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On-shell Renormalization with Vector-like Leptons, One-loop Muon–Higgs Coupling and Muon $g-2$

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Abstract

Models with vector-like leptons can strongly modify the lepton mass generation mechanism and lead to correlated effects in lepton–Higgs couplings and lepton dipole moments. Here we begin an analysis of higher-order corrections in such models by setting up a renormalization scheme with full on-shell conditions on the lepton self energies, masses and fields. A minimal set of fundamental parameters is renormalized in the $\overline{\text{MS}}$ scheme. We provide a detailed discussion of lepton mixing and redundancies at higher orders, show how the relevant counterterms can be obtained from the renormalization conditions, and determine the β-functions corresponding to the scheme. As a first application we calculate the one-loop effective muon–Higgs coupling and analyse its correlation with the muon anomalous magnetic moment $\Delta a_\mu^{\text{VLL}}$. In the interesting case of large masses and opposite-sign coupling, the lowest-order correlation implies a fixed value of $\Delta a_\mu^{\text{VLL}}$ around 22.5×10^{-10}, while the higher-order corrections significantly reduce this value to the interval $(10 \ldots 18) \times 10^{-10}$.

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1 Introduction

In recent years vector-like lepton (VLL) models have gained increasing interest as a potential answer to some of the long standing open questions in particle physics. We focus on a model where the Standard Model (SM) is extended by additional vector-like leptons which can have fundamental gauge-invariant Dirac masses and mix with the SM leptons. Such VLLs have been considered in the context of the muon $g-2$ (also in combination with extended scalar sectors) [1–10], charged lepton flavor violation (cLFV) [3, 11, 12], and electroweak precision observables [1, 2, 13].

The key feature of such VLL models lies in their change of the lepton mass generation mechanism: The masses of the SM-like leptons result in part (or even entirely) from mixing, rather than through their fundamental interactions with the Higgs only. In general, SM fermion mass terms require a source of chirality flips and a breaking of electroweak gauge invariance. These two requirements are shared by other observables such as effective Higgs-lepton couplings $\lambda_{ll'}$ and lepton anomalous magnetic moments $a_l = (g-2)_l/2$. The implications of these relations for a_μ are discussed in Refs. [14, 15] as well as in Refs. [2, 16] with focus on the correlation between a_μ, $\lambda_{\mu\mu}$ and further observables in a wide range of models. Many extensively studied models such as two-Higgs-doublet (2HDM), supersymmetric (SUSY) and leptoquark (LQ) models introduce modifications in the scalar sector, which lead to new chirality flips at the loop level. In contrast, VLL models provide new sources of chirality flips already at tree-level via the Dirac masses and Yukawa couplings of the VLL fields and imply distinct correlations between the observables involving chirality flips [2, 10, 17].

These correlations can be understood as a result of effective dimension-6 operators of the form $\bar{l}lHH^\dagger H$ that are generated at tree-level in VLL models. Such operators can lead to $\mathcal{O}(1)$ tree-level corrections to $\lambda_{\mu\mu}$ that turn out to be strongly correlated with contributions to a_μ (for a general study of these operators see Ref. [18]). On the other hand, experimental constrains restrict $|\lambda_{\mu\mu}|$ to be close to the SM value. This allows either $\lambda_{\mu\mu} \approx \lambda_{\mu\mu}^{\rm SM}$ in which case $\Delta a_\mu^{\rm VLL} \approx 0$ or, intriguingly, $\lambda_{\mu\mu} \approx -\lambda_{\mu\mu}^{\rm SM}$ in which case the correlation implies some fixed value $\Delta a_\mu^{\rm VLL} \neq 0$. For example, in the case of one generation of heavy VLLs coupling to the muon discussed in Refs. [1, 2], the tree-level correlation predicts

$$a_\mu^{\rm VLL}|_{(\lambda_{\mu\mu}=-\lambda_{\mu\mu}^{\rm SM})} \simeq 22.5 \times 10^{-10}\,. \tag{1}$$

This is intriguingly close to the deviation between the Fermilab Run 1,2,3 measurements [19, 20] and the 2020 white paper SM theory value based on [21–41], $a_\mu^{\rm exp} - a_\mu^{\rm SM} = (24.9 \pm 4.8) \times 10^{-10}$. Clearly, scrutiny of SM predictions (see Refs. [21, 42] for an overview of results and prospects) and Fermilab Run 4,5,6 results might change this value. Hence it is an open question whether the deviation in Eq. (1) remains a viable value.

The aim of the present paper is to provide a starting point for higher-order studies of VLL models. Here, we focus on the SM extended by one generation of VLLs that mix with all of the SM leptons. We develop an on-shell renormalization scheme and discuss the treatment of the lepton mixing at higher orders. As a first application of the scheme, we determine the modification of Eq. (1) from including one-loop corrections to the effective muon-Higgs coupling $\lambda_{\mu\mu}$.

On-shell renormalization has already been studied in a variety of scenarios beyond the Standard Model (BSM) and has often turned out to be highly nontrivial. This is in contrast to the electroweak SM where there is a one-to-one correspondence between fundamental parameters and masses or similarly simple on-shell observables. E.g. the on-shell renormalization of the Minimal Supersymmetric Standard Model (MSSM) has been studied in Refs. [43–45] with focus on the SUSY sectors; here, SUSY relations between masses imply that only a subset of masses can be on-shell renormalized. In addition, the renormalization of parameters such as $\tan\beta$ or mixing angles is required and troublesome [46, 47]. The on-shell renormalization of the two-Higgs doublet model (2HDM) has been studied in Refs. [48–54] with increasing focus on mixing angles and similar parameters for which an on-shell scheme is not suitable. For these parameters $\overline{\text{MS}}$ renormalization conditions can be advantageous, but in such mixed schemes the renormalization of the vacuum expectation values/of tadpoles becomes critical [46, 50, 52–58].

In the VLL model considered in this paper, the 5×5 lepton mixing is responsible for the intriguing phenomenology, but it also complicates the discussion of higher-order corrections. Nonetheless, with the steady increase in experimental precision as well as spectacular predictions like Eq. (1), increasing the theoretical accuracy is desirable. This requires a proper definition of a renormalization scheme, where an on-shell scheme allows to study correlations between observables in a particularly transparent way.

We begin with a concise review of the model in Sec. 2, including the definition, the conventions for the Dirac masses, the Yukawa interaction parameters, and the unitary mixing matrix for lepton mass eigenstates. In Sec. 3 we develop a renormalization scheme for this model where all masses and fields are on-shell renormalized and additional $\overline{\text{MS}}$-conditions are imposed on original parameters of the gauge-invariant Lagrangian. We discuss in detail the counterterm structure and solution to the on-shell condition as well as the residual renormalization scale dependence of the fundamental parameters. Afterwards, in Sec. 4 we extend the definition of the effective muon-Higgs coupling to one-loop order and apply the renormalization scheme to compute the corresponding renormalized higher-order contributions. Finally, in Sec. 5 we summarize the results and our conclusion.

2 Model Definition

	SM			VL	
	$l_L^i = \begin{pmatrix}\nu_L^i\\ e_L^i\end{pmatrix}$	e_R^i	$H = \begin{pmatrix}\phi^+\\ v+\frac{1}{\sqrt{2}}(h+i\phi^0)\end{pmatrix}$	$L_{L/R} = \begin{pmatrix}L^0_{L/R}\\ L^-_{L/R}\end{pmatrix}$	$E_{L/R}$
$\text{SU}(2)_\text{L}$	**2**	**1**	**2**	**2**	**1**
$\text{U}(1)_\text{Y}$	$-\frac{1}{2}$	-1	$\frac{1}{2}$	$-\frac{1}{2}$	-1

Table 1: Definition of the SM and VL lepton fields (with generation index $i = 1, 2, 3$) and their corresponding quantum numbers (where the electric charge is related to $\text{SU}(2)_L$ and $\text{U}(1)_Y$ quantum numbers as $Q = T^3 + Y$). The fields h, ϕ^0, ϕ^+ correspond to the physical Higgs field and the neutral and charged Goldstone bosons, and $v \approx 174$ GeV. The VLL fields are defined as $L_{L/R} = P_{L/R}L$ and $E_{L/R} = P_{L/R}E$, where L are $SU(2)$ doublets and E $SU(2)$ singlets with the same quantum numbers as the SM lepton doublets and singlets respectively.

The SM lepton sector consists of three generations of left-handed $\text{SU}(2)_L$ doublets l_L^i and right-handed singlets e_R^i. In addition, we consider one generation of vector-like leptons (VLL) comprised of both left- and right handed doublets $L_{L/R}$ and singlets $E_{L/R}$ with gauge-quantum numbers identical to those of the SM fields (see Table 1). This model has previously been studied in the

muon-specific limit in Refs. [1, 2] and in the context of charged lepton flavor violation in Ref. [3]. It is useful to combine the SM and VL lepton fields with identical quantum numbers into the 4-tuples

$$l_L^a = (l_L^i, L_L)^{\mathrm{T}}, \qquad e_R^a = (e_R^i, E_R)^{\mathrm{T}} \qquad \text{for} \qquad a = 1, 2, 3, 4. \tag{2}$$

Viewed in this way, the model contains four generations of the usual chiral leptons l_L^a and e_R^a as well as one generation of opposite-chirality fields L_R and E_L. Using these definitions, the most general renormalizable Lagrangian can be written as

$$\begin{aligned}\mathcal{L} \supset\ & \bar{l}_L^a i \not{D} l_L^a + \bar{e}_R^a i \not{D} e_R^a + \bar{L}_R i \not{D} L_R + \bar{E}_L i \not{D} E_L \\ & - \left(\bar{l}_L^a M_L^a L_R + \bar{E}_L M_E^a e_R^a + \bar{l}_L^a Y_{ab} e_R^b H + \bar{E}_L \bar{\lambda} L_R H^\dagger + h.c.\right),\end{aligned} \tag{3}$$

where the $\mathrm{SU(2)_L} \times \mathrm{U(1)_Y}$ covariant derivative is given by $D_\mu = \partial_\mu - i g_W \frac{\sigma^i}{2} W_\mu^i + i g_Y Y B_\mu$. In the present paper we assume CP invariance and consider only real parameters. M_L and M_E are 4-tuple vectors of mass parameters, Y is a 4×4 Yukawa matrix and $\bar{\lambda}$ a single Yukawa coupling. Similar to the SM Yukawa sector, the Lagrangian Eq. (3) contains a redundancy and not all parameters carry physical information. This redundancy results from the invariance of the Lagrangian under the following simultaneous transformation of the fields and parameters

$$l_L^a \to (V_L l_L)^a, \quad e_R^a \to (V_R e_R)^a, \quad M_L^a \to (V_L M_L)^a, \quad M_E^a \to (M_E^{\mathrm{T}} V_R^\dagger)^a, \quad Y_{ab} \to (V_L Y V_R^\dagger)_{ab}, \tag{4}$$

where $V_{L/R}$ are arbitrary unitary 4×4 matrices. As a result, 12 of the Yukawa and mass parameters can be absorbed into a redefinition of the lepton fields. In particular, it is convenient to absorb the off-diagonal Dirac masses together with the off-diagonal SM Yukawa couplings, such that w.l.o.g. the mass and coupling matrices can be written as

$$M_L^a = (0, 0, 0, m_L)^{\mathrm{T}}, \qquad M_E^a = (0, 0, 0, m_E)^{\mathrm{T}}, \qquad Y_{ab} = \begin{pmatrix} y_i \delta_{ij} & \lambda_i^E \\ \lambda_j^L & \lambda \end{pmatrix}. \tag{5}$$

The remaining 12 physical parameters in this sector are thus taken as the 3 SM Yukawa couplings y_i, the 6 new Yukawa couplings λ_i^L, λ_i^E which are responsible for the interactions between the SM and VLL fields, the Yukawa coupling λ involving the VLL fields, and the 2 Dirac masses m_L and m_E. We note that the 13th new physical parameter $\bar{\lambda}$ cannot be absorbed by any field redefinition. The vacuum expectation value (vev) of the Higgs field spontaneously breaks the $\mathrm{SU(2)_L} \times \mathrm{U(1)_Y}$ symmetry and generates additional lepton mass terms in the Lagrangian. After electroweak symmetry breaking (EWSB), the 5×5 mass matrices for the charged leptons and neutrinos are given by

$$(\bar{e}_L^a, \bar{E}_L) \begin{pmatrix} (Yv)_{ab} & M_L^a \\ M_E^b & \bar{\lambda} v \end{pmatrix} \begin{pmatrix} e_R^b \\ L_R \end{pmatrix}, \qquad (\bar{\nu}_L^a, 0) \begin{pmatrix} 0 & M_L^a \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ L_R^0 \end{pmatrix}. \tag{6}$$

