\text{SUSY}}^2}}_{\sim\epsilon^2},
\tag{A.8}
$$

where

$$
\left(M^{(0)}\right)^2 = \begin{pmatrix} m_A^2 \sin^2\beta & -m_A^2 \cos\beta\sin\beta & 0 & 0 \\ -m_A^2\cos\beta\sin\beta & m_A^2\cos^2\beta & 0 & 0 \\ 0 & 0 & 4(M_B^D)^2 + m_S^2 & 0 \\ 0 & 0 & 0 & m_T^2 + 4(M_W^D)^2 \end{pmatrix},
\tag{A.9}
$$

$$
\left(M^{(1)}\right)^2 = \begin{pmatrix} 0 & 0 & -v\left(g' M_B^D + \sqrt{2}\lambda\mu\right)\cos\beta & v\left(g M_W^D + \Lambda\mu\right)\cos\beta \\ 0 & 0 & v\left(g' M_B^D + \sqrt{2}\lambda\mu\right)\sin\beta & -v\left(g M_W^D + \Lambda\mu\right)\sin\beta \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},
\tag{A.10}
$$

$$\left(M^{(2)}\right)^2 = \begin{pmatrix} \frac{1}{4}(g'^2+g^2)v^2\cos^2\beta & -\frac{1}{8}(g'^2+g^2)v^2\sin(2\beta) & 0 & 0 \\ -\frac{1}{8}(g'^2+g^2)v^2\sin(2\beta) & \frac{1}{4}(g'^2+g^2)v^2\sin^2\beta & 0 & 0 \\ 0 & 0 & \frac{v^2\lambda^2}{2} & -\frac{v^2\lambda\Lambda}{2\sqrt{2}} \\ 0 & 0 & -\frac{v^2\lambda\Lambda}{2\sqrt{2}} & \frac{v^2\Lambda^2}{4} \end{pmatrix}. \tag{A.11}$$

In this discussion, the assumptions of (7.1) are made, where $v_s \approx 0$ and $v_t \approx 0$. This presents no contradiction to the tree level solutions for v_s and v_t, which imply $v_{s,t} \propto \epsilon^3$, since this discussion does not go beyond $\mathcal{O}(\epsilon^2)$.
The eigenvalues are the entries of the D matrices, they read up to $\mathcal{O}(\epsilon^2)$

$$D^{(0)} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & m_A^2 & 0 & 0 \\ 0 & 0 & 4(M_B^D)^2+m_S^2 & 0 \\ 0 & 0 & 0 & m_T^2+4(M_W^D)^2 \end{pmatrix}, \tag{A.12}$$

$$D^{(1)} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \tag{A.13}$$

$$D^{(2)} = \begin{pmatrix} \frac{1}{4}(g'^2+g^2)v^2\cos^2(2\beta) - v^2\left(\frac{(g'M_B^D+\sqrt{2}\lambda\mu)^2}{4(M_B^D)^2+m_S^2} + \frac{(gM_W^D+\Lambda\mu)^2}{m_T^2+4(M_W^D)^2}\right)\cos^2(2\beta) & 0 & 0 & 0 \\ 0 & (2,2) & 0 & 0 \\ 0 & 0 & (3,3) & 0 \\ 0 & 0 & 0 & (4,4) \end{pmatrix} \tag{A.14}$$

with

$$\begin{aligned}(2,2) = {} & v^2 \frac{\sin^2(2\beta)}{4\left(m_A^2-4(M_B^D)^2-m_S^2\right)\left(m_A^2-m_T^2-4(M_W^D)^2\right)} \\ & \times \Big[g^2\left(m_A^2-4(M_B^D)^2-m_S^2\right)\left(m_A^2-m_T^2\right)+g'^2\left(m_A^2-m_S^2\right)\left(m_A^2-m_T^2-4(M_W^D)^2\right) \\ & + 8\sqrt{2}\,g'\,M_B^D\left(m_A^2-m_T^2-4(M_W^D)^2\right)\lambda\mu + 8g\left(m_A^2-4(M_B^D)^2-m_S^2\right)M_W^D\Lambda\mu \\ & + 4\left(2\left(m_A^2-m_T^2-4(M_W^D)^2\right)\lambda^2+\left(m_A^2-4(M_B^D)^2-m_S^2\right)\Lambda^2\right)\mu^2\Big],\end{aligned} \tag{A.15}$$

