

Second Order Post Newtonian Equations of Light Propagation in Multiple Systems

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Abstract

The first order post Newtonian scheme in multiple systems presented by Damour-Soffel-Xu is extended to the second order one for light propagation without changing the advantage of the scheme on the linear partial differential equations of potential and vector potential. The spatial components of the metric tensor are extended to the second order level both in the global coordinates (q_{ij}/c^4 term) and in a local coordinates (Q_{ab}/c^4 term). The equations of q_{ij} (or Q_{ab}) are deduced from Einstein field equations. The linear relationship between q_{ij} and Q_{ab} are presented also. The 2PN equations of light ray based on the extended scheme are deduced by means of the iterative method. We also use parametrized second post Newtonian metric tensor to substitute into the null geodetic equations to obtain the parametrized second order equations of light ray which might be useful in the observation and measurement in the future space missions.

Keywords 2PN Approximate Method, Light Propagation, Multiple Systems

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1 Introduction

Recently, in terms of advanced technology a series of space missions are proposed and planned to launch, e.g. LISA (Laser Interferometer Space Antenna) [1], aimed to detect low-frequency (10^{-4} -1 Hz) gravitational wave with a strain sensitivity of $4 \times 10^{-21}/(Hz)^{1/2}$, GAIA (Global Astrometric Interferometer for Astrophysics) [2] planned to measure the position and the parallaxes of celestial bodies with the precision of few μas (micro arc second), ASTROD [3] and ASTROD-I [4] (Astro-dynamical Space Test of Relativity using Optical Devices) in a lower frequency range and higher sensitivity for the gravitational wave detection compared to LISA, LATOR (Laser Astrometric Test Of Relativity) to measure the bending of light near the sun to an accuracy of $0.02 \mu\text{as}$ [5] and so on. In such a requirement of experimental precision the second order post Newtonian contribution to light propagation has to be taken into account. On other hand, when the light pass nearby the sun or any planet, the potential produced by 2^N multipole moment is $V_{\text{mono}}(l/r)^N$ [6], where V_{mono} is the potential created by the mass monopole, l the length deviated to monopole and r the distance of the action. For the quadrupole of the sun ($N = 2$) $(l/r)^2 < \epsilon^2$, where ϵ^2 is the small parameter of post Newtonian expansion (nearby the sun $\epsilon^2 \sim 10^{-6}$), l can be estimated from the oblateness of the sun [7]. Therefore if we consider the second order post Newtonian (2PN) problem, the first post Newtonian (1PN) quadrupole has to be taken into account. Because the trajectory of light ray normally is calculated in the global coordinates, but the relativistic multipole moments have to be computed in the local coordinates. In 1PN level, the problem has been thoroughly solved in the scheme presented by Damour-Soffel-Xu [8, 9, 10, 11], in which a complete 1PN general relativistic celestial mechanics for N arbitrarily composed and shaped, rotating deformable bodies is described. Their scheme is widely abbreviated as the DSX scheme [12]. A similar results has been deduced by means of a different way: a surface integral derivation [13].

As for light propagation, we should also mention the other methods. The method of Lorentz covariant theory employed to study the problems related to the propagation of light ray in a gravitational field is based on the solution of the null geodesic equations by means of the first Minkowskian approximation [14, 15]. In the second order approximation the Lorentz covariant theory has to be extended also. Numerical simulation is also an important tool to study the light propagation in the gravitational field of the moving bodies [16]. A more rigorous approaches (e.g. the second post Newtonian (2PN) scheme) might be helpful for such kind of simulations.

The 2PN contribution for light ray has been discussed for a long time (early 80's) by Epstein and Shapiro [17], Richter and Matzner [18], and by others (later). But all of them consider only in one global coordinate system, therefore they can not calculate the relativistic contribution from multipole moments which should be calculated in the local coordinates. As we know the relativistic theory of reference system established only after 1991^[10], although it is only 1PN approximation.