We introduce the lepton mass eigenstate fields as the 5-tuples

$$\hat{e}_L = U_L^\dagger \begin{pmatrix} e_L^a \\ E_L \end{pmatrix}, \qquad \hat{e}_R = U_R^\dagger \begin{pmatrix} e_R^a \\ L_R \end{pmatrix}, \qquad \hat{\nu}_L = U_L^{\nu\dagger} \begin{pmatrix} \nu_L^a \\ 0 \end{pmatrix}, \qquad \hat{\nu}_R = U_R^{\nu\dagger} \begin{pmatrix} 0 \\ L_R^0 \end{pmatrix}, \tag{7}$$

where the unitary 5×5 matrices $U_{L/R}$ and $U_{L/R}^\nu$ are chosen such that the mass matrices are diagonalized as

$$\left[U_L^\dagger \begin{pmatrix} Yv & M_L \\ M_E^{\mathrm{T}} & \bar{\lambda} v \end{pmatrix} U_R \right]_{ab} := m_a \delta_{ab}, \qquad \left[U_L^{\nu\dagger} \begin{pmatrix} 0 & M_L \\ 0 & 0 \end{pmatrix} U_R^\nu \right]_{ab} := m_L \delta_{a4} \delta_{b4} \tag{8}$$

Note that in our convention the only massive neutrino is $\hat{\nu}^4$ with mass m_L, while the SM neutrinos $\hat{\nu}_i$ remain massless and $\hat{\nu}_5 = 0$. The masses of the charged leptons are given approximately by

$$m_i = y_i v + \frac{\bar{\lambda}\lambda_i^E \lambda_i^L v^3}{m_E m_L} + \mathcal{O}(v^4) \tag{9a}$$

$$m_{4,5} = m_{L,E} + \mathcal{O}(v^2)\,, \tag{9b}$$

where y_i is defined in Eq. (5). In the mass eigenstate basis the lepton mass terms and physical Higgs-lepton Yukawa interactions are given by the following terms in the classical Lagrangian

$$\mathcal{L} \supset -m_a \bar{\hat{e}}_L^a \hat{e}_R^a - m_L \bar{\hat{\nu}}_L^4 \hat{\nu}_R^4 - \tfrac{1}{\sqrt{2}} \bar{\hat{e}}_L^a Y_{ab}^h \hat{e}_R^b h + h.c., \tag{10}$$

where we have introduced the effective 5×5 Yukawa coupling matrix

$$Y^h = U_L^\dagger \begin{pmatrix} Y & 0 \\ 0 & \bar{\lambda} \end{pmatrix} U_R\,. \tag{11}$$

Our convention differs slightly from the one used in Refs. [1, 2] where $U_{L/R}^{(\nu)} = \mathbb{1}$ in the limit $v \to 0$, but which results in less symmetric expressions for the coupling and counterterm matrices. All coupling matrices together with the Feynman rules are listed in appendix A.

3 Renormalization

In this section we present a renormalization scheme developed for the VLL model with full on-shell conditions on the lepton, Higgs- and gauge-boson self-energies. These on-shell conditions allow for a transparent computation of higher-order corrections to observables in terms of the physical pole masses and well-defined input parameters of the VLL model which satisfy Eq. (8) at all orders. Typically, difficulties arise when the on-shell scheme is applied to the renormalization of BSM scenarios since there is often no one-to-one correspondence between masses and parameters. For example in the chargino and neutralino sector of the MSSM a simultaneous on-shell renormalization of all masses is impossible, as 3 of the 6 masses are fixed by the remaining input parameters [44]. In the 2HDM a complete on-shell renormalization is possible, but is made difficult by the appearance of additional parameters such as mixing angles and several possible schemes have been studied extensively in Refs. [48, 50, 52–54]. Our approach to the renormalization of the VLL model is similar to the treatment of the chargino and neutralino sector discussed in Refs. [43, 44]. The important difference is that in our case a complete on-shell renormalization of all lepton masses is possible.

3.1 Gauge and Higgs Sector

Here, we briefly list our notation for the one-loop on-shell (OS) renormalization of the gauge and Higgs sector. It is chosen to be identical to the one of the SM; hence we adopt the conventions and results from Refs. [55, 59]. The renormalization transformation of the bare fields and parameters is done on the level of mass-eigenstate quantities,

$$h \to \left(1 + \tfrac{1}{2}\delta Z_h\right) h, \qquad M_h^2 \to M_h^2 + \delta M_h^2, \tag{12a}$$

$$\begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix} \to \begin{pmatrix} 1 + \frac{1}{2}\delta Z_{AA} & \frac{1}{2}\delta Z_{AZ} \\ \frac{1}{2}\delta Z_{ZA} & 1 + \frac{1}{2}\delta Z_{ZZ} \end{pmatrix} \begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix}, \qquad M_Z^2 \to M_Z^2 + \delta M_Z^2, \tag{12b}$$

$$W_\mu^\pm \to \left(1 + \tfrac{1}{2}\delta Z_{WW}\right) W_\mu^\pm, \qquad M_W^2 \to M_W^2 + \delta M_W^2, \tag{12c}$$

in terms of the renormalized quantities and renormalization constants δZ_h, $\delta Z_{VV'}$ (with $V = A, Z, W$) as well as δM_h, δM_Z and δM_W. Denoting the unrenormalized Higgs and gauge-boson self-energies by

$$\Sigma_h(p^2), \qquad \Sigma^{\mu\nu}_{VV'}(q) = \Sigma^T_{VV'}(q^2)\big(\eta^{\mu\nu} - \tfrac{q^\mu q^\nu}{q^2}\big) + \Sigma^L_{VV'}(q^2)\tfrac{q^\mu q^\nu}{q^2}, \tag{13}$$

the OS conditions on the renormalized Higgs and gauge-boson self-energies result in the following expressions for the renormalization constants,

$$\delta Z_h = -\widetilde{\mathrm{Re}}\tfrac{d}{dp^2}\Sigma_h(p^2)\big|_{p^2=M_h^2}, \qquad \delta M_h^2 = \widetilde{\mathrm{Re}}\Sigma_h(M_h^2), \tag{14a}$$

$$\delta Z_{VV} = -\widetilde{\mathrm{Re}}\tfrac{d}{dq^2}\Sigma^T_{VV}(q^2)\big|_{p^2=M_V^2}, \qquad \delta M_V^2 = \widetilde{\mathrm{Re}}\Sigma^T_{VV}(M_V^2), \tag{14b}$$

$$\delta Z_{AZ} = -\tfrac{2}{M_Z^2}\widetilde{\mathrm{Re}}\Sigma^T_{AZ}(M_Z^2), \qquad \delta Z_{ZA} = \tfrac{2}{M_Z^2}\widetilde{\mathrm{Re}}\Sigma^T_{AZ}(0), \tag{14c}$$

where $\widetilde{\mathrm{Re}}$ removes the imaginary parts of any appearing loop integrals. From charge universality it follows that the renormalization of the electric charge is given by

$$e \to e + \delta e, \qquad \tfrac{\delta e}{e} = -\tfrac{1}{2}\big(\delta Z_{AA} + \tfrac{s_W}{c_W}\delta Z_{ZA}\big). \tag{15}$$

In this setup the original parameters of the Lagrangian, i.e. in particular the Higgs vev v, are treated as a combination of the boson masses and electric charge e. Consequently, the corresponding renormalization constant is fixed by the above counterterms as

$$v \to v + \delta v, \qquad \tfrac{\delta v}{v} = \tfrac{\delta M_W^2}{2M_W^2} + \tfrac{c_W^2}{s_W^2}\Big(\tfrac{\delta M_Z^2}{2M_Z^2} - \tfrac{\delta M_W^2}{2M_W^2}\Big) - \tfrac{\delta e}{e}. \tag{16}$$

Finally, we discuss the renormalization of tadpole diagrams and the implication for the renormalization of v. All previous equations and relations hold independently of the specific choice for the tadpole counterterm δt, provided that the contribution of 1-particle reducible tadpoles T and the tadpole counterterm to the self-energies in Eq. (14) are included. However, different definitions of δt lead to different values of the on-shell counterterms and subsequently δv. While this is inconsequential in the pure SM, this choice can become important in BSM scenarios where not all parameters are necessarily renormalized on-shell. A discussion of this issue for the MSSM can be found in Ref. [46] and further details along with several different schemes are given in Refs. [52, 53, 56, 58]. Here we choose the parameter-renormalized tadpole scheme (PRTS) in which the renormalized tadpole diagrams T are set to zero,

$$T + \delta t^{\mathrm{PRTS}} = 0. \tag{17}$$

With this requirement v corresponds to the true minimum of the loop-corrected effective potential, but δv is gauge-dependent. This scheme was found particularly advantageous in Ref. [46] in conjunction with the $\overline{\mathrm{MS}}$ renormalization of other parameters. Further discussions of the vev renormalization in general theories up to the two-loop level can be found in Refs. [60, 61].

3.2 Lepton Sector

The renormalization of the lepton sector of the VLL model at one-loop order is the central part of our work. We begin by generating the counterterm Lagrangian with the correct structure of renormalization constants needed to fully absorb all appearing divergences and fulfil complete OS conditions. Afterwards, we proceed to set up and evaluate the OS conditions on the lepton self-energies and finally discuss the renormalization of the remaining parameters that are defined in the $\overline{\mathrm{MS}}$-scheme.

3.2.1 Renormalization Transformation and Counterterm Lagrangian

A complete counterterm Lagrangian reflecting the symmetries of the theory is most easily obtained by a renormalization transformation of the fields and parameters in the classical Lagrangian. To determine the appropriate renormalization transformations, one must first choose a specific basis for the fields and independent parameters like e.g. the gauge-eigenstate basis Eq. (3) (for the SM this was worked out in Ref. [62]) or the mass-eigenstate basis Eq. (10) (for the SM this was worked out in Refs. [59, 63]). As mentioned in the Introduction, in many BSM theories, setting up an OS renormalization scheme is significantly less straightforward than in the SM. In our case, the main complication is the 5×5 lepton mixing. There is enough freedom to treat all masses as free parameters but the remaining parameters cannot be written as simple mixing angles (in contrast to e.g. the 2HDM) and are affected by the redundancy Eq. (4) between fields and parameters.

In our setup we follow the approach outlined in Secs. 4.2 and 4.3 of Ref. [43] and used for the renormalization of the chargino and neutralino sector of the MSSM in Ref. [44]. This approach is advantageous because it does not require a renormalization of the mixing matrices and allows for a straight forward combination of OS and $\overline{\text{MS}}$ renormalization conditions.

