

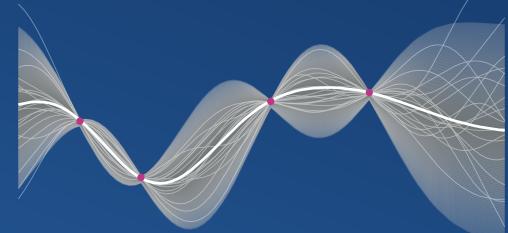


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Gaussian Process Regression for Multi-Modal Fatigue Crack Growth Identification in Textile-Reinforced Composites

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### Introduction

#### Context

- ► Textile-reinforced composites: lightweight, high-performance, used in aerospace & wind energy
- ► Fatigue crack growth governs lifetime and structural integrity mainly delamination

#### Challenge

- Crack growth is non-uniform [5, 2, 4, 6]
- Small increments interrupted by crack jumps
- ▶ Regimes of slow growth alternating with accelerated propagation
- ▶ Patterns are partly periodic but also irregular, due to textile architecture
- ▶ Multi-modal fatigue data e.g. load-displacement, DIC is noisy and scarce
- ▶ Difficult to detect and model small-scale crack growth regimes with classical window-sized kernels or parametric regression-type models

Machine Learning based data-driven approach like **Gaussian Process Regression with different sparse or uncertain data** 

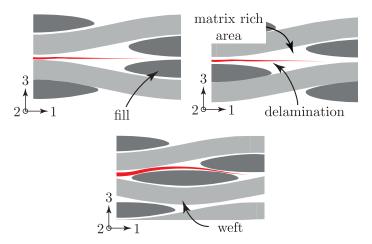


Figure: Idealized textile layer configurations of plain weave textiles

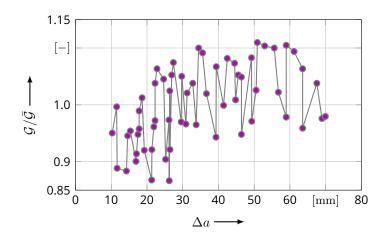


Figure: Exemplary R-curve from quasi-static DCB-test normalized to mean ERR  $\bar{\mathcal{G}}$ 







## **Material | Selection & Manufacturing**

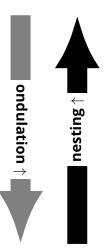
- ▶ Areal weight: higher values lead to increased undulation and reduced nesting (and vice versa) [2, 1]
- ► Manufacturing: resin transfer molding (RTM-6, Hexcel®) in a closed tool of (530 × 530 × 4) mm³
- ▶ Pre-crack insert: Teflon® foil for DCB specimens
- ► Adhesive: load blocks bonded with 3M DP490

Table: Material selection for investigating the influence of the fabric architecture on the delamination propagation

| Туре | Color | ECC-Style <sup>†</sup> | Areal Density       | Spacing      | Yarn                       | Idealized Yarn Geometry <sup>‡</sup> |
|------|-------|------------------------|---------------------|--------------|----------------------------|--------------------------------------|
| I    | _     | 447                    | $160 \frac{g}{m^2}$ | 4threads/cm  |                            |                                      |
| II   | _     | 450                    | $200 \frac{g}{m^2}$ | 5 threads/cm | Pyrofil <sup>®</sup> TR30S |                                      |
| III  |       | 460-5                  | $245 \frac{g}{m^2}$ | 6 threads/cm |                            |                                      |



<sup>‡:</sup> images are normalized to column width, do not represent actual geometry of fabrics; created with TexGen and Blender









## **Fatigue Experiments | Set-Up**

▶ Double Cantilever Beam (DCB) Test: according to ASTM D5528

► **Testing Machine**: PSB100 (Schenck)

► Load cell: 100 kN

► Maximum load amplitude: 50 mm at 10 Hz

► Force transducer: 2 kN

#### Optical Measurement System

Camera: Nikon DSLR D7000 with (6000 × 4000) Px<sup>2</sup>

► Lens: AF-S Micro NIKKOR 60 mm

Camera control: gPhoto2 on Nvidia Jetson Nano (USB-Interface)

► Triggering: GPIO with digital input from testing machine

▶ Post-Processing with RCNN and ZEISS Correlate

#### ► Fatigue Experiment Control

• Pre-cracking step: displacement-controlled ( $\delta_u$ )

lmage acquisition before each load block (n = 1000 cycles)