In this paper we will extend the 1PN DSX scheme to 2PN for the discussion on the propagation of light ray, i.e. we will extend the metric g_{ij} in global coordinates (and G_{ab} in local coordinates) to $O(6)$ and deduce the corresponding equations from Einstein field equation. In the section 3, we deduce the second order post Newtonian equation of light ray by means of the iterative method. In the section 4, we discuss the parametrized second post Newtonian formalism (PP²N) which is a tentatively expansion of Ref.[19] and the corresponding equations of 1PN light ray. Some conclusion remark is made in Sec. 5.

2 The extension of DSX scheme

In this section, we will extend 1PN DSX scheme to 2PN scheme for discussion on the second order contribution in light propagation. Our symbols and signature follow the DSX scheme. Here we summarize the notation in DSX paper which we will use in this paper. The signature $-+++$ is taken; spacetime indices go from 0 to 3 and denoted by Greek indices, while space indices (1 to 3) are denoted by Latin indices. We use Einstein's summation convection for both types of indices, whatever the position of repeated indices. The flat metric is denoted by $f_{\mu\nu}$, with components $\text{diag}(-1,+1,+1,+1)$ in Lorentzian coordinates. In post Newtonian expansions we shall often abbreviate the order symbol $O(c^{-n})$ simply by $O(n)$. The "global" (or "common view") coordinates used for describing the overall dynamics of the system will be denoted by $(x^\mu) \equiv (ct, x^i)$. By contrast, each of

the ‘‘local’’ coordinate systems A , used for describing the internal dynamics of each body, will be denoted by $(X_A^\alpha) \equiv (cT_A, x_A^a)$. We distinguish the second part of the Latin alphabet (i,j,k, ...) for global space coordinates from the first part of the Latin alphabet (a,b,c, ...) for local space coordinates as done in the DSX scheme. A spatial multi-index containing l indices is simply denoted L (and K for k indices, etc.), i.e., $L \equiv i_1 i_2 \dots i_l$. A multi-summation is always understood for repeated multi-index $S_L T_L = \Sigma_{i_1 \dots i_l} S_{i_1 \dots i_l} T_{i_1 \dots i_l}$. Given a spatial vector, v^i , its l th tensorial power is denoted by $v^L \equiv v^{i_1} v^{i_2} \dots v^{i_l}$. Also, $\partial_L \equiv \partial_{i_1} \dots \partial_{i_l}$. The symmetric and trace-free (STF) part of a spatial tensor will be denoted by angular brackets (or by a caret when no ambiguity arises): $STF_{i_1 \dots i_l}(T_{i_1 \dots i_l}) \equiv T_{(i_1 \dots i_l)} \equiv \hat{T}_L$. The coordinate transformation between the local coordinate system X_A^α and the global one x^μ reads (omitting a labelling index A on all quantities pertaining to the local frame):

$$x^\mu = f^\mu(X^a) = z^\mu(X^0) + e_a^\mu(X^0)Y^a(X^0, X^b) + \xi^\mu$$

where

$$Y^a(X^0, X^b) \equiv X^a + (1/c^2)[(1/2)A^a(X^2) - X^a(A_b X^b)].$$

Moreover, one has the definitions

$$A_a(S) = f_{\mu\nu} e_a^\mu(S) \frac{d^2 z^\nu}{d\tau^2}, \quad v^i = c \frac{dz^i}{dz^0}, \quad e_0^i = c^{-1} e_0^0 v^i, \quad e_a^i(S) = e_a^i \frac{dz^i}{dS} + O(4) \quad (1)$$

and

$$e_0^0(S) = 1 + \frac{1}{c^2} \left(\frac{1}{2} v^2 + \bar{w} \right) + \frac{1}{c^4} \left[\frac{3}{8} v^4 + \frac{1}{2} (\bar{w})^2 + \frac{5}{2} \bar{w} v^2 - 4 \bar{w}_i v^i \right] + O(6), \quad (2)$$

$$e_a^0(S) = R_a^i \left\{ \frac{v^i}{c} \left[1 + \frac{1}{c^2} \left(\frac{1}{2} v^2 + 3\bar{w} \right) \right] - \frac{4}{c^4} \bar{w}_i \right\} + O(5), \quad (3)$$

$$e_a^i(S) = \left[1 - \frac{1}{c^2} \bar{w} \right] \left[\delta^{ij} + \frac{1}{2c^2} v^i v^j \right] R_a^i + O(4), \quad (4)$$