The approach proceeds in two steps. First, the fields and parameters in the gauge-eigenstate basis are renormalized. Secondly, an additional finite transformation of only the fields into mass-eigenstate fields is performed. In combination, a redundancy-free counterterm Lagrangian is obtained which contains renormalization constants of fundamental parameters and matrix-valued field renormalization constants for mass-eigenstate fields, that allow to fulfil complete OS conditions. In detail, starting from the original gauge-eigenstate Lagrangian in Eq. (3) we first introduce the renormalization transformation for the field multiplets as

$$l_L^a \to (Z_L^l)_{ab}^{\frac{1}{2}}\, l_L^b, \qquad L_R \to (Z_R^L)^{\frac{1}{2}} L_R, \tag{18a}$$

$$e_R^a \to (Z_R^e)_{ab}^{\frac{1}{2}}\, e_R^b, \qquad E_L \to (Z_L^E)^{\frac{1}{2}} E_L, \tag{18b}$$

where matrix-valued field renormalization constants allow mixing between fields with identical quantum numbers. The Dirac mass and Yukawa parameters in Eq. (3) are renormalized as

$$M_L^a \to M_L^a + \delta M_L^a, \qquad M_E^a \to M_E^a + \delta M_E^a, \tag{19a}$$

$$Y_{ab} \to Y_{ab} + \delta Y_{ab}, \qquad \bar{\lambda} \to \bar{\lambda} + \delta\bar{\lambda}. \tag{19b}$$

At this point, as a consequence of the redundancy Eq. (4), there is an ambiguity between the field and parameter counterterms. Despite assuming the tree-level diagonalization Eq. (5), the same symmetry still implies an invariance of the bare Lagrangian under the following transformation of the renormalization constants

$$(Z_L^l)^{\frac{1}{2}} \to (\mathbb{1} + \delta V_L)(Z_L^l)^{\frac{1}{2}}, \tag{20a}$$

$$(Z_R^e)^{\frac{1}{2}} \to (\mathbb{1} + \delta V_R)(Z_R^e)^{\frac{1}{2}}, \tag{20b}$$

$$M_L + \delta M_L \to (\mathbb{1} + \delta V_L)(M_L + \delta M_L), \tag{20c}$$

$$M_E^T + \delta M_E^T \to (M_E^T + \delta M_E^T)(\mathbb{1} + \delta V_R^\dagger), \tag{20d}$$

$$Y + \delta Y \to (\mathbb{1} + \delta V_L)(Y + \delta Y)(\mathbb{1} + \delta V_R^\dagger), \tag{20e}$$

where $\mathbb{1} + \delta V_{L/R}$ can be arbitrary unitary matrices. We make use of this transformation to bring the counterterm matrices into the same form as the tree-level couplings in Eq. (5), i.e. we assume

that the 13 independent parameter renormalization constants in the sector have the form

$$\delta M_L^a = (0,0,0,\delta m_L)^T, \qquad \delta M_E^a = (0,0,0,\delta m_E)^T, \qquad \delta Y_{ab} = \begin{pmatrix} \delta y_i \delta_{ij} & \delta\lambda_i^E \\ \delta\lambda_j^L & \delta\lambda \end{pmatrix}, \qquad \delta\bar{\lambda}. \tag{21}$$

This removes the ambiguity and disentangles field and parameter counterterms. However, it is important to keep this choice in mind when comparing to results in the literature. For example, for the calculation of the β-functions of general gauge theories in Refs. [64, 65] this ambiguity was removed by assuming hermitian field renormalization matrices, which comes at the cost of off-diagonal counterterms $\delta M_{L/E}^i$ and δy_{ij}. We will discuss the consequences and our treatment in more detail in Sec. 3.3.

This structure of the renormalization constants corresponding to the gauge-eigenstate basis is sufficient to cancel all UV divergences but not sufficient to fulfil full OS conditions. Hence, as a second step we perform a finite transformation to mass-eigenstate fields. We require that the diagonalization condition in Eq. (8) holds exactly, i.e. at all orders for the renormalized mass matrix. This provides an exact definition of the unitary matrices $U_{L,R}$, $U^\nu_{L,R}$ as well as of the mass eigenvalues m_a as functions of the fundamental renormalized parameters. In contrast, Eq. (7) cannot be required at all orders if full OS conditions are desired. Instead, the mass-eigenstate fields are defined via the following higher-order generalization of Eq. (7),

$$\begin{pmatrix} e_L^a \\ E_L \end{pmatrix} = U_L \mathcal{R}_L \hat{e}_L, \quad \begin{pmatrix} e_R^a \\ L_R^- \end{pmatrix} = U_R \mathcal{R}_R \hat{e}_R, \quad \begin{pmatrix} \nu_L^a \\ 0 \end{pmatrix} = U_L^\nu \mathcal{R}_L^\nu \hat{\nu}_L, \quad \begin{pmatrix} 0 \\ L_R^0 \end{pmatrix} = U_R^\nu \mathcal{R}_R^\nu \hat{\nu}_R. \tag{22}$$

Here, we have introduced additional finite and invertible matrices $\mathcal{R}^{(\nu)}_{L/R} = \mathbb{1} + \delta\mathcal{R}^{(\nu)}_{L/R}$ (see also Sec. 4.2 of Ref. [43]). In total, the combination of the field transformations (18) and (22) amounts to a renormalization transformation from bare gauge-eigenstate fields to renormalized mass-eigenstate fields in terms of products of three matrices. Like in Refs. [43, 44] it is useful rewrite these products in terms of final field renormalization constants which effectively renormalize mass-eigenstate fields. Thus, the ultimate form of the renormalization transformation from gauge-eigenstate fields to mass-eigenstate fields is written as

$$\begin{pmatrix} e_L^a \\ E_L \end{pmatrix} \to \begin{pmatrix} (Z_L^l)^{\frac{1}{2}} & 0 \\ 0 & (Z_L^E)^{\frac{1}{2}} \end{pmatrix} U_L \mathcal{R}_L \hat{e}_L \equiv U_L (Z_L)^{\frac{1}{2}} \hat{e}_L, \tag{23a}$$

$$\begin{pmatrix} e_R^a \\ L_R^- \end{pmatrix} \to \begin{pmatrix} (Z_R^e)^{\frac{1}{2}} & 0 \\ 0 & (Z_R^L)^{\frac{1}{2}} \end{pmatrix} U_R \mathcal{R}_R \hat{e}_R \equiv U_R (Z_R)^{\frac{1}{2}} \hat{e}_R, \tag{23b}$$

$$\begin{pmatrix} \nu_L^a \\ 0 \end{pmatrix} \to \begin{pmatrix} (Z_L^l)^{\frac{1}{2}} & 0 \\ 0 & 1 \end{pmatrix} U_L^\nu \mathcal{R}_L^\nu \hat{\nu}_L \equiv U_L^\nu (Z_L^\nu)^{\frac{1}{2}} \hat{\nu}_L, \tag{23c}$$

$$\begin{pmatrix} 0 \\ L_R^0 \end{pmatrix} \to \begin{pmatrix} \mathbb{1} & 0 \\ 0 & (Z_R^L)^{\frac{1}{2}} \end{pmatrix} U_R^\nu \mathcal{R}_R^\nu \hat{\nu}_R \equiv U_R^\nu (Z_R^\nu)^{\frac{1}{2}} \hat{\nu}_R, \tag{23d}$$

where we have introduced the four 5×5 matrices $Z_{L/R}$, $Z^\nu_{L/R}$ which effectively implement the matrix-valued field renormalization of the mass-eigenstate fields. We reiterate that the diagonalization matrices $U_{L/R}$, $U^\nu_{L/R}$ split off on the left always remain of tree-level order.

This completes the setup of the renormalization transformation. In summary, the transformations are given by Eq. (19) for the fundamental parameters (with the constraints (4), (21)) and by Eq. (23) (far right) for the fields. This renormalization transformation is applied to the gauge-eigenstate basis Lagrangian Eq. (3), using the definition of the diagonalization matrices $U^{(\nu)}_{L/R}$ in

Eq. (8). The result is the correct structure of the bare Lagrangian expressed in terms of renormalized fundamental parameters and mass-eigenstate fields. It can be written as

$$\begin{aligned}\mathcal{L} \supset \bar{\hat{e}}^a \Big\{ & i\partial\!\!\!/ \Big[Z_L^{\frac{1}{2}\dagger} Z_L^{\frac{1}{2}} \mathrm{P_L} + Z_R^{\frac{1}{2}\dagger} Z_R^{\frac{1}{2}} \mathrm{P_R} \Big]_{ab} - \Big[Z_R^{\frac{1}{2}\dagger} (m+\delta m)^\dagger Z_L^{\frac{1}{2}} \mathrm{P_L} + Z_L^{\frac{1}{2}\dagger} (m+\delta m) Z_R^{\frac{1}{2}} \mathrm{P_R} \Big]_{ab} \\ & - \tfrac{1}{\sqrt{2}} \Big[Z_R^{\frac{1}{2}\dagger} (Y^h + \delta Y^h)^\dagger Z_L^{\frac{1}{2}} \mathrm{P_L} + Z_L^{\frac{1}{2}\dagger} (Y^h + \delta Y^h) Z_R^{\frac{1}{2}} \Big]_{ab} Z_h h \Big\} \hat{e}^b \\ + \bar{\hat{\nu}}^a \Big\{ & i\partial\!\!\!/ \Big[Z_L^{\nu\frac{1}{2}\dagger} Z_L^{\nu\frac{1}{2}} \mathrm{P_L} + Z_R^{\nu\frac{1}{2}\dagger} Z_R^{\nu\frac{1}{2}} \mathrm{P_R} \Big]_{ab} - (m_L + \delta m_L) \Big[Z_{R,a4}^{\nu\frac{1}{2}\dagger} Z_{L,4b}^{\nu\frac{1}{2}} \mathrm{P_L} + Z_{L,a4}^{\nu\frac{1}{2}\dagger} Z_{R,4b}^{\nu\frac{1}{2}} \mathrm{P_R} \Big] \Big\} \hat{\nu}^b,\end{aligned} \tag{24}$$

which includes a bare counterpart of the tree-level Lagrangian Eq. (10), where the mass and Yukawa matrices effectively renormalize as

$$m_a \delta_{ab} + \delta m_{ab} = m_a \delta_{ab} + \left[U_L^\dagger \begin{pmatrix} \delta(Yv) & \delta M_L \\ \delta M_E^T & \delta(\bar{\lambda} v) \end{pmatrix} U_R \right]_{ab} \tag{25a}$$

$$Y_{ab}^h + \delta Y_{ab}^h = Y_{ab}^h + \left[U_L^\dagger \begin{pmatrix} \delta Y & 0 \\ 0 & \delta\bar{\lambda} \end{pmatrix} U_R \right]_{ab} \tag{25b}$$

We reiterate that the fundamental parameters are taken as the ones of Eq. (3) and the fundamental renormalization constants are the ones of Eq. (19) and Eq. (23) (far right). Other quantities, in particular $U_{L/R}$ and m_a as well as the renormalization constants δm_{ab} and δY^h are derived from these definitions and are given by Eq. (8) and Eq. (25), respectively.

3.2.2 One-loop On-Shell Renormalization

Having established the structure of the counterterm Lagrangian we set out to solve the OS renormalization conditions on the lepton self-energies for the renormalization constants at one-loop order. In terms of the renormalized charged lepton self-energy matrix $\hat{\Sigma}_{ab}$ these conditions read

$$\widetilde{\mathrm{Re}}\hat{\Sigma}_{ab}(p\!\!\!/) u_b(p)\big|_{p^2=m_b^2} = 0, \qquad \lim_{p^2 \to m_a^2} \tfrac{1}{p\!\!\!/ - m_a} \widetilde{\mathrm{Re}}\hat{\Sigma}_{aa}(p\!\!\!/) u_a(p) = 0, \qquad a,b = 1,...,5 \tag{26}$$

where $\widetilde{\mathrm{Re}}$ discards the imaginary parts resulting from the loop integration. To solve for the renormalization constants we first write the renormalized self-energy in terms of the covariant decomposition

$$i\hat{\Sigma}_{ab}(p\!\!\!/) = ip\!\!\!/ \Big(\hat{\Sigma}_{ab}^R(p^2) \mathrm{P_R} + \hat{\Sigma}_{ab}^L(p^2) \mathrm{P_L} \Big) + i \Big(\hat{\Sigma}_{ab}^{SR}(p^2) \mathrm{P_R} + \hat{\Sigma}_{ab}^{SL}(p^2) \mathrm{P_L} \Big), \tag{27}$$

where hermiticity of the (counterterm) Lagrangian relates the coefficient functions by

$$\widetilde{\mathrm{Re}}\hat{\Sigma}_{ab}^R(p^2) = \widetilde{\mathrm{Re}}\hat{\Sigma}_{ba}^{R*}(p^2), \quad \widetilde{\mathrm{Re}}\hat{\Sigma}_{ab}^L(p^2) = \widetilde{\mathrm{Re}}\hat{\Sigma}_{ba}^{L*}(p^2), \quad \widetilde{\mathrm{Re}}\hat{\Sigma}_{ab}^{SR}(p^2) = \widetilde{\mathrm{Re}}\hat{\Sigma}_{ba}^{SL*}(p^2). \tag{28}$$

Next, we expand the charged lepton field renormalization matrices in Eq. (24) to one-loop order

$$Z_{L/R}^{\frac{1}{2}} \approx \mathbb{1} + \tfrac{1}{2} \delta Z_{L/R}. \tag{29}$$