$$\begin{aligned}(3,3) = {} & \frac{1}{2}v^2\Big\{\lambda^2 + \frac{g'^2(M_B^D)^2+2\sqrt{2}g'M_B^D\lambda\mu+2\lambda^2\mu^2}{m_A^2-4(M_B^D)^2-m_S^2}\left(-1+\cos(4\beta)\right) \\ & + \frac{g'^2(M_B^D)^2+2\sqrt{2}g'M_B^D\lambda\mu+2\lambda^2\mu^2}{4(M_B^D)^2+m_S^2}\left(1+\cos(4\beta)\right)\Big\},\end{aligned} \tag{A.16}$$

$$\begin{aligned}(4,4) = {} & \frac{1}{4}v^2\Big\{\Lambda^2 - \frac{4\left(gM_W^D+\Lambda\mu\right)^2\left(m_A^2-2\left(m_T^2+4(M_W^D)^2\right)+m_A^2\cos(4\beta)\right)}{\left(m_T^2+4(M_W^D)^2\right)\left(-m_A^2+m_T^2+4(M_W^D)^2\right)} \\ & + \frac{4m_A^2\left(gM_W^D+\Lambda\mu\right)^2\sin^2(2\beta)}{\left(-m_A^2+m_T^2+4(M_W^D)^2\right)^2} \\ & + 4\left(m_T^2+4(M_W^D)^2\right)\left(gM_W^D+\Lambda\mu\right)^2\left(-\frac{\cos^2(2\beta)}{\left(m_T^2+4(M_W^D)^2\right)^2}-\frac{\sin^2(2\beta)}{\left(-m_A^2+m_T^2+4(M_W^D)^2\right)^2}\right)\Big\}.\end{aligned} \tag{A.17}$$

The eigenvectors are given in the matrices U. However, $U^{(1)}$ and $U^{(2)}$ are analytically rather large and will not be listed here explicitly, as they are not relevant for the present discussion.

$U^{(0)}$ reads

$$U^{(0)} = \begin{pmatrix} \cos\beta & -\sin\beta & 0 & 0 \\ \sin\beta & \cos\beta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{A.18}$$

The upper left 2x2 block corresponds to the MSSM result.

A.3. Simple one loop setup

In order to genuinely expand the Coleman-Weinberg potential up to fourth order in v, the mass eigenvalues m^2 need to be calculated in the perturbative framework up to fourth order in v/m_{SUSY} under the assumptions listed in section 9.1. All contributions can be split according to

$$\frac{M^2}{m_{\text{SUSY}}^2} = \underbrace{\frac{\left(M^{(0)}\right)^2}{m_{\text{SUSY}}^2}}_{\sim\epsilon^0} + \underbrace{\frac{\left(M^{(1)}\right)^2}{m_{\text{SUSY}}^2}}_{\sim\epsilon} + \underbrace{\frac{\left(M^{(2)}\right)^2}{m_{\text{SUSY}}^2}}_{\sim\epsilon^2}, \tag{A.19}$$

where $\epsilon = v/M_{SUSY}$.

A.3.1. CP-even Higgs

The CP-even Higgs mass Matrix $M_{H^0}^2$ is

$$M_{H^0}^2 = \begin{pmatrix} m_{H_d}^2 & 0 & 0 & 0 \\ 0 & m_{H_u}^2 & 0 & 0 \\ 0 & 0 & m_S^2 + \frac{v^2\lambda^2}{2} & -\frac{v^2\lambda\Lambda}{2\sqrt{2}} \\ 0 & 0 & -\frac{v^2\lambda\Lambda}{2\sqrt{2}} & m_T^2 + \frac{v^2\Lambda^2}{4} \end{pmatrix}. \tag{A.20}$$

It can be dissected into

$$\begin{aligned} \left(M^{(0)}\right)^2 &= \begin{pmatrix} m_{H_d}^2 & 0 & 0 & 0 \\ 0 & m_{H_u}^2 & 0 & 0 \\ 0 & 0 & m_S^2 & 0 \\ 0 & 0 & 0 & m_T^2 \end{pmatrix}, \\ \left(M^{(1)}\right)^2 &= \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \\ \left(M^{(2)}\right)^2 &= \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{v^2\lambda^2}{2} & -\frac{v^2\lambda\Lambda}{2\sqrt{2}} \\ 0 & 0 & -\frac{v^2\lambda\Lambda}{2\sqrt{2}} & \frac{v^2\Lambda^2}{4} \end{pmatrix}. \end{aligned} \tag{A.21}$$