where $R_a^i(S)$ is a slowly changing rotation matrix $R_a^i R_a^j = \delta^{ij}$, $R_a^i R_b^i = \delta^{ab}$, $cdR_a^i/dS = O(2)$. All of above three equations are already shown in Eq.(5.21b)–(5.21d) of Ref.[8], as for Eq.(4) will be extended to $O(6)$ later (see Eq.(51)). Now, in this paper, a fixed-star coordinates has been chosen, therefore $R_a^i = \delta_a^i$, z^μ represents each central world line $\mathcal{L}_A(X^a = 0)$. \bar{w} is the external potential of body A, v^i is the coordinate three-velocity of the central world line \mathcal{L}_A measured in the global coordinate system, and V^a defined by $v^i = R_a^i V^a$ or $V^a = R_i^a v^i$.

As we mentioned early, DSX scheme is the first post Newtonian approximation for particle motion. The metric tensor in a global coordinates is written in the form

$$g_{00} = -\exp\left(-\frac{2w}{c^2}\right) + O(6), \quad (5)$$

$$g_{0i} = -\frac{4w_i}{c^3} + O(5), \quad (6)$$

$$g_{ij} = \delta_{ij} \exp\left(\frac{2w}{c^2}\right) + O(4), \quad (7)$$

and they satisfy the conformal isotropic condition

$$g_{00}g_{ij} = -\delta_{ij} + O(4). \quad (8)$$

The metric tensor in the local coordinates A has a similar form as in global, one only needs to change the small letters into capital letters

$$G_{00} = -\exp\left(-\frac{2W}{c^2}\right) + O(6), \quad (9)$$

$$G_{0a} = -\frac{4W_a}{c^3} + O(5), \quad (10)$$

$$G_{ab} = \delta_{ab} \exp\left(\frac{2W}{c^2}\right) + O(4), \quad (11)$$

and they also satisfy the conformal isotropic condition

$$G_{00}G_{ab} = -\delta_{ab} + O(4). \quad (12)$$

The potential W and vector potential W_a for body A can be divided into two parts (self part W^+ , W_a^+ and external part \overline{W} , \overline{W}_a)

$$W = W^+ + \overline{W}, \quad (13)$$

$$W_a = W_a^+ + \overline{W}_a. \quad (14)$$

As in DSX scheme, we use W_α to represent W ($\alpha = 0$) and W_a ($\alpha = a$). From Einstein field equation, we obtain equations to be satisfied by W and W_a (compare Eq.(4.3a), (4.3b) and (4.4) of Ref.[8])

$$\square_X W + \frac{4}{c^2} \partial_T (\partial_T W + \partial_b W_b) = -4\pi G \Sigma + O(4), \quad (15)$$

$$\Delta_X W_a - \partial_a (\partial_T W + \partial_b W_b) = -4\pi G \Sigma^a + O(2), \quad (16)$$

where $\square_X = \Delta_X - c^{-2} \partial_T^2$ with $\Delta_X = \partial^2 / \partial X^a \partial X^a$, and where the source terms

$$\Sigma^a \equiv (\Sigma, \Sigma^a) \equiv \left| \frac{T^{00} + T^{aa}}{c^2}, \frac{T^{0a}}{c} \right| \quad (17)$$

are now defined by components of the stress-energy tensor in the X^a coordinate system.

In harmonic gauge

$$\partial_T W + \partial_a W_a = 0. \quad (18)$$

We have (see Eq.(4.51) of Ref.[8])

$$\square_X W_\alpha^+ = -4\pi G \Sigma_\alpha. \quad (19)$$

By means of Blanchet-Damour multipole moments [20] we obtain the solutions

$$W^{+A}(T, \mathbf{X}) = G \sum_{l \geq 0} \frac{(-1)^l}{l!} \partial_L [R^{-1} M_L^A(T \pm R/c)] + \frac{1}{c^2} \partial_T \Lambda^A + O(4), \quad (20)$$