The covariant coefficients in Eq. (27) can now be expressed in terms of the corresponding coefficients of the unrenormalized one-loop self-energy matrix Σ_{ab} and the one-loop counterterm contributions

$$\hat{\Sigma}^R_{ab}(p^2) = \Sigma^R_{ab}(p^2) + \tfrac{1}{2}(\delta Z_R)_{ab} + \tfrac{1}{2}(\delta Z_R^\dagger)_{ab} \tag{30a}$$

$$\hat{\Sigma}^L_{ab}(p^2) = \Sigma^L_{ab}(p^2) + \tfrac{1}{2}(\delta Z_L)_{ab} + \tfrac{1}{2}(\delta Z_L^\dagger)_{ab} \tag{30b}$$

$$\hat{\Sigma}^{SR}_{ab}(p^2) = \Sigma^{SR}_{ab}(p^2) - \tfrac{1}{2}m_a(\delta Z_R)_{ab} - \tfrac{1}{2}(\delta Z_L^\dagger)_{ab}m_b - (\delta m)_{ab} \tag{30c}$$

$$\hat{\Sigma}^{SL}_{ab}(p^2) = \Sigma^{SL}_{ab}(p^2) - \tfrac{1}{2}m_a(\delta Z_L)_{ab} - \tfrac{1}{2}(\delta Z_R^\dagger)_{ab}m_b - (\delta m^\dagger)_{ab}. \tag{30d}$$

Finally, the OS conditions Eq. (26) can be solved and imply the following expression for the renormalization constants of the charged leptons

$$\text{for } a = b \quad \begin{cases} (\delta m)_{aa} &= \frac{1}{2}\,\widetilde{\mathrm{Re}}\left[m_a\Sigma^R_{aa} + m_a\Sigma^L_{aa} + \Sigma^{SR}_{aa} + \Sigma^{SL}_{aa}\right]\Big|_{p^2=m_a^2} \\ (\delta Z_R)_{aa} &= -\widetilde{\mathrm{Re}}\left[\Sigma^R_{aa} + m_a\left(\Sigma^{SR\prime}_{aa} + \Sigma^{SL\prime}_{aa}\right) + m_a^2\left(\Sigma^{R\prime}_{aa} + \Sigma^{L\prime}_{aa}\right)\right]\Big|_{p^2=m_a^2} \\ (\delta Z_L)_{aa} &= -\widetilde{\mathrm{Re}}\left[\Sigma^L_{aa} + m_a\left(\Sigma^{SR\prime}_{aa} + \Sigma^{SL\prime}_{aa}\right) + m_a^2\left(\Sigma^{R\prime}_{aa} + \Sigma^{L\prime}_{aa}\right)\right]\Big|_{p^2=m_a^2} \end{cases} \tag{31a}$$

$$\text{for } a \neq b \quad \begin{cases} (\delta Z_R)_{ab} &= \frac{2}{m_a^2-m_b^2}\Big[m_b^2\widetilde{\mathrm{Re}}\Sigma^R_{ab} + m_a m_b\widetilde{\mathrm{Re}}\Sigma^L_{ab} + m_a\widetilde{\mathrm{Re}}\Sigma^{SR}_{ab} \\ &\qquad + m_b\widetilde{\mathrm{Re}}\Sigma^{SL}_{ab} - m_a(\delta m)_{ab} - m_b(\delta m^\dagger)_{ab}\Big]\Big|_{p^2=m_b^2} \\ (\delta Z_L)_{ab} &= \frac{2}{m_a^2-m_b^2}\Big[m_b^2\widetilde{\mathrm{Re}}\Sigma^L_{ab} + m_a m_b\widetilde{\mathrm{Re}}\Sigma^R_{ab} + m_a\widetilde{\mathrm{Re}}\Sigma^{SL}_{ab} \\ &\qquad + m_b\widetilde{\mathrm{Re}}\Sigma^{SR}_{ab} - m_a(\delta m^\dagger)_{ab} - m_b(\delta m)_{ab}\Big]\Big|_{p^2=m_b^2} \end{cases} \tag{31b}$$

The neutral lepton sector can be treated analogously. The main difference is that the structure of the theory guarantees that there are three massless and one massive neutrino and the mixing is trivial. Hence, the The corresponding conditions on the neutrino self-energies result in

$$\text{for } a = b \quad \begin{cases} \delta m_L &= \frac{1}{2}\,\widetilde{\mathrm{Re}}\left[m_L\Sigma^{\nu R}_{44} + m_L\Sigma^{\nu L}_{44} + \Sigma^{\nu SR}_{44} + \Sigma^{\nu SL}_{44}\right]\Big|_{p^2=m_L^2} \\ (\delta Z^\nu_R)_{aa} &= -\widetilde{\mathrm{Re}}\left[\Sigma^{\nu R}_{aa} + \delta_{a4}m_L\left(\Sigma^{\nu SR\prime}_{aa} + \Sigma^{\nu SL\prime}_{aa} + m_L\Sigma^{\nu R\prime}_{aa} + m_L\Sigma^{\nu L\prime}_{aa}\right)\right]\Big|_{p^2=\delta_{a4}m_L^2} \\ (\delta Z^\nu_L)_{aa} &= -\widetilde{\mathrm{Re}}\left[\Sigma^{\nu L}_{aa} + \delta_{a4}m_L\left(\Sigma^{\nu SR\prime}_{aa} + \Sigma^{\nu SL\prime}_{aa} + m_L\Sigma^{\nu R\prime}_{aa} + m_L\Sigma^{\nu L\prime}_{aa}\right)\right]\Big|_{p^2=\delta_{a4}m_L^2} \end{cases} \tag{32a}$$

$$\text{for } a,b \neq 4 \begin{cases} (\delta Z^\nu_{L/R})_{a4} &= -\frac{2}{m_L}\widetilde{\mathrm{Re}}\left[m_L\Sigma^{\nu L/R}_{a4}(m_L^2) + \Sigma^{\nu SR/SL}_{a4}(m_L^2)\right] \\ (\delta Z^\nu_{L/R})_{4b} &= \frac{2}{m_L}\widetilde{\mathrm{Re}}\Sigma^{\nu SL/SR}_{4b}(0) \end{cases}. \tag{32b}$$

Note that the OS conditions on the off-diagonal neutrino self-energies for $a, b \neq 4$ are fulfilled automatically and do not provide any non-trivial constraints for renormalization constants.

3.2.3 Renormalization of the Fundamental Parameters

The OS conditions listed above do not fix all fundamental renormalization constants of Eq. (19) and Eq. (23) unambiguously. The remaining freedom has to be fixed by imposing additional conditions.

In order to motivate our choice we start by writing the relation Eq. (25) for the diagonal mass renormalization constants as

$$\delta m_{aa} = \sum_{b,c} (U_L^\dagger)_{ab} (U_R)_{ca} \delta M_{bc}, \qquad a = 1, ..., 5, \qquad \delta M = \begin{pmatrix} \delta(Yv) & \delta M_L \\ \delta M_E^{\mathrm{T}} & \delta(\bar{\lambda} v) \end{pmatrix}. \tag{33}$$

The OS conditions fix the left-hand side for all $a = 1 \ldots 5$ via Eq. (31a). In addition, the neutral lepton OS condition (32a) directly fixes δm_L. Hence, 6 of the 13 renormalization constants δm_L, δm_E, δY_{ab} and $\delta\bar{\lambda}$ can be determined from OS conditions but further conditions need to be imposed on the remaining renormalization constants. Some care has to be taken in order to avoid absorbing large corrections of order of the VLL masses $m_{4,5} \gg v$ into e.g. $\delta(\lambda v)$ or $\delta(\bar{\lambda} v)$. Such corrections are instead naturally absorbed by δm_E and δm_L. In principle, this implies we should solve Eq. (33) for δm_L and δm_E in addition to 3 of the Yukawa counterterms. However, since δm_L is already fixed by the neutrino OS conditions we are forced to choose a fourth Yukawa counterterm instead. Fortunately, the difference between δm_L obtained from the neutral lepton OS conditions and from solving Eq. (33), like the difference between m_4 and m_L, is generated by EWSB effects. It is therefore of order v and can safely be absorbed into e.g. $\delta(\lambda v)$ or $\delta(\bar{\lambda} v)$. An obvious choice for the four Yukawa counterterms is δy_i (to allow for the OS renormalized SM limit) and either $\delta\lambda$ or $\delta\bar{\lambda}$. Here we choose $\delta\lambda$, such that Eq. (33) has to be solved for $\delta(Y_{aa} v)$ and δm_E. For simplicity, we fix the remaining renormalization constants in the $\overline{\mathrm{MS}}$ scheme. To summarize, the parameter counterterms are determined in the following way:

- δm_E, $\delta(y_i v)$ and $\delta(\lambda v)$ are fixed via the charged lepton masses by Eq. (33) together with Eq. (31a)
- δm_L is fixed via the neutrino mass by Eq. (32a)
- $\delta\lambda_i^L, \delta\lambda_i^E$ and $\delta\bar{\lambda}$ are fixed by the $\overline{\mathrm{MS}}$ scheme
- δv is fixed by Eq. (16) and the tadpole is fixed by Eq. (17).

This provides sufficient conditions to fix all δM_{ab} and in turn all δm_{ab} and δY_{ab}^h unambiguously. To solve for the set of renormalization constants, we first write Eq. (33) in matrix notation

$$\boldsymbol{\delta}\mathbf{m} = \boldsymbol{\kappa} \cdot \boldsymbol{\delta}\mathbf{M} + \boldsymbol{\delta}\mathbf{u}, \qquad \text{where} \quad \boldsymbol{\delta}\mathbf{M} = \{\delta(y_1 v), \delta(y_2 v), \delta(y_3 v), \delta(\lambda v), \delta m_E\}, \tag{34}$$

with the coefficient matrix $\boldsymbol{\kappa}$ and constant vector $\boldsymbol{\delta}\mathbf{u}$ given by

$$\boldsymbol{\kappa}_{ab} = \begin{cases} (U_L^\dagger)_{ab} (U_R)_{ba} & b = 1, ..., 4 \\ (U_L^\dagger)_{a5} (U_R)_{4a} & b = 5 \end{cases}, \qquad \boldsymbol{\delta}\mathbf{u}_a = (U_L^\dagger)_{a5} (U_R)_{5a} \delta M_{55} + \sum_{\substack{b \neq c \\ b \neq 5}} (U_L^\dagger)_{ab} (U_R)_{ca} \delta M_{bc}. \tag{35}$$

This allows us to write the solution in the following compact form

$$\boldsymbol{\delta}\mathbf{M} = \boldsymbol{\kappa}^{-1} (\boldsymbol{\delta}\mathbf{m} - \boldsymbol{\delta}\mathbf{u}), \qquad \delta Y_{aa} = \tfrac{1}{v} \big(\delta M_{aa} - Y_{aa} \delta v \big). \tag{36}$$

These equations hold separately for the finite and UV divergent parts and can be evaluated numerically or using perturbative expressions for the rotation matrices such as those given in [2]. The remaining $\overline{\mathrm{MS}}$ parameter counterterms are taken from the literature [64, 65].

3.3 Relation to $\overline{\text{MS}}$ and Renormalization Group Running

In the renormalization scheme defined by the renormalization transformations Eq. (19) and Eq. (23) (right), the on-shell conditions Eq. (26) and by the relation Eq. (36), the mass-eigenstate fields are on-shell renormalized and the diagonalized mass matrices Eq. (8) correspond to their respective physical pole masses. However, in this way only a subset of the fundamental parameters are determined by physical on-shell conditions while the rest are defined in the $\overline{\text{MS}}$ scheme with the constraint (21). Here we describe how to determine the values of the $\overline{\text{MS}}$ counterterms and the resulting running of all renormalized parameters. We begin with the well-known β-functions derived for general gauge theories in the $\overline{\text{MS}}$ scheme, see e.g. [65] and references therein. As mentioned in the previous section, these β-functions are derived assuming hermitian field renormalization matrices which resolves the redundancy between field and parameter renormalization. We instead removed this redundancy by imposing Eq. (21) on the parameter counterterms, allowing for general field renormalization matrices. As a result, the renormalization constants including the $\frac{1}{\epsilon}$ poles, and consequently the β-functions, change. We have explicitly computed the required matrices $\delta V_{L/R}$ of Eq. (20) that absorb the off-diagonal parameter counterterms into the field renormalization matrices. The resulting β-functions corresponding to $\overline{\text{MS}}$ with condition Eq. (21) are listed in App. B. These β-functions unambiguously determine the $\overline{\text{MS}}$ renormalization constants $\delta\lambda_i^E$, $\delta\lambda_j^L$, $\delta\bar{\lambda}$ required in our scheme.