▶ Load block: force-controlled ( $P_y$ ) with R = 0.1 at 1 Hz

▶ Up to k = 500 load blocks

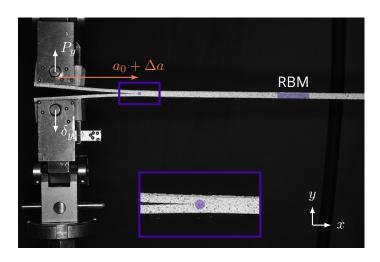


Figure: Experimental set-up and notation

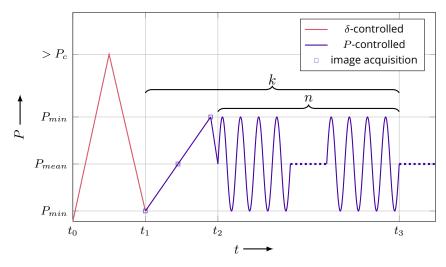


Figure: Fatigue experiment control and monitoring.







## Fatigue Experiments | Evaluation of $\mathcal G$ via MCC

- ► Modified Compliance Calibration (MCC) from quasi-static tests with optical crack length observations
- ▶ **Linear regression** on the calibration dataset  $\mathcal{D}^{(c)} = \{(\mathbb{C}_i, a_i)\}_{i=1}^m$  for each material  $c \in \{\mathsf{I}, \mathsf{II}, \mathsf{III}\}$ :

$$\frac{a}{h} = \zeta \sqrt[3]{\mathbb{C}} + \kappa$$

 Uncertainty estimation via standard error of the modified CC regression fit

|          | Type I          | Type II | Type III |
|----------|-----------------|---------|----------|
| ζ        | 29.902<br>0.528 | 28.143  | 27.907   |
| $\sigma$ | 0.528           | 0.310   | 0.511    |

- ► **Compliance in fatigue experiments** obtained from hysteresis curve (unloading branch)
- ► **Calculation of** *G* via MCC:

$$\mathcal{G}_{\mathrm{min/max}} = \frac{3\,P_{\mathrm{min/max}}^2\,\sqrt[3]{\mathbb{C}^2}}{2\,\zeta\,wh}$$

► Energy release rate range:

$$\Delta \mathcal{G} = \mathcal{G}_{\mathsf{max}} - \mathcal{G}_{\mathsf{min}},$$

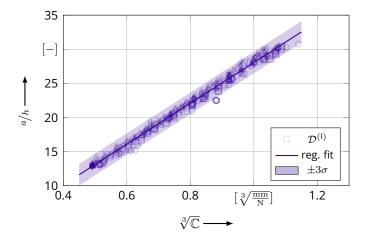


Figure: Results of CC for a  $\mathcal{D}^{(1)} = \{(N_i, a_i)\}_{i=1}^{m_k}$ .

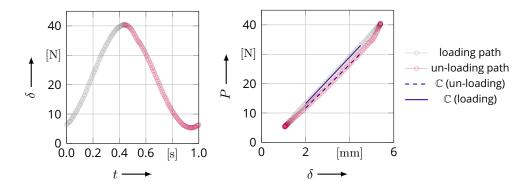


Figure: Exemplary compliance extraction from fatigue hysteresis curve (N=300)







## Fatigue Experiments | Evaluation of a via optical methods

- Crack tip detection using a tailored recurrent convolutional network with a YOLO-like head (to be published)
- Optical crack tip measurements discretized every 1000 load cycles
- ▶ Post-processing in Zeiss Correlate with rigid body motion tools
- ▶ **Estimating the uncertainty:** The standard error of the optical crack tip detection is derived from the mean validation  $L^1$  loss of  $\approx 0.1$  Px, corresponding to a spatial deviation of  $\approx 0.2$  mm.

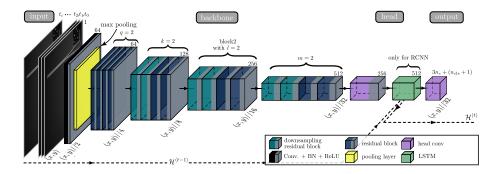


Figure: Schematic Illustration of RCNN

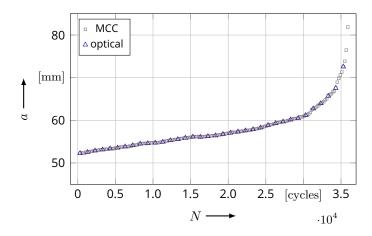


Figure: Exemplary results of the obtained a-N-curve for type I







## **Gaussian Process Regression**

- ▶ Advantages over kernel-size heuristics or parametric regression:
  - ▶ No need to predefine window size or filtering parameters
  - ► Non-parametric ⇒ adapts complexity to data
  - Provides predictive uncertainty (confidence intervals)
  - ► Applicable also for sparse data
- ▶ Gaussian Process Regression with  $\mathcal{D} = \{(N_i, a_i)\}_{i=1}^m = \{(\mathbf{x}, \mathbf{y})\}_{i=1}^m$