$$W_a^{+A}(T, \mathbf{X}) = -G \sum_{l \geq 1} \frac{(-1)^l}{l!} \left[\partial_{L-1} \left(R^{-1} \frac{d}{dT} M_{aL-1}^A(T \pm R/c) \right) + \frac{l}{l+1} \epsilon_{abc} \partial_{bL-1} [R^{-1} S_{cL-1}^A(T \pm R/c)] \right] - \frac{1}{4} \partial_a \Lambda^A + O(2), \quad (21)$$

where

$$\Lambda^A \equiv 4\pi \sum_{l \geq 0} \frac{(-1)^l}{(l+1)!} \frac{2l+1}{2l+3} \partial_L [R^{-1} \mu_L^A(T \pm R/c)], \quad (22)$$

$$\mu_L^A(T) \equiv \int_A d^3 X \hat{X}^{bL} \Sigma^b(T, \mathbf{X}). \quad (23)$$

In Eqs.(20)–(23) the \pm sign in a function of the local time, T , divided by the coordinate distance to the origin of the local system, $R \equiv |\mathbf{X}|$, denotes the average

$$\frac{F(T \pm R/c)}{R} \equiv \frac{1}{2} \left(\frac{F(T - R/c)}{R} + \frac{F(T + R/c)}{R} \right), \quad (24)$$

which is the well-known time-symmetric solution, with spherical symmetry, of the wave equation.

The “mass” (M_L^A) and “spin” (S_L^A) multipole moments of body A appearing in Eqs.(20), (21) are the STF Cartesian tensors defined by the same expressions of the matter distribution variables [20] for closed

gravitationally self-interacting system, but now restricted to an integration over the volume of body A, using the local-system matter variables Eq.(17): i.e.,

$$M_L^A(T) \equiv \int_A d^3X \hat{X}^L \Sigma(T, \mathbf{X}) + \frac{1}{2(2l+3)c^2} \frac{d^2}{dT^2} \left(\int_A d^3X \hat{X}^L \mathbf{X}^2 \Sigma(T, \mathbf{X}) \right) - \frac{4(2l+1)}{(l+1)(2l+3)c^2} \frac{d}{dT} \left(\int_A d^3X \hat{X}^a \Sigma^a(T, \mathbf{X}) \right) + O(4) \quad (l \geq 0), \quad (25)$$

$$S_L^A(T) \equiv \int_A d^3X \epsilon^{ab<c_l} \hat{X}^{L-1>a} \Sigma^b(T, \mathbf{X}) + O(2) \quad (l \geq 1). \quad (26)$$

In other hand, from the definition of the external part in the global coordinates, we have in the global coordinates for N bodies system

$$\bar{w}_\mu^A = \sum_{B \neq A}^N w_\mu^B, \quad (27)$$

where w_μ^B is the four potentials of the body B. From now on we will omit the labelling index A on all quantities pertaining to the local frame. Based on \bar{w}_μ , we can calculate \bar{W}_α with the following equations

$$w = \left(1 + \frac{2V^2}{c^2} \right) W + \frac{4}{c^2} V^a W_a + \frac{c^2}{2} \ln(A_0^0 A_0^0 - A_a^0 A_a^0) + O(4), \quad (28)$$

$$w_i = v^i W + R_a^i W_a + \frac{c^3}{4} (A_0^0 A_0^i - A_a^0 A_a^i) + O(2). \quad (29)$$

Then from some calculation we can get \bar{W} and \bar{W}_a also. Up to now we have briefly reviewed the main story of DSX scheme. But as we mentioned early that DSX scheme is 1PN approximation method (for the equations of motion of bodies). If we want to extend DSX scheme to apply to the second order post Newtonian (2PN) approximation of light propagation, we have to extend the metric g_{ij} to $O(6)$ level rather than $O(4)$ [21], i.e.

$$G_{00} = -\exp\left(-\frac{2W}{c^2}\right) + O(6), \quad (30)$$

$$G_{0a} = -\frac{4W_a}{c^3} + O(5), \quad (31)$$

$$G_{ab} = \delta_{ab} \exp\left(\frac{2W}{c^2}\right) + \frac{Q_{ab}}{c^4} + O(6). \quad (32)$$

The contravariant metric tensor reads

$$G^{00} = -\exp\left(\frac{2W}{c^2}\right) + O(6), \quad (33)$$

$$G^{0a} = -\frac{4W_a}{c^3} + O(5), \quad (34)$$

$$G^{ab} = \delta_{ab} \exp\left(-\frac{2W}{c^2}\right) - \frac{Q_{ab}}{c^4} + O(6). \quad (35)$$

The metric tensor in the global coordinate system has a similar form, just change $G_{\alpha\beta}$, W , W_a and Q_{ab} by $g_{\mu\nu}$, w , w_i and q_{ij} .