The running of parameters in our (partial) OS scheme is further modified by the presence of finite contributions $\delta\Theta_\alpha^{[0]}$ to the renormalization constants $\delta\Theta_\alpha = \frac{1}{\epsilon}\delta\Theta_\alpha^{[1]} + \delta\Theta_\alpha^{[0]}$ for some of the fundamental model parameters collectively denoted by Θ_α. Specifically, for our choices in the previous section, these finite contributions relate the model parameters in our scheme to the ones in the pure $\overline{\text{MS}}$ scheme by

$$m_L^{\overline{\text{MS}}} = m_L + \delta m_L^{[0]}, \qquad m_E^{\overline{\text{MS}}} = m_E + \delta m_E^{[0]}, \qquad Y_{aa}^{\overline{\text{MS}}} = Y_{aa} + \delta Y_{aa}^{[0]} \tag{37}$$

The resulting running of the parameters Θ_α in our scheme is therefore given by

$$\mu\frac{d\Theta_\alpha}{d\mu} \equiv \beta^{\text{eff}}(\Theta_\alpha) \approx \beta(\Theta_\alpha) - \mu\frac{\partial\delta\Theta_\alpha^{[0]}}{\partial\mu} + \mathcal{O}(2L) \tag{38}$$

For purely OS renormalized parameters, like m_L, the effective β-function vanishes. However, because of the mixing between OS and $\overline{\text{MS}}$ parameters when going to the mass basis the parameters, Y_{aa} and m_E necessarily retain some residual μ dependence. We also note that due to the mixing between $\overline{\text{MS}}$ and OS counterterms in Eq. (36) the usual (one-loop) OS relation

$$\mu\frac{\partial\delta\Theta_\alpha^{[0]}}{\partial\mu} = 2\delta\Theta_\alpha^{[1]} \tag{39}$$

does not hold for these parameters.

4 One-Loop Effective Higgs Coupling and Correlation with a_μ

Having established a renormalization scheme that addresses the higher-order mixing between the lepton fields in the VLL model and establishes OS conditions on masses and fields, in this section we test its validity and discuss a first application to the phenomenological impact of higher-order corrections.

In particular, we focus on the effective muon–Higgs coupling $\lambda_{\mu\mu}$ in the muon-specific case, where the VLL couplings to the first and third generation of SM leptons are zero, $\lambda_{1,3}^E = \lambda_{1,3}^L = 0$. The

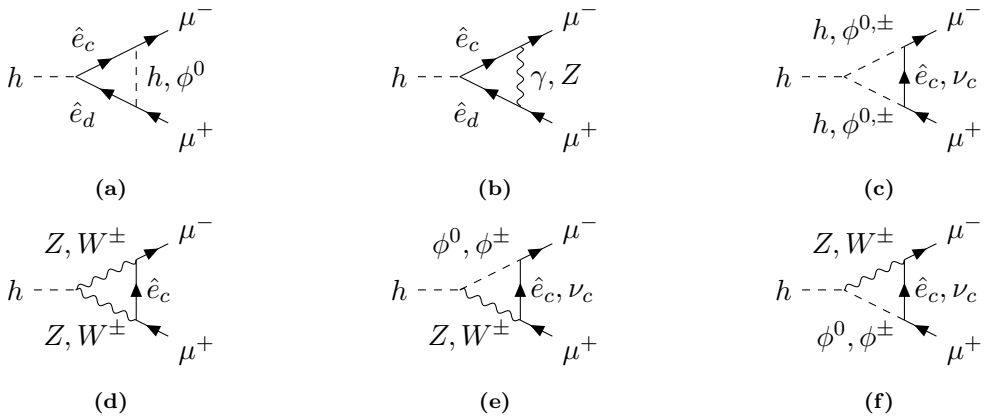


Figure 1: One-loop virtual contributions to $h \to \mu^+\mu^-$ in the VLL model.

effective coupling $\lambda_{\mu\mu}$ is one of the most interesting observables in the VLL model as it receives large chirally enhanced tree-level corrections from the mixing between SM and VL leptons. Furthermore, these corrections are directly correlated with the one-loop VLL contributions to the anomalous magnetic moment a_μ. For this reason the tree-level effective coupling has been studied extensively in the muon-specific VLL model in search for an explanation of the tentative experimental deviation of Δa_μ from the SM prediction [1, 2, 6, 10]. Here, we calculate the one-loop corrections to $\lambda_{\mu\mu}$, and we show how these higher-order contributions can significantly modify the correlation between $\lambda_{\mu\mu}$ and $\Delta a_\mu^{\rm VLL}$ found at tree-level, see Eq. (1).

4.1 Definition and Vertex Renormalization

At tree-level, the effective muon–Higgs coupling $\lambda_{\mu\mu}$ can be directly defined by the partial decay width of an on-shell Higgs into a muon/anti-muon pair

$$\Gamma(h \to \mu^-\mu^+) \approx \frac{M_h}{8\pi^2}|\lambda_{\mu\mu}^{\rm tree}|^2 + \mathcal{O}(1L). \tag{40}$$

Neglecting the muon mass, it is related to the tree-level matrix element by

$$|\lambda_{\mu\mu}^{\rm tree}|^2 = \tfrac{1}{M_h^2}\overline{|\mathcal{M}|^2}(h \to \mu^-\mu^+)_{\rm tree} = (Y_{22}^h)^2. \tag{41}$$

This definition can be extended to the one-loop level in the following simple way. In general, the loop-corrected on-shell matrix element can be written as

$$\mathcal{M}\big(h(q) \to \mu^-(p)\mu^+(p')\big) = -i\bar{u}(p)\Big[\zeta(q^2,p^2,p'^2)^*{\rm P_L} + \zeta(q^2,p^2,p'^2){\rm P_R}\Big]v(p'). \tag{42}$$

At next-to-leading order (NLO) the coefficient function ζ develops an infrared (IR) divergence due to the vertex and self-energy corrections from soft photon loops. This divergence can be factored out into a universal gauge-invariant quantity αB [66],

$$\zeta = \exp(\alpha B)\zeta_0, \tag{43}$$

such that ζ_0 is IR finite, and where (in dimensional regularization)

$$\begin{aligned}\alpha B = \frac{e^2}{16\pi^2}\Big\{&4 + B_0(0,m_\mu^2,m_\mu^2) - B_0(M_h^2,m_\mu^2,m_\mu^2) + 4m_\mu^2 B_0'(m_\mu^2,0,m_\mu^2)\\ &- 2(M_h^2 - 2m_\mu^2)C_0(M_h^2,m_\mu^2,m_\mu^2,m_\mu^2,m_\mu^2,0)\Big\}.\end{aligned} \tag{44}$$

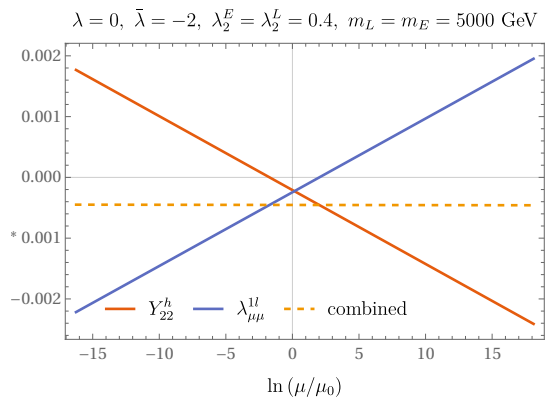


Figure 2: Example of cancellation between the explicit scale dependence of the one-loop contributions to the effective muon–Higgs coupling $\lambda_{\mu\mu}^{1l}(\mu)$ and the (linearized) running of the tree-level effective coupling $Y_{22}^h(\mu)$.

We can thus define an IR finite effective muon–Higgs coupling at the loop-level by[1]

$$\lambda_{\mu\mu} \equiv \sqrt{2}\zeta_0(M_h^2, m_\mu^2, m_\mu^2) = \sqrt{2}\exp(-\alpha B)\zeta(M_h^2, m_\mu^2, m_\mu^2). \tag{45}$$

At one-loop order this expression can be written explicitly as the tree-level value plus the virtual corrections ζ^{1l} from the diagrams depicted in Fig. 1 and the counterterm contribution generated by the Lagrangian Eq. (24).

$$\lambda_{\mu\mu}^{\text{tree}+1l} = Y_{22}^h(1-\alpha B) + \sqrt{2}\zeta^{1l}(M_h^2, m_\mu^2, m_\mu^2) + \delta Y_{22}^h + \tfrac{1}{2}(\delta Z_L^\dagger Y^h + Y^h \delta Z_R + \delta Z_h Y^h)_{22}. \tag{46}$$

To compute the numerical results, the amplitudes for the diagrams in Fig. 1 were obtained using `FeynArts` [67] and `FeynCalc` [68–71] and the appearing loop function were evaluated using `LoopTools` [72]. In `LoopTools` IR divergent loop functions are automatically regulated using a finite photon mass. We have checked that our results are independent of the artificial photon mass within the numerical precision.

Importantly, we have also confirmed numerically that the UV divergences in Eq. (46) cancel. This cancellation is non-trivial since the renormalization constants are defined using a combination of self-energy diagrams and explicit $1/\epsilon$ poles for the $\overline{\text{MS}}$ counterterms derived from the modified β-functions in App. B, while the UV divergences of ζ^{1l} are obtained from the genuine 3-point loop diagrams in Fig. 1. The cancellation thus constitutes a significant check of the renormalization scheme.

4.2 Cancellation of the Renormalization Scale Dependence

A second important check for the validity of the scheme is the cancellation of the renormalization scale dependence in physical observables. As discussed in Sec. 3.3, due to the combination of OS and $\overline{\text{MS}}$ conditions, the fundamental model parameters retain a residual renormalization scale μ-dependence. At the same time the one-loop contributions to the effective Higgs coupling in Eq. (45) depend on the renormalization scale μ explicitly. In total, the explicit and implicit μ-dependences must cancel exactly at the one-loop level.

[1] It would be straightforward to extend this definition to include the real radiation corrections that remove the IR divergences from the physical decay width. However, like the soft singularities, these contributions cancel in the ratio $\lambda_{\mu\mu}/\lambda_{\mu\mu}^{\text{SM}}$ and have no impact on our further analysis.

The implicit μ-dependence is governed by the modified renormalization group equation Eq. (38) which can, in principle, be solved to obtain the one-loop running of the fundamental gauge-eigenstate parameters $Y_{ab}(\mu)$ and $m_E(\mu)$. To check the cancellation of the scale dependence in the physical observable, i.e. in Eq. (45), we are interested in the effect of this running on the effective coupling matrix $Y^h_{22}(\mu)$ in the mass-eigenstate basis. By construction, the diagonalized lepton mass matrices represent the physical pole masses and are independent of μ. However, the effective Yukawa coupling matrix inherits a scale dependence from both the running couplings and the scale dependence of the rotation matrices $U_{L/R}$. In general, solving the RGE and switching to the mass basis results in a complicated non-linear dependence of the coupling matrices on the renormalization scale. Since the cancellation between the implicit and the explicit μ-dependence can only be seen up to the considered loop order, it is useful to truncate the higher-order terms analytically. Working at one-loop order this means we should only consider the linear dependence on $\ln(\mu)$ in both the running couplings and loop corrections. In this approximation, the renormalization scale dependence of the fundamental parameters can be obtained directly from Eq. (38) by integration

$$\Theta_\alpha(\mu) \approx \Theta_\alpha(\mu_0) + \beta^{\text{eff}}(\Theta_\alpha(\mu_0)) \ln\left(\tfrac{\mu}{\mu_0}\right) + \mathcal{O}(2L). \tag{47}$$

From this, the linearized running of the effective couplings in the mass basis can be obtained analytically using the perturbative expressions of the diagonalization matrices given in Ref. [2]

$$Y^h_{22}(\mu) \approx Y^h_{22}(\mu_0) + \frac{2v^2\bar{\lambda}\lambda^E_2\lambda^L_2}{m_4 m_5}\left(\frac{\beta^{\text{eff}}(\bar{\lambda})}{\bar{\lambda}} + \frac{\beta^{\text{eff}}(\lambda^E_2)}{\lambda^E_2} + \frac{\beta^{\text{eff}}(\lambda^L_2)}{\lambda^L_2}\right)\ln\left(\tfrac{\mu}{\mu_0}\right) + \mathcal{O}(v^4). \tag{48}$$

Fig. 2 shows an example of how this linearized running of the tree-level coupling exactly cancels the explicit one-loop logarithms appearing in the renormalized (IR-subtracted) vertex corrections.