The matrix of eigenvalues D up to ϵ^4 reads

$$D = \begin{pmatrix} m_{H_d}^2 & 0 & 0 & 0 \\ 0 & m_{H_u}^2 & 0 & 0 \\ 0 & 0 & m_S^2 + \frac{1}{2}v^2\lambda^2\left(1 + \frac{v^2\Lambda^2}{4m_S^2 - 4m_T^2}\right) & 0 \\ 0 & 0 & 0 & m_T^2 + \frac{1}{4}v^2\left(1 - \frac{v^2\lambda^2}{2m_S^2 - 2m_T^2}\right)\Lambda^2 \end{pmatrix}. \tag{A.22}$$

A.3.2. CP-odd Higgs

The CP-odd Higgs mass Matrix M_A^2 is

$$M_A^2 == \begin{pmatrix} m_{H_d}^2 & 0 & 0 & 0 \\ 0 & m_{H_u}^2 & 0 & 0 \\ 0 & 0 & m_S^2 + \frac{v^2\lambda^2}{2} & -\frac{v^2\lambda\Lambda}{2\sqrt{2}} \\ 0 & 0 & -\frac{v^2\lambda\Lambda}{2\sqrt{2}} & m_T^2 + \frac{v^2\Lambda^2}{4} \end{pmatrix}. \tag{A.23}$$

It can be dissected into

$$\begin{aligned} \left(M^{(0)}\right)^2 &= \begin{pmatrix} m_{H_d}^2 & 0 & 0 & 0 \\ 0 & m_{H_u}^2 & 0 & 0 \\ 0 & 0 & m_S^2 & 0 \\ 0 & 0 & 0 & m_T^2 \end{pmatrix}, \\ \left(M^{(1)}\right)^2 &= \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \\ \left(M^{(2)}\right)^2 &= \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{v^2\lambda^2}{2} & -\frac{v^2\lambda\Lambda}{2\sqrt{2}} \\ 0 & 0 & -\frac{v^2\lambda\Lambda}{2\sqrt{2}} & \frac{v^2\Lambda^2}{4} \end{pmatrix}. \end{aligned} \tag{A.24}$$

The matrix of eigenvalues D up to ϵ^4 reads

$$D = \begin{pmatrix} m_{H_d}^2 & 0 & 0 & 0 \\ 0 & m_{H_u}^2 & 0 & 0 \\ 0 & 0 & m_S^2 + \dfrac{v^2\lambda^2}{2} + \dfrac{v^4\lambda^2\Lambda^2}{8m_S^2 - 8m_T^2} & 0 \\ 0 & 0 & 0 & m_T^2 + \dfrac{v^2\Lambda^2}{4} - \dfrac{v^4\lambda^2\Lambda^2}{8m_S^2 - 8m_T^2} \end{pmatrix}. \tag{A.25}$$

A.3.3. Charged Higgs

The Charged Higgs mass Matrix $M_{H^\pm}^2$ is

$$M_{H^\pm}^2 = \begin{pmatrix} m_{H_d}^2 & 0 & 0 & 0 \\ 0 & m_{H_u}^2 & 0 & 0 \\ 0 & 0 & m_T^2 + 2(M_W^D)^2 & 2(M_W^D)^2 \\ 0 & 0 & 2(M_W^D)^2 & m_T^2 + 2(M_W^D)^2 + \frac{v^2\Lambda^2}{2} \end{pmatrix}. \tag{A.26}$$

It can be dissected into

$$\begin{aligned}
\left(M^{(0)}\right)^2 &= \begin{pmatrix} m_{H_d}^2 & 0 & 0 & 0 \\ 0 & m_{H_u}^2 & 0 & 0 \\ 0 & 0 & m_T^2 & 0 \\ 0 & 0 & 0 & m_T^2 \end{pmatrix}, \\
\left(M^{(1)}\right)^2 &= \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 2(M_W^D)^2 & 2(M_W^D)^2 \\ 0 & 0 & 2(M_W^D)^2 & 2(M_W^D)^2 \end{pmatrix}, \\
\left(M^{(2)}\right)^2 &= \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{v^2\Lambda^2}{2} \end{pmatrix}.
\end{aligned} \tag{A.27}$$

The matrix of eigenvalues D up to ϵ^4 reads

$$D = \begin{pmatrix} m_{H_d}^2 & 0 & 0 & 0 \\ 0 & m_{H_u}^2 & 0 & 0 \\ 0 & 0 & m_T^2 + \frac{v^2\Lambda^2}{4} - \frac{v^4\Lambda^4}{64(M_W^D)^2} & 0 \\ 0 & 0 & 0 & m_T^2 + \frac{v^2\Lambda^2}{4} + \frac{v^4\Lambda^4}{64(M_W^D)^2} \end{pmatrix}. \tag{A.28}$$

A.3.4. Neutral-R Higgs

The CP-odd Higgs mass Matrix M_R^2 is

$$M_R^2 = \begin{pmatrix} m_{R_d}^2 & 0 \\ 0 & m_{R_u}^2 + \frac{1}{4}v^2\left(2\lambda^2 + \Lambda^2\right) \end{pmatrix}, \tag{A.29}$$

which is already diagonal.