The relations between w , w_i , q_{ij} and W , W_a , Q_{ab} will be studied in the following. In DSX scheme, since the metric in the global coordinate system as well as in every local coordinate systems has the similar form, so that, if we deduce any equation in one local coordinate system, then we have it in every local coordinates as well as in the global coordinates. Now in this paper, the situation is similar, Therefore we need only to treat in a local coordinate system A, but we omit the index A always. The spatial conformal isotropic condition Eq.(12) is revised as

$$G_{ab} G_{00} = -\delta_{ab} - \frac{Q_{ab}}{c^4} + O(6). \quad (36)$$

If we attribute Q_{ab}/c^4 to $O(4)$, it is just the form in DSX scheme. Therefore Eq.(32) is an extension of Eq.(7). Q_{ab} also can be taken as a spatial anisotropic contribution in second order (see Eq.(36)).

The metric tensor G_{0a} is taken as $O(3)$ here, in fact it is only as small as $O(4)$ because of the relatively slow rotation rate in the sun [18, 22]. To keep the uniformity with DSX scheme therefore we take such formula of G_{0a} as well as for g_{0i} .

We have to calculate the equations satisfied by W , W_a and Q_{ab} from the Einstein field equations. On the first step, the Christoffel symbols can be calculated from the metric tensor

$$\begin{aligned}
\Gamma_{00}^0 &= -\frac{W_{,t}}{c^3} + O(5), \\
\Gamma_{0a}^0 &= -\frac{W_{,a}}{c^2} + O(6), \\
\Gamma_{00}^a &= -\frac{W_{,a}}{c^2} + \frac{4WW_{,a}}{c^4} - \frac{4W_{a,t}}{c^4} + O(6), \\
\Gamma_{ab}^0 &= \delta_{ab}\frac{W_{,t}}{c^3} + \frac{4}{c^3}W_{(a,b)} + O(5), \\
\Gamma_{0b}^a &= -\frac{4}{c^3}W_{[a,b]} + \frac{W_{,t}}{c^3}\delta_{ab} + O(5), \\
\Gamma_{bc}^a &= \frac{1}{c^2}(\delta_{ab}W_{,c} + \delta_{ac}W_{,b} - \delta_{bc}W_{,a}) + \frac{1}{2c^4}(Q_{ab,c} + Q_{ac,b} - Q_{bc,a}) + O(6),
\end{aligned} \tag{37}$$

where two indices enclosed in a parentheses is denoted as symmetrization, and in a square bracket means antisymmetrization.

Then we can deduce the Ricci tensor

$$R^{00} = -\frac{\nabla^2 W}{c^2} - \frac{1}{c^4}(3\partial_t\partial_t W + 4\partial_t\partial_d W_d) + O(6), \tag{38}$$

$$R^{0a} = -\frac{2}{c^3}(\nabla^2 W_a - \partial_a\partial_d W_d - \partial_t\partial_a W) + O(5), \tag{39}$$

$$\begin{aligned}
R^{ab} &= -\frac{1}{c^2}\delta_{ab}\nabla^2 W + \frac{1}{c^4}[4\delta_{ab}W\nabla^2 W + \delta_{ab}W_{,tt} + 4W_{(a,b),t} - 2W_{,a}W_{,b} \\
&\quad + \frac{1}{2}(Q_{ad,bd} + Q_{bd,ad} - Q_{ab,dd} - Q_{dd,ab})] + O(6),
\end{aligned} \tag{40}$$

and the scalar curvature

$$R = -\frac{2}{c^2}\nabla^2 W + \frac{1}{c^4}(4W\nabla^2 W + 6W_{,tt} + 8W_{a,at} - 2W_{,a}W_{,a} + Q_{ab,ab} - Q_{aa,bb}) + O(6). \tag{41}$$