$$f(\mathbf{x}) \sim \mathcal{GP}(m(\mathbf{x}), k(\mathbf{x}, \mathbf{x}'))$$

► Training by minimizing the **negative log marginal likelihood (NLML)**:

$$\mathcal{L}(\theta) = -\log p(\mathbf{y} \,|\, \mathbf{X}, \theta)$$

**Preprocessing:** standardize to  $\mathcal{N}(0,1)$  for stability

$$\tilde{\mathbf{x}} = \frac{\mathbf{x} - \mu}{\sigma}, \quad \tilde{\mathbf{y}} = \frac{\mathbf{y} - \mu}{\sigma}$$

▶ Implementation in **GPyTorch** [3] for PyTorch-based GPU-acceleration

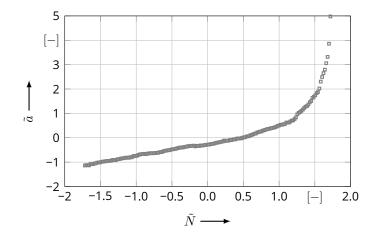


Figure: Normalized dataset 
$$\mathcal{D} = \{(\tilde{N}_i, \tilde{a}_i)\}_{i=1}^m = \{(\tilde{\mathbf{x}}, \tilde{\mathbf{y}})\}_{i=1}^m \sim \mathcal{N}(0, 1)$$

- ▶ Physics-informed prior (mean function:  $m(\mathbf{x})$ ): encode expected exponential growth in the GP mean and let the kernel model residuals
- ▶ **Kernel function**  $k(\mathbf{x}, \mathbf{x}')$  must still be carefully chosen, as it encodes prior assumptions about smoothness and correlation.







## **Gaussian Process Regression | Kernel & Mean Function Selection**

Physics-informed mean (prior):

$$m(\mathbf{x}) = \alpha \exp(\beta x)$$

- Kernel functions:
  - RBF (squared exponential)

$$k_{\text{RBF}}(\mathbf{x}, \mathbf{x}') = \sigma_f^2 \exp\left(-\frac{\|\mathbf{x} - \mathbf{x}'\|^2}{2\ell^2}\right)$$

Periodic (sinusoidal)

$$k_{\mathrm{Per}}(\mathbf{x}, \mathbf{x}') = \sigma_f^2 \exp\left(-\frac{2}{\ell^2} \sin^2\left(\frac{\pi}{p} \|\mathbf{x} - \mathbf{x}'\|\right)\right)$$

Combined kernel (as used):

$$k(\mathbf{x}, \mathbf{x}') = k_{\text{RBF}}^{(1)}(\mathbf{x}, \mathbf{x}') + k_{\text{RBF}}^{(2)}(\mathbf{x}, \mathbf{x}') k_{\text{Per}}(\mathbf{x}, \mathbf{x}')$$

- ► Training (GP):
  - Optimizer: Adam with learning rate  $lr = 1 \times 10^{-2}$
  - ► Maximum number of iterations: 1000
  - ► Early stopping when improvement in marginal log-likelihood satisfies

$$|\mathcal{L}_{i-1} - \mathcal{L}_i| < 10^{-4}$$

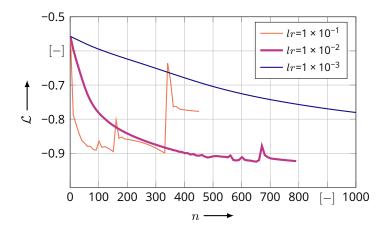


Figure: Training results of  $\mathcal{GP}(m(\mathbf{x}), k(\mathbf{x}, \mathbf{x}'))$ 

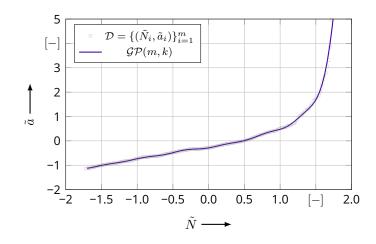


Figure: Inference results of  $\mathcal{GP}(m(\mathbf{x}), k(\mathbf{x}, \mathbf{x}'))$ 