4.3 Qualitative Discussion and Muon $g-2$

Having confirmed the validity of the renormalization scheme, we move on to discuss the qualitative impact of the one-loop corrections. In particular we focus on the correlation between $\lambda_{\mu\mu}$ and the VLL contributions to the muon anomalous magnetic moment $\Delta a_\mu^{\text{VLL}}$. For the calculation of $\Delta a_\mu^{\text{VLL}}$, our implementation agrees with the formula presented e.g. in Ref. [2]. For simplicity, we summarize here only the main behaviour, see Eq. (39) in Ref. [2],

$$\Delta a_\mu^{\text{VLL}} \approx -c(m_{L,E})\frac{\bar{\lambda}\lambda^E_2\lambda^L_2 v^3}{m_L m_E m_\mu} \times 22.5 \times 10^{-10}\,, \tag{49}$$

where in our convention the coefficient $c(m_{L,E})$ is approximately $+1$ (-1) for heavy (EW-scale) VLL masses. Unlike in most other models where the chirally enhanced contributions to the effective Higgs coupling and a_μ arise simultaneously at the one-loop level (cf. Ref. [16]), in the VLL model, new contributions to m_μ and to $\lambda_{\mu\mu}$ are already generated at tree-level by the lepton mixing [1–3, 17]. Hence, there is a very peculiar correlation between the effective Higgs coupling $\lambda_{\mu\mu}$ and $\Delta a_\mu^{\text{VLL}}$. For heavy VLL masses $m_{L,E} \gg v$ it can be written as

$$\left|\frac{\lambda_{\mu\mu}}{\lambda^{\text{SM}}_{\mu\mu}}\right|^{\text{tree}} \approx \quad \left|1 - \frac{2\Delta a_\mu^{\text{VLL}}}{22.5\times 10^{-10}}\right|. \tag{50}$$

This equation is the origin of Eq. (1) quoted in the Introduction.

Previous studies [1, 2] focused in particular on the region of small VLL masses $m_{L,E} \sim v$ where $c(m_{L,E}) \approx -1$ and the sign in the correlation changes, such that $\Delta a_\mu^{\text{VLL}}$ of order of the Fermilab

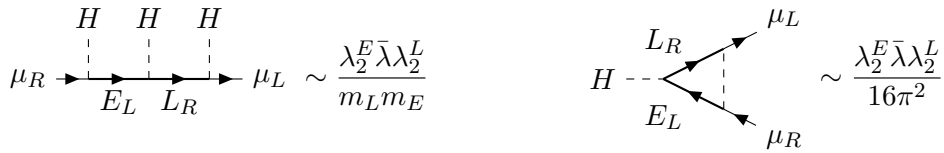


Figure 3: *Left:* matching diagram for $C_{\mu H}$ at tree-level. *Right:* matching diagram for $y_\mu^{\rm EFT}$ at one-loop order. The thick lines represent the heavy internal VLL that are integrated out.

result [20, 21] would require a large deviation $|\lambda_{\mu\mu}/\lambda_{\mu\mu}^{\rm SM}| \sim 3$. After these studies, LHC constraints on the Higgs coupling have become much more stringent, restricting $|\lambda_{\mu\mu}/\lambda_{\mu\mu}^{\rm SM}| \simeq 1$. This has been used as a motivation for extended VLL models, where the correlation Eq. (50) is modified e.g. by couplings to additional scalar particles [4–6]. On the other hand, for large VLL masses the Fermilab result and LHC data are perfectly compatible with Eq. (50) if $\lambda_{\mu\mu}/\lambda_{\mu\mu}^{\rm SM} \simeq -1$. It is therefore of interest to better understand how one-loop corrections to the effective Higgs coupling affect the above correlation.

The effective field theory (EFT) approach used in Refs. [1, 2] to study the correlation due to the lepton mixing also nicely extends to the one-loop level. After integrating out the VLLs, the resulting Lagrangian contains two chirality-flipping terms, a SM-like Yukawa interaction and a dimension-6 operator with three Higgs fields,

$$\mathcal{L}_{EFT} \supset -y_\mu^{\rm EFT} \bar{l}_L^2 \mu_R H - C_{\mu H}\ \bar{l}_L^2 \mu_R H (H^\dagger H) + h.c.\,. \tag{51}$$

Both $y_\mu^{\rm EFT}$ and the Wilson coefficient $C_{\mu H}$ are determined by matching and admit an expansion into tree-level and loop contributions (see Fig. 3)

$$y_\mu^{\rm EFT} = y_\mu + \mathcal{O}\Big(\frac{\bar{\lambda}\lambda_2^E\lambda_2^L}{16\pi^2}\Big), \qquad C_{\mu H} = \frac{\bar{\lambda}\lambda_2^E\lambda_2^L}{m_L m_E}\Big(1+\mathcal{O}(1L)\Big). \tag{52}$$

Both the tree-level matching contribution to $C_{\mu H}$ as well as the one-loop matching contribution to $y_\mu^{\rm EFT}$ are chirally enhanced compared to the fundamental Yukawa coupling y_μ, with the difference that the tree-level contribution is suppressed by $1/(m_L m_E)$ while the one-loop contribution is suppressed by $1/(16\pi^2)$. In the EFT, the muon pole mass m_μ and the physical muon–Higgs coupling $\lambda_{\mu\mu}$ are given by

$$m_\mu = y_\mu^{\rm EFT} v + \quad C_{\mu H} v^3 + \Delta m_\mu^{\rm 1L,EFT}, \tag{53a}$$

$$\lambda_{\mu\mu} = y_\mu^{\rm EFT} \quad + 3 C_{\mu H} v^2 + \Delta\lambda_{\mu\mu}^{\rm 1L,EFT}. \tag{53b}$$

The important factor 3 arises from combinatorics and the last terms correspond to loop corrections within the EFT. Those EFT loop corrections can arise e.g. from loops with gauge or Higgs bosons and cannot contain additional chiral enhancements. Chiral enhancements only appear in the matching coefficients as discussed above. Interestingly, for VLL masses above the TeV scale the one-loop corrections entering via $y_\mu^{\rm EFT}$ can be larger than the tree-level corrections entering via $C_{\mu H}$, which provides further motivation for a one-loop analysis.

Eliminating $y_\mu^{\rm EFT}$ between the previous two equations, using Eqs. (49) and (52) and neglecting higher orders leads to the lowest-order correlation formula (50). Taking into account higher orders, we find that no new chiral enhancements beyond those present at tree-level are introduced by the

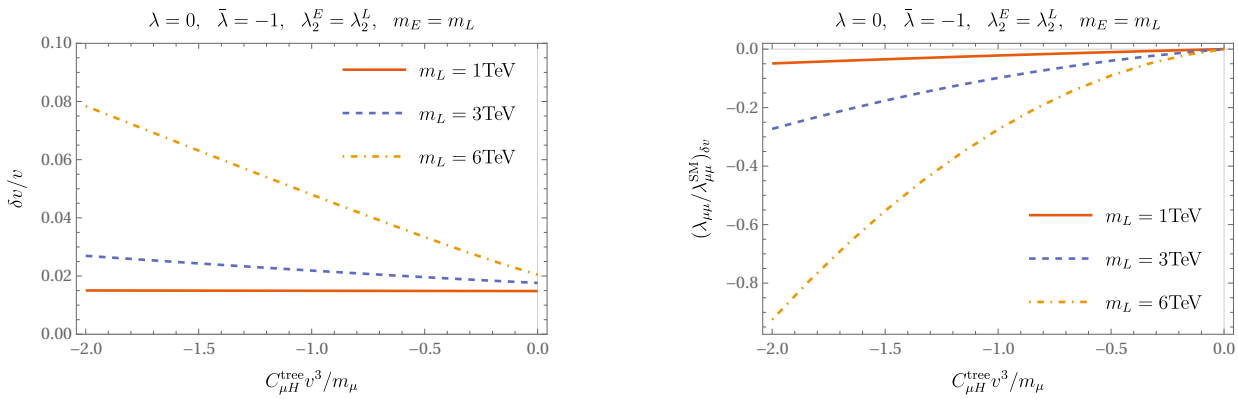


Figure 4: *Left:* Plot of $\delta v/v$ as a function of $C_{\mu H}$ for several fixed values of the VLL masses. *Right:* Plot of the corresponding contribution to $\lambda_{\mu\mu}/\lambda_{\mu\mu}^{\rm SM}$.

loop corrections and the one-loop corrected correlation for large VLL masses is of the form[2]

$$\left|\frac{\lambda_{\mu\mu}}{\lambda_{\mu\mu}^{\rm SM}}\right|^{\rm tree+1L} \approx \quad \left|1 - \frac{2\Delta a_\mu^{\rm VLL}}{22.5\times 10^{-10}} \times \Big(1 + \mathcal{O}(1{\rm L})\Big)\right|. \tag{54}$$

This can also be compared with the analysis of Ref. [16], where generic models without tree-level mixing were considered, such that $C_{\mu H}$ arises only at the one-loop level. Going through similar steps, a correlation like Eq. (54) can be derived, but Δa_μ would only appear multiplied with $\mathcal{O}(1{\rm L})$ effects (i.e. the "1+" term in the round brackets would be absent in Eq. (54)).

Eq. (54) defines our expectation for the behaviour of the explicit one-loop results discussed below. Despite the absence of new chiral enhancements at one-loop order, it turns out that the magnitude of the one-loop corrections can be expected to be large, due to logarithms of the VLL masses and mixing effects. To provide a qualitative estimate, we mention that one particular contribution that exhibits both of these effects is the correction due to the renormalization constants δv and δZ_h. These are obtained from gauge- and Higgs-boson self-energy diagrams, where loops with heavy fermions typically result in large logarithmically enhanced contributions which enter the effective Higgs coupling through the Yukawa counterterm and field renormalization. For example, the leading VLL Yukawa contribution to δv in the limit $m_L \simeq m_E \gg v$ is given by

$$\frac{\delta v}{v}\Big|_{\rm VLL} \simeq \frac{1}{32\pi^2}\Big(\lambda^2 + \bar{\lambda}^2 + (\lambda_2^E)^2 + (\lambda_2^L)^2\Big)\ln\left(\frac{m_L^2}{\bar{\mu}^2}\right) \tag{55}$$

Similarly, the contribution due to the top quark reads

$$\frac{\delta v}{v}\Big|_{m_t} \approx \frac{3y_t^2}{32\pi^2}\left[\ln\left(\frac{m_t^2}{\bar{\mu}^2}\right) + \frac{1-2s_W^2}{2s_W^2}\right]. \tag{56}$$

Both contributions involve potentially large coefficients and a logarithmic scale dependence, such that $\delta v/v$ will always result in large contributions to the effective Higgs coupling, regardless of the choice of renormalization scale.

If the lepton masses were generated purely by the Higgs mechanism as in the SM, only the combination $\delta Z_h/2 - \delta v/v$ would appear in the one-loop effective Higgs coupling (this combination

[2]If other dimension-6 operators besides $\bar{l}^2\mu H^3$ are included, additional mass-suppressed terms can appear independently of $\Delta a_\mu^{\rm VLL}$. In our case none of the possible operators introduce new chiral enhancements and are therefore neglected.