From Einstein field equation we can derive equations of W , W_a and Q_{ab}

$$\nabla^2 W + \frac{1}{c^2}(3W_{,tt} + 4\partial_t\partial_a W_a) = -4\pi G\Sigma + O(4), \tag{42}$$

$$\nabla^2 W_a - \partial_a\partial_b W_b - \partial_t\partial_a W = -4\pi G\Sigma^a + O(2), \tag{43}$$

$$\begin{aligned}
&-2\delta_{ab}W_{,tt} + 4(W_{(a,b),t} - \delta_{ab}W_{(d,d),t}) - 2W_{,a}W_{,b} + \delta_{ab}W_{,d}W_{,d} \\
&\quad + \frac{1}{2}(Q_{ad,bd} + Q_{bd,ad} - Q_{ab,dd} - Q_{dd,ab} - \delta_{ab}Q_{dc,dc} + \delta_{ab}Q_{dd,cc}) = 8\pi GT^{ab},
\end{aligned} \tag{44}$$

where $\Sigma = (T^{00} + T^{aa})/c^2$ and $\Sigma^a = T^{0a}/c$ (the same definition in DSX scheme). Eq.(42) and (43) are linear PDE, which can be solved in certain suitable gauge conditions, then Eq.(44) will be solved also. We shall point out that, T^{ab} can be expressed by Σ and Σ^a (to see Eq.(A13) of Ref.[23]). Therefore it is self-consistent within the framework of DSX scheme. Similar field equations in the global coordinate system can be obtained easily by substituting capital letters by small letters.

From Eqs.(42) and (43) we see that Q_{ab} does not appear in the field equations of W and W_a , and they are the same as ones in DSX scheme (keeping linear equations). The solutions of W and W_a related to relativistic multipole moments are still valid as before in DSX scheme.

Now, we shall discuss the transformation relations of the potential, the vector potential and Q_{ab} (or q_{ij}) from the global coordinate system to a local coordinate system and vice versa. The coordinate transformation law reads

$$g^{\mu\nu} = \frac{\partial x^\mu}{\partial X^\alpha} \frac{\partial x^\nu}{\partial X^\beta} G^{\alpha\beta}. \quad (45)$$

Considering g^{00} , we obtain

$$w = \left(1 + \frac{2V^2}{c^2}\right) W + \frac{4}{c^2} V^a W_a + \frac{c^2}{2} \ln(A_0^0 A_0^0 - A_a^0 A_a^0) + O(4), \quad (46)$$

where $A_a^\mu = \partial x^\mu / \partial X^a$, $V^a = R_i^a v^i$ (R_i^a is a slowly changing rotation matrix and v^i is the coordinate three-velocity of the central world line measured in the global coordinate system).

For g^{0i} , it turns out to be

$$w_i = v^i W + R_a^i W_a + \frac{c^3}{4} (A_0^0 A_0^i - A_a^0 A_a^i) + O(2). \quad (47)$$

In Eqs.(46) and (47), Q_{ij} does not enter these equations, and the relations between w , w_i and W , W_a are the same as in DSX scheme (to keep linear relation). As we know, all of the discussion about the theory of reference systems are based on the linear PDE of potentials (Eq.(42), (43)) and linear relation between potentials in the global coordinates and in the local coordinates ((46) and (47)). Since these equations are the same as before (in DSX scheme), the theory of reference system in DSX scheme (for potential and vector potential) is valid in this paper. Also in Eqs.(42), (43), (46) and (47) Q_{ab} does not appear, therefore self-parts W^+ , W_a^+ and external parts \bar{W} , \bar{W}_a can be obtained just like in the DSX scheme [8, 9].

Finally, for g^{ij} , we get

$$\begin{aligned} q_{ij} = & -2W(2V^2 \delta_{ij} - R_a^i R_b^j V^a V^b) - 8V^a W_a \delta_{ij} + 8R_a^i R_b^j W^a V^b + R_a^i R_b^j Q_{ab} \\ & + 2W c^2 (A_a^i A_a^j - \delta_{ij}) + c^4 \{A_0^i A_0^j - A_a^i A_a^j + \delta_{ij} [1 - \ln(A_0^0 A_0^0 - A_a^0 A_a^0)]\} + O(2). \end{aligned} \quad (48)$$

Here we should emphasize that Eq.(48) has only formally given for completeness.