A.3.5. Charged R Higgses

Since the charged R-Higgses do not mix, the contribution is directly given by

$$m_{R^+}^2 = m_{R_u}^2 + \frac{\Lambda^2 v^2}{2} \tag{A.30}$$

A.3.6. Neutralinos

The squared Neutralino mass Matrix $M_\chi^T M_\chi$ is effectively 2×2 under the assumptions of this analysis

$$M_\chi^T M_\chi = \begin{pmatrix} \frac{v^2\lambda^2}{2} & -\frac{v^2\lambda\Lambda}{2\sqrt{2}} \\ -\frac{v^2\lambda\Lambda}{2\sqrt{2}} & \frac{v^2\Lambda^2}{4} \end{pmatrix}. \tag{A.31}$$

It can be dissected into

$$\begin{aligned}
\left(M^{(0)}\right)^2 &= \begin{pmatrix} (M_B^D)^2 & 0 \\ 0 & (M_W^D)^2 \end{pmatrix}, \\
\left(M^{(1)}\right)^2 &= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \\
\left(M^{(2)}\right)^2 &= \begin{pmatrix} \frac{v^2\lambda^2}{2} & -\frac{v^2\lambda\Lambda}{2\sqrt{2}} \\ -\frac{v^2\lambda\Lambda}{2\sqrt{2}} & \frac{v^2\Lambda^2}{4} \end{pmatrix}.
\end{aligned} \tag{A.32}$$

The matrix of eigenvalues D up to ϵ^4 reads

$$D = \begin{pmatrix} (M_B^D)^2 + \frac{v^2\lambda^2}{2} + \frac{v^4\lambda^2\Lambda^2}{8(M_B^D)^2 - 8(M_W^D)^2} & 0 \\ 0 & (M_W^D)^2 + \frac{v^2\Lambda^2}{4} - \frac{v^4\lambda^2\Lambda^2}{8(M_B^D)^2 - 8(M_W^D)^2} \end{pmatrix}. \tag{A.33}$$

A.3.7. ρ Charginos

Under the assumptions of this analysis, only the ρ-Charginos contribute. The squared mass Matrix $M_{\rho^-}^T M_{\rho^-}$ is

$$M_{\rho^-}^T M_{\rho^-} = \begin{pmatrix} (M_W^D)^2 + \frac{v^2\Lambda^2}{2} & 0 \\ 0 & 0 \end{pmatrix}, \tag{A.34}$$

which is already diagonal.

A.3.8. stop

Under the assumptions of this analysis, the mass matrix for the stop reads

$$M_{\tilde{t}}^2 = \begin{pmatrix} m_{Q,33}^2 + \frac{v^2 y_t^2}{2} & 0 \\ 0 & m_{u,33}^2 + \frac{v^2 y_t^2}{2} \end{pmatrix}, \tag{A.35}$$

which is already diagonal.

B. Benchmark points

Table B.1 includes Benchmark points 1-3 from [12].

	BMP1	BMP2	BMP3
$\tan\beta$	3	10	40
B_μ	500^2	300^2	200^2
$\lambda_d,\ \lambda_u$	$1.0,\ -0.8$	$1.1,\ -1.1$	$0.15,\ -0.15$
$\Lambda_d,\ \Lambda_u$	$-1.0,\ -1.2$	$-1.0,\ -1.0$	$-1.0,\ -1.15$
M_B^D	600	1000	250
$m_{R_u}^2$	2000^2	1000^2	1000^2
$\mu_d,\ \mu_u$	$400,\ 400$		
M_W^D	500		
M_O^D	1500		
$m_T^2,\ m_S^2,\ m_O^2$	$3000^2,\ 2000^2,\ 1000^2$		
$m_{Q;1,2}^2,\ m_{Q;3}^2$	$2500^2,\ 1000^2$		
$m_{D;1,2}^2,\ m_{D;3}^2$	$2500^2,\ 1000^2$		
$m_{U;1,2}^2,\ m_{U;3}^2$	$2500^2,\ 1000^2$		
$m_L^2,\ m_E^2$	1000^2		
$m_{R_d}^2$	700^2		

Table B.1.: Benchmark points BMP1–BMP3 from [12].

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