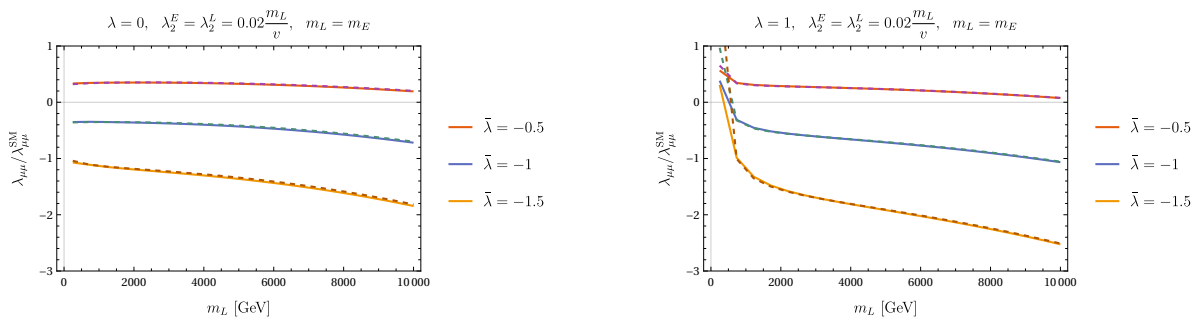


Figure 5: Comparison between the exact result (solid) and fit (dashed) of the one-loop results for $\lambda_{\mu\mu}/\lambda_{\mu\mu}^{\rm SM}$ as a function of m_L. We have set $m_E = m_L$ and $\lambda_2^E = \lambda_2^L = 0.02\frac{m_L}{v}$. The colours correspond to different values of $\bar\lambda$ as indicated, while $\lambda = 0$ (left) and $\lambda = 1$ (right).

corresponds to the quantity studied in Refs. [60, 61]). In this combination the logarithmically enhanced terms cancel and the remaining correction is at most $\mathcal{O}(1\%)$, even if heavy new particles are present. However, in the VLL model additional terms $\propto \delta v$ exist due to the tree-level lepton mixing. This is most directly seen by applying the transformation $v \to v + \delta v$ and the Higgs field renormalization in the tree-level result,

$$\lambda_{\mu\mu}^{\rm tree} = \frac{m_\mu}{v} + 2C_{\mu H}v^2 \quad \Longrightarrow \quad \lambda_{\mu\mu}^{\rm 1L}\big|_{\delta v} = \big(4C_{\mu H}v^2 - \frac{m_\mu}{v}\big)\frac{\delta v}{v} + \lambda_{\mu\mu}^{\rm tree}\tfrac{1}{2}\delta Z_h. \tag{57}$$

Simplifying, this results in the following contribution to the one-loop effective muon–Higgs coupling,

$$\lambda_{\mu\mu}^{\rm 1L}\big|_{\delta v} = \lambda_{\mu\mu}^{\rm tree}(\tfrac{1}{2}\delta Z_h - \tfrac{\delta v}{v}) + 6C_{\mu H}v^2\frac{\delta v}{v}, \tag{58}$$

where the first term is small but the second term is doubly enhanced by the prefactor 6 and the large contributions within δv. In particular in the region where $\lambda_{\mu\mu}/\lambda_{\mu\mu}^{\rm SM} \approx -1$ and the VLL masses are $\gtrsim 1$ TeV, the contribution of $\delta v/v$ can account for several $\mathcal{O}(10\%)$ corrections to the tree-level value. This effect is illustrated in Fig. 4 where the value of δv (left) and corresponding correction to $\lambda_{\mu\mu}$ (right) are plotted for several different VLL masses.

Including the contributions from all loop and counterterm diagrams also beyond δv, the leading logarithmic one-loop VLL Yukawa contribution to the effective Higgs coupling is given by

$$\frac{\lambda_{\mu\mu}}{\lambda_{\mu\mu}^{\rm SM}}\bigg|_{\rm 1l,VLL} \simeq \frac{\lambda_2^E\lambda_2^L\bar\lambda v^3}{32\pi^2 m_L^2 m_\mu}\left[9\Big(\lambda^2 + \bar\lambda^2 + (\lambda_2^E)^2 + (\lambda_2^L)^2\Big) - 12\lambda\bar\lambda\right]\ln\left(\frac{m_L^2}{M_h^2}\right). \tag{59}$$

Note that the terms in the square brackets are always positive and thus lead to an increase of $|\lambda_{\mu\mu}|$ compared to the tree-level result.

4.4 Numerical Results

Finally we present numerical results on the complete one-loop corrections to $\lambda_{\mu\mu}$ in our renormalization scheme. We find good agreement with the qualitative discussion above and determine the numerical impact on the correlation Eq. (54) between $\lambda_{\mu\mu}$ and $\Delta a_\mu^{\rm VLL}$. We note that the updated analysis of Ref. [73] on the EW precision constrains stemming from Z-pole observables translates into the following bounds on λ_2^E and λ_2^L

$$\left|\frac{\lambda_2^L v}{m_L}\right| \lesssim 0.02, \qquad \left(\frac{\lambda_2^E v}{m_E}\right)^2 + 1.16\left(\frac{\lambda_2^L v}{m_L}\right)^2 \lesssim 0.00054. \tag{60}$$

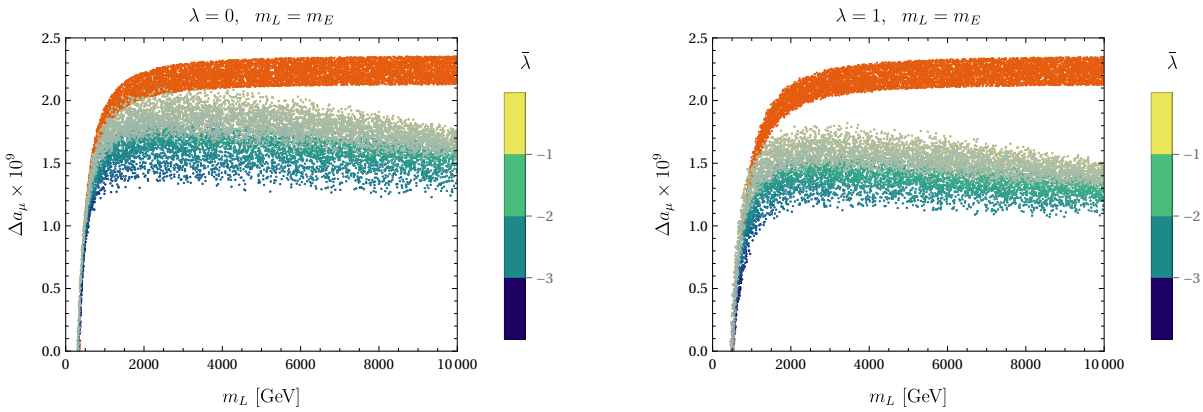


Figure 6: Scatter plot of the correlation between $\Delta a_\mu^{\rm VLL}$ and m_L for $\lambda_{\mu\mu}/\lambda_{\mu\mu}^{\rm SM} = -1 \pm 0.1$. The red points show the results using the tree-level value of $\lambda_{\mu\mu}$ while the one-loop results are colour-coded to highlight the dependence on $\bar{\lambda}$. The greyed-out points violate the EW constraints Eq. (60). The data was obtained from a random scan with $m_L = m_E \in [250, 10000]$ GeV and $\lambda = 0$ (left), $\lambda = 1$ (right).

In particular, in order to achieve $\lambda_{\mu\mu}^{\rm tree} = -\lambda_{\mu\mu}^{\rm SM}$ these bounds require $|\bar{\lambda}| \gtrsim 1.5$.[3]

We have performed a grid and a random scan for $m_L = m_E \in [250, 10000]$ GeV, $\bar{\lambda} \in [-4, 0]$, $\lambda \in [-1, 1]$ and $\lambda_2^{L/E}$ in the range allowed by the above constraints. Using the grid scan data together with an informed ansatz following the discussion in the previous section, we obtain the following approximate numerical formula for the one-loop corrected normalized effective Higgs-coupling

$$\begin{aligned}\frac{\lambda_{\mu\mu}}{\lambda_{\mu\mu}^{SM}} \approx 1 + \frac{\lambda_2^E \lambda_2^L \bar{\lambda} v^3}{m_L^2 m_\mu} \Bigg\{ & 2 + 0.056 - 0.032 \ln\left(\frac{m_L^2}{M_h^2}\right) + \frac{v^2}{m_L^2}\left[4\bar{\lambda}\lambda - (\lambda_2^E)^2 - (\lambda_2^L)^2\right] \\ & + 0.028\left[\bar{\lambda}^2 + \lambda^2 + (\lambda_2^L)^2 + (\lambda_2^E)^2\right] \ln\left(\frac{m_L^2}{M_h^2}\right) - 0.038\lambda\bar{\lambda} \ln\left(\frac{m_L^2}{M_h^2}\right) \\ & - 0.035\left[\bar{\lambda}^2 + \lambda^2 + (\lambda_2^L)^2\right] + 0.033(\lambda_2^E)^2 + 0.013\lambda\bar{\lambda} \Bigg\}.\end{aligned} \tag{61}$$

The "2" in the brackets corresponds to lowest order, all other terms are one-loop corrections. The logarithmically enhanced terms correspond to the analytical result (59). In the fit we have also included $\mathcal{O}(v^2/m_L^2)$ corrections corresponding to the tree-level expansion of Y_{22}^h to improve the convergence at small VLL masses. In Fig. 5 we have plotted a comparison between the numerical fit (dashed) and the results obtained from the full loop calculation and tree-level couplings from the exact diagonalization (solid) as a function of m_L. The plot shows good agreement at large VLL masses $\gtrsim 1000$ GeV, but a deviation at small masses (in particular for $\lambda \neq 0$) which can be attributed to the missing terms of even higher orders in v/m_L.

Finally, in Fig. 6 we have plotted points from the random scan in the m_L–$\Delta a_\mu^{\rm VLL}$ plane, for which $\lambda_{\mu\mu}/\lambda_{\mu\mu}^{\rm SM} = -1 \pm 0.1$. The red points show the results using $\lambda_{\mu\mu}^{\rm tree}$ and thus reflect the correlation Eq. (50) leading to the asymptotic value $\Delta a_\mu^{\rm VLL} \approx 22.5 \times 10^{-10}$. In contrast, including the one-loop results (color-coded according to the value of $\bar{\lambda}$) leads to a significantly lower asymptotic value of $\Delta a_\mu^{\rm VLL}$. This can be understood as a consequence of the logarithmic enhancement of the one-loop corrections. According to Eq. (61), in combination the logarithmic terms can easily increase the effective Higgs coupling by 10–50% compared to the tree-level value, which in turn requires smaller

[3] These constraints are potentially slightly modified by inclusion of the one-loop corrections to the muon–Z coupling which, however, are beyond the scope of this paper.

couplings to fulfil $\lambda_{\mu\mu}/\lambda_{\mu\mu}^{\rm SM} \approx -1$ at one-loop order. Consequently, with the asymptotic behavior of $\Delta a_\mu^{\rm VLL}$ determined by Eq. (49), the one-loop corrections to the correlation result in an overall decrease of the predicted value of the deviation.

5 Conclusion

In models with vector-like leptons, the lepton mass generation mechanism can be strongly modified. Similar to the neutrino seesaw mechanism, SM-like lepton masses can receive significant contributions from mixing with VLL with fundamental gauge-invariant Dirac mass terms. The modified mass generation mechanism is accompanied by potentially large effects in the effective lepton–Higgs couplings and in the anomalous magnetic moment a_μ, which are strongly correlated in a peculiar way.

Here we have started an analysis of higher-order effects in a VLL model where one VLL generation can mix with all SM leptons via additional Yukawa couplings. As the first theoretical step we have developed a renormalization scheme. Our scheme addresses the mixing between the SM and vector-like leptons and implements full on-shell conditions on the lepton self-energies. It thus establishes a direct physical interpretation of mass parameters and fields and allows a transparent computation of observables. The remaining freedom is fixed by imposing $\overline{\rm MS}$ conditions on a judiciously chosen set of fundamental parameters of the gauge-eigenstate Lagrangian. Here we use the redundancy Eq. (4) between parameters and fields to minimize the number of such parameters.

Because of the combination of on-shell and $\overline{\rm MS}$ conditions, there is a residual renormalization-scale dependence of the fundamental model parameters. The corresponding β functions differ from standard ones because of the finite parts of counterterms and because of the use of the redundancy. Both changes have been evaluated. The cancellation of divergences as well as the cancellation of the renormalization-scale dependence in the computation of observables provides a strong check of the scheme and its implementation.