We should point that in Ref.[21], our calculation is incomplete since in Eq.(48) A_a^j must be calculated up to $O(6)$ level, but in fact A_a^j is related with e_a^j as

$$A_a^j = e_a^j + \frac{\partial \xi^j}{\partial x^a}, \quad (49)$$

where e_a^j is the value of A_a^j at the central world line of body A. But precision of e_a^j is only $O(4)$ level (see Eq.(4)), therefore we need to extend e_a^j to $O(6)$. We have the formula (see Eq.(5.19) of Ref.[8])

$$\bar{g}_{\alpha\beta} e_a^\alpha e_b^\beta = \delta_{ab}. \quad (50)$$

From Eq.(50) and (4), we have

$$e_a^i = R_a^i - \frac{R_a^i}{c^2} \bar{w}|_A + \frac{1}{2c^2} v_A^i V_a + \frac{b_a^i}{c^4} + O(6), \quad (51)$$

where

$$b_a^i = \frac{1}{2} \bar{w}^2|_A R_a^i + \frac{3}{8} V^2 V_a v^i + \frac{3}{2} \bar{w}|_A V_a v^i - \frac{1}{2} \bar{q}_{ij}|_A R_a^j. \quad (52)$$

where $|_A$ denote the value at the central world line of body A. In Eq.(44), W and W_a can be taken as known functions. By means of harmonic gauge ($\partial_a W_a + \partial_t W = 0$), Eq.(44) becomes

$$\begin{aligned} & 2\delta_{ab} W_{,tt} + 4W_{(a,b),t} - 2W_{,a} W_{,b} + \delta_{ab} W_{,d} W_{,d} \\ & + \frac{1}{2} (Q_{ad,bd} + Q_{bd,ad} + \delta_{ab} Q_{dd,cc} - Q_{ab,dd} - Q_{dd,ab} - \delta_{ab} Q_{dc,dc}) = 8\pi G T^{ab}, \end{aligned} \quad (53)$$

We also divide Q_{ab} into self-part and external part:

$$Q_{ab} = Q_{ab}^+ + \bar{Q}_{ab}. \quad (54)$$

In DSX scheme, they took the “weak effacement of post Newtonian external gravitational potentials in the local frame”, i.e. the “external” PN potentials $\bar{W}_\alpha^a(T_A, X_A)$ in the local A frame vanish for all T_A times, at the origin of the frame (at the central world line):

$$\forall T_A, \quad \bar{W}_\alpha^A(T_A, 0, 0, 0) = 0 \quad (55)$$

(see Eq.(5.12) of Ref.[8]). We assume “weak effacement” condition to be valid also for \bar{Q}_{ab} , i.e.

$$\forall T_A, \quad \bar{Q}_{ab}^A(T_A, 0, 0, 0) = 0. \quad (56)$$

With the weak effacement condition, we directly write out equations satisfied by Q_{ab}^+ and \bar{Q}_{ab} from Eq.(53):

$$2\delta_{ab}W_{,tt}^+ + 4W_{(a,b),t}^+ - 2W_{,a}^+W_{,b}^+ - 2(W_{,a}^+\bar{W}_{,b} + \bar{W}_{,a}W_{,b}^+) + \delta_{ab}W_{,d}^+W_{,d}^+ + 2\delta_{ab}W_{,d}^+\bar{W}_{,d} \\ + \frac{1}{2} \left(Q_{ad,bd}^+ + Q_{bd,ad}^+ + \delta_{ab}Q_{dd,cc}^+ - Q_{dd,ab}^+ - Q_{dd,ab}^+ - \delta_{ab}Q_{dc,dc}^+ \right) = 8\pi GT^{ab}, \quad (57)$$

$$2\delta_{ab}\bar{W}_{,tt} + 4\bar{W}_{(a,b),t} - 2\bar{W}_{,a}\bar{W}_{,b} + \delta_{ab}\bar{W}_{,d}\bar{W}_{,d} \\ + \frac{1}{2} \left(\bar{Q}_{ad,bd} + \bar{Q}_{bd,ad} + \delta_{ab}\bar{Q}_{dd,cc} - \bar{Q}_{ab,dd} - \bar{Q}_{dd,ab} - \delta_{ab}\bar{Q}_{dc,dc} \right) = 0. \quad (58)$$