## Results | type |

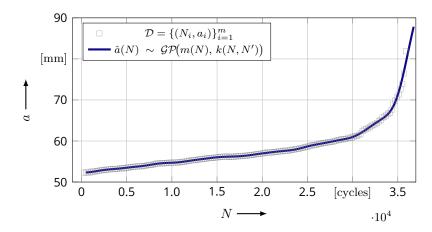


Figure: a - N-curve for type I

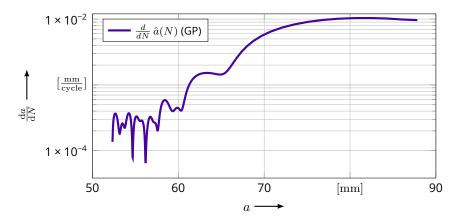


Figure:  $a - \frac{da}{dN}$ -curve for type I

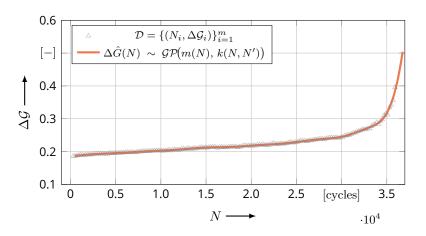


Figure:  $N - \Delta G$ -curve for type I

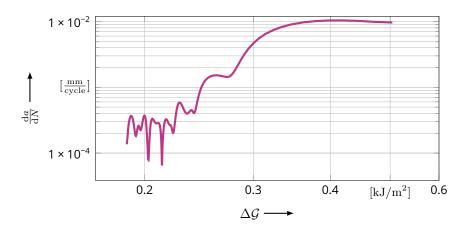


Figure:  $\Delta G - \frac{\mathrm{d}a}{\mathrm{d}N}$ -curve for type I







### **Results**

- Comparative analysis of material types I–III under identical load amplitude ( $P_{\rm max} \sim 0.8\,{\cal G}_c$ )
- ▶ Observed trend in crack-growth behaviour: lower  $\frac{\mathrm{d}a}{\mathrm{d}N}$  with decreasing ondulation length, indicating a retardation effect of textile waviness
- ► Clear distinction of crack-growth regimes (**crack jumps**) visible in the  $\Delta \mathcal{G} \frac{da}{dN}$  relationship
- Data-driven GPR modelling highlights sensitivity of fatigue life to reinforcement architecture
- ightharpoonup Small oscillations during the first  $\sim 1000$  cycles indicate running-in effects  $\Rightarrow$  assignment of higher uncertainty in this region
- ► Additional data required for a more robust statistical analysis, in particular for higher cycle counts or loads

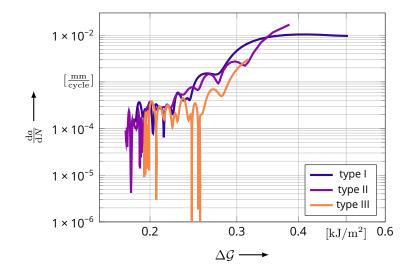


Figure:  $\Delta G - \frac{\mathrm{d}a}{\mathrm{d}N}$ -curve for type I-III







### Conclusion

- ▶ Developed a data-driven framework using Gaussian Process Regression (GPR) with physics-informed mean functions as priors for crack propagation in DCB tests on plain-weave composites
- ► Fused multimodal data: mechanical compliance curves (higher uncertainty) and optical crack-tip measurements (sparser but more precise)
- ► Kernel design (RBF, sinusoidal) with exponential mean function (physics-informed prior) shown sufficient to represent the characteristic crack-growth behaviour
- ► Enabled both predictive interpolation and physics-consistent extrapolation of fatigue crack growth
- Comparative analysis across three plain-weave textile composites with varying areal weight

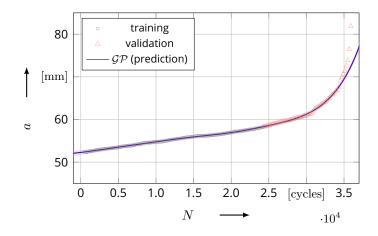


Figure: Extrapolation inference of  $\mathcal{GP}(m(\mathbf{x}), k(\mathbf{x}, \mathbf{x}'))$ 

#### Outlook:

- ► Analysis of a higher number of tests for data-driven differentiation of textile material behaviour
- extension to ENF and further advanced fatigue experiments [6]

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