As the first phenomenological application we have calculated the one-loop corrections to the effective muon-Higgs coupling $\lambda_{\mu\mu}$ and its correlation with the VLL contribution to the muon magnetic moment $\Delta a_\mu^{\rm VLL}$. We focused in particular on the interesting region $\lambda_{\mu\mu} \simeq -\lambda_{\mu\mu}^{\rm SM}$, where the tree-level correlation predicts $\Delta a_\mu^{\rm VLL}$ roughly of order of the current tentative experimental deviation. We found that, despite not introducing new chiral enhancements, the one-loop corrections can significantly modify the results found at tree-level and in particular lead to a smaller predicted value of $\Delta a_\mu^{\rm VLL}$. This is mainly due to the logarithmic enhancements present in the one-loop results for $\lambda_{\mu\mu}$, together with the large couplings required to achieve $\lambda_{\mu\mu} \simeq -\lambda_{\mu\mu}^{\rm SM}$.

Quantitatively, the lowest-order correlation Eq. (1) predicts a rigidly fixed, large value for $\Delta a_\mu^{\rm VLL}$ of around 22.5×10^{-10}, but the loop-corrected correlation is incompatible with such large values of $\Delta a_\mu^{\rm VLL}$ and predicts instead a range between around $(10 \ldots 18) \times 10^{-10}$ for masses in the multi-TeV region. It will be interesting to compare this range to future updates of the Fermilab a_μ measurement and the SM theory prediction.

As a further outlook, given the large corrections found here it will be motivated to explore the phenomenological impact of one-loop corrections and the properties of the renormalization scheme in a broader range of observables. The renormalization scheme can be used to evaluate further observables in the lepton/Higgs/EW sector at the loop level, for example to obtain more accurate EW constraints on the model from Z-pole measurements. Moreover, the scheme is not restricted to the flavour-conserving case but also allows to compute e.g. lepton-flavour violating Higgs-, Z-, or muon decays.

Acknowledgments

We acknowledge financial support by the German Research Foundation (DFG) under grant number STO 876/7-2.

A Feynman Rules

Here we list Feynman rules of the VLL model in the mass-eigenstate basis defined in Eq. (7). The SM vertices are taken from Ref. [74]. For the gauge sector we adapt the sign convention from [59] who define the gauge-boson mass eigenstates by

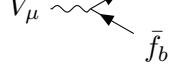
$$\begin{pmatrix} W_\mu^+ \\ W_\mu^- \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{i}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} W_\mu^1 \\ W_\mu^2 \end{pmatrix}, \qquad \begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix} = \begin{pmatrix} c_W & -s_W \\ s_W & c_W \end{pmatrix} \begin{pmatrix} B_\mu \\ W_\mu^3 \end{pmatrix}, \tag{62}$$

and the covariant derivative as

$$D_\mu = \partial_\mu - i\tfrac{g_W}{\sqrt{2}}\left(t^+ W_\mu^+ + t^- W_\mu^-\right) - i\tfrac{g_W}{c_W}(t^3 - Q s_W^2) Z_\mu + ieQA_\mu, \tag{63}$$

where $t^\pm = (t^1 \pm i t^2)$ (for doublets, $t^i = \frac{\sigma^i}{2}$), $Q = t^3 + Y$ and $e = g_Y c_W = g_W s_W$.

Gauge Interactions



$$= i\gamma_\mu \Big(C_R \mathrm{P_R} + C_L \mathrm{P_L} \Big)_{ab}$$

$V\bar{f}f$	$A\,\bar{\hat{e}}^b\hat{e}^a$	$Z\,\bar{\hat{e}}^b\hat{e}^a$	$Z\,\bar{\hat{\nu}}^b\hat{\nu}^a$	$W\bar{\hat{e}}^b\hat{\nu}^a$	$W\bar{\hat{\nu}}^b\hat{e}^a$
C_R^V	$e\delta_{ab}$	$(g_R^Z)_{ab}$	$(g_R^{Z\nu})_{ab}$	$(g_R^W)_{ab}$	$(g_R^W)^*_{ba}$
C_L^V	$e\delta_{ab}$	$(g_L^Z)_{ab}$	$(g_L^{Z\nu})_{ab}$	$(g_L^W)_{ab}$	$(g_L^W)^*_{ba}$

Figure 7: Feynman rules for gauge interactions

where

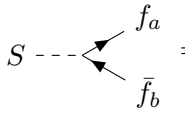
$$g_{L/R}^Z = \tfrac{e}{c_W s_W} U_{L/R}^\dagger \cdot \left(s_W^2 - \tfrac{1}{2} T_{L/R}\right) \cdot U_{L/R}, \tag{64a}$$

$$g_{L/R}^{Z\nu} = \tfrac{e}{c_W s_W} U_{L/R}^{\nu\dagger} \cdot \left(\tfrac{1}{2} T_{L/R}\right) \cdot U_{L/R}^\nu \tag{64b}$$

$$g_{L/R}^W = \tfrac{e}{\sqrt{2} s_W} U_{L/R}^{\nu\dagger} \cdot \left(T_{L/R}\right) \cdot U_{L/R} \tag{64c}$$

with $T_R = \mathrm{diag}(0,0,0,0,1)$ and $T_L = \mathrm{diag}(1,1,1,1,0)$.

Yukawa Interactions



$$= -\tfrac{i}{\sqrt{2}} \Big(C_R^S \mathrm{P_R} + C_L^S \mathrm{P_L} \Big)_{ab}$$

$S\bar{f}f$	$h\,\bar{\hat{e}}^b\hat{e}^a$	$\phi^0\,\bar{\hat{e}}^b\hat{e}^a$	$\phi^+\bar{\hat{e}}^b\hat{\nu}^a$	$\phi^-\bar{\hat{\nu}}^b\hat{e}^a$
C_R^S	Y_{ab}^h	$iY_{ab}^{\phi^0}$	$\sqrt{2}Y_{ab}^{\phi^+}$	$\sqrt{2}Y_{ab}^{\phi^-}$
C_L^S	Y_{ba}^{h*}	$-iY_{ba}^{\phi^0 *}$	$\sqrt{2}Y_{ba}^{\phi^- *}$	$\sqrt{2}Y_{ba}^{\phi^+ *}$

Figure 8: Feynman rules for Yukawa interactions

where

$$Y^h = U_L^\dagger \begin{pmatrix} Y_{ab} & 0 \\ 0 & \bar{\lambda} \end{pmatrix} U_R, \tag{65a}$$

$$Y^{\phi^0} = U_L^\dagger \begin{pmatrix} Y_{ab} & 0 \\ 0 & -\bar{\lambda} \end{pmatrix} U_R, \tag{65b}$$

$$Y^{\phi^+} = U_L^{\nu\dagger} \begin{pmatrix} Y_{ab} & 0 \\ 0 & 0 \end{pmatrix} U_R, \tag{65c}$$

$$Y^{\phi^-} = U_L^\dagger \begin{pmatrix} 0 & 0 \\ 0 & \bar{\lambda} \end{pmatrix} U_R^\nu \tag{65d}$$

B Renormalization Group Equations

Here we list the one-loop β-functions of the model assuming the off-diagonal counterterms $\delta M_L^i, \delta M_E^i$ and δy_{ij} have been absorbed into δV_L and δV_R, see Eq. (21). In general, if we consider a theory with a set of bare parameters $\Theta_\alpha^0 = \mu^{\xi_\alpha \epsilon}(\Theta_\alpha + \delta\Theta_\alpha)$, where the μ-dependent prefactor is introduced to generate a D-dimensional bare Lagrangian while keeping parameters integer-dimensional, the β-functions in the $\overline{\text{MS}}$-scheme are given by

$$\beta(\Theta_\alpha) = -\xi_\alpha \delta\Theta_\alpha^{[1]} + \sum_\beta \frac{\partial \delta\Theta_\alpha^{[1]}}{\partial \Theta_\beta} \xi_\beta \Theta_\beta \tag{66}$$

where $\delta\Theta_\alpha^{[1]}$ denotes the coefficient of the $1/\epsilon$ pole in $\delta\Theta_\alpha$. Using this relationship and using the abbreviations $g_W = e/s_W$, $g_Y = e/c_W$ and

$$\delta\Upsilon = 2\bar{\lambda}^2 + 2\lambda^2 + 2\sum_j \left((\lambda_j^E)^2 + (\lambda_j^L)^2 + y_j^2 + 3(y_j^u)^2 + 3(y_j^d)^2 \right) - \frac{9}{2} g_W^2 - \frac{15}{2} g_Y^2, \tag{67}$$

the Yukawa β-functions are given by

$$(32\pi^2)\,\beta(y_i) = \; y_i\left[\delta\Upsilon + 3y_i^2 + 2(\lambda_i^L)^2 + (\lambda_i^E)^2\right] - 4\lambda_i^E\lambda_i^L\bar{\lambda}\frac{m_E^2+2m_L^2}{m_E m_L} \tag{68a}$$

$$\begin{aligned}(32\pi^2)\beta(\lambda_i^L) = \lambda_i^L&\left[d\Upsilon + 3(\lambda_i^L)^2 + 4y_i^2 + \lambda\Big(\lambda - \bar{\lambda}\tfrac{8m_L}{m_E}\Big)\right] + 2\lambda_i^E y_i\left[\lambda + \bar{\lambda}\tfrac{2m_E}{m_L}\right]\\ &+ \sum_{k\neq i}\tfrac{\lambda_k^L}{y_i^2-y_k^2}\left[\lambda_i^L\lambda_k^L\Big(5y_i^2 - y_k^2\Big) + 2\lambda_i^E\lambda_k^E y_i y_k - 4\bar{\lambda}\Big(\lambda_i^L\lambda_k^E y_k + \lambda_i^E\lambda_k^L y_i\Big)\tfrac{m_E^2+2m_L^2}{m_E m_L}\right]\end{aligned} \tag{68b}$$

$$\begin{aligned}(32\pi^2)\beta(\lambda_i^E) = \lambda_i^E&\left[d\Upsilon + 3(\lambda_i^E)^2 + 5y_i^2 + 2\lambda\Big(\lambda - \bar{\lambda}\tfrac{2m_E}{m_L}\Big)\right] + 4\lambda_i^L y_i\left[\lambda + \bar{\lambda}\tfrac{2m_L}{m_E}\right]\\ &+ \sum_{k\neq i}\tfrac{\lambda_k^E}{y_i^2-y_k^2}\left[\lambda_i^E\lambda_k^E\Big(4y_i^2 - 2y_k^2\Big) + 4\lambda_i^L\lambda_k^L y_i y_k - 4\bar{\lambda}\Big(\lambda_i^L\lambda_k^E y_i + \lambda_i^E\lambda_k^L y_k\Big)\tfrac{m_E^2+2m_L^2}{m_E m_L}\right]\end{aligned} \tag{68c}$$

$$(32\pi^2)\;\beta(\lambda) = \lambda\;\left[\delta\Upsilon + 3\lambda^2 + \sum_i\Big(4(\lambda_i^E)^2 + 5(\lambda_i^L)^2\Big)\right] + \sum_i\left[6y_i\lambda_i^E\lambda_i^L + 4\bar{\lambda}\tfrac{m_E^2(\lambda_i^E)^2+2m_L^2(\lambda_i^L)^2}{m_E m_L}\right] \tag{68d}$$

$$(32\pi^2)\;\beta(\bar{\lambda}) = \bar{\lambda}\;\left[\delta\Upsilon + 3\bar{\lambda}^2\right]. \tag{68e}$$

Similarly, the β-functions of the Dirac masses are given by

$$(32\pi^2)\beta(m_L) = 4\bar{\lambda}\lambda m_E + \Big(\lambda^2 + \bar{\lambda}^2 + \sum_j(\lambda_j^L)^2 - 9g_W^2 - 3g_Y^2\Big)m_L \tag{69a}$$

$$(16\pi^2)\beta(m_E) = 4\bar{\lambda}\lambda m_L + \Big(\lambda^2 + \bar{\lambda}^2 + \sum_j(\lambda_j^E)^2 - 6g_Y^2\Big)m_E. \tag{69b}$$

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