The weak effacement condition is automatically satisfied for Eq.(58). In principle from Eqs.(57) and (58) and boundary condition we could solve Q_{ab}^+ and \bar{Q}_{ab} numerically. But for the problem of the light propagation in the solar system it becomes much simple. Since the velocity of the relative motion is small inside the sun ($v^2/c^2 < \epsilon^2$), especially the shape of the sun is close to a monopole, therefore if the origin of the coordinate system is taken as the center of the solar mass (dipole always equal to zero), the quadrupole terms (and higher multipole moments) in 2PN contribution on Q_{ab} can be ignored, i.e.

$$Q_{ab} = \delta_{ab}Q/3 \quad (59)$$

in the local coordinates of the sun. As for other local coordinates of each planets 2PN contribution is too small ($10^{-19} - 10^{-18}$) to be calculated. In the local coordinates of the sun the external potential is negligible, then $W = W^+$.

Substituting Eq.(59) into Eq.(53) and taking PN gauge ($3\partial_t W + 4\partial_a W_a = O(2)$), Q satisfies the equation as following:

$$\left(\frac{Q}{3} + \frac{1}{2}W^2 \right)_{,dd} = 8\pi G \left(T^{aa} - \frac{W\Sigma}{2} \right). \quad (60)$$

The right hand side of above equation has a compact support source and the solution of Eq.(60) is

$$Q = - \int \frac{6G(T^{aa} - W\Sigma/2)}{r} d^3X - \frac{3}{2}W^2. \quad (61)$$

From Eq.(61) it is clear that Q itself does not have compact support neither is linear in W (because of the last term). We should point that if we use PN gauge, W and W_a still can use B-D moments to expand, but one needs to do the gauge transformation from the harmonic gauge to the PN gauge by means of choosing a suitable Λ in Eqs.(20) and (21).

In the global coordinates of the solar system, we have a similar equation and solution. Since there is very small difference between the global coordinates of the solar system and the local coordinates of the sun (additional 10^{-3} in c^{-4} level) we can always ignore the distinction between q and Q of the sun. Therefore, for the situation of the solar system, it may be unnecessary to solve Eqs.(57) and (58) directly, but in the following discussion we will still keep q_{ij} in the equations for the generality.

3 2PN Equations of Light Ray

In this section we will deduce the 2PN equations of light ray in the global coordinates by the iterative method. Certainly, the result can be also used in the local coordinates by substituting the quantities in the global coordinates for the corresponding quantities in the local coordinates. For example, by replacing w , w_i , q_{ij} , $g_{\mu\nu}$ by W , W_a , Q_{ab} , G_{ab} . We start the basic equations of light ray:

$$g_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda} = 0, \quad (62)$$

$$\frac{d^2 x^\mu}{d\lambda^2} + \Gamma_{\nu\sigma}^\mu \frac{dx^\nu}{d\lambda} \frac{dx^\sigma}{d\lambda} = 0, \quad (63)$$

where λ is an ‘‘affine’’ parameter. Normally we replace λ by time t in terms of $\mu = 0$ component of Eq.(63) (see Ref.[7]), then Eq.(62) and (63) become

$$\frac{d^2 x^i}{dt^2} = \left(\frac{1}{c} \Gamma_{\nu\sigma}^0 \frac{dx^i}{dt} - \Gamma_{\nu\sigma}^i \right) \frac{dx^\nu}{dt} \frac{dx^\sigma}{dt}, \quad (64)$$

and

$$g_{\mu\nu} \frac{dx^\mu}{dt} \frac{dx^\nu}{dt} = 0. \quad (65)$$

In fact Eqs.(64) and (65) are our real basic equations of light ray. For 1PN equation of light ray, we use 1PN metric (neglect g_{0i} and all $O(4)$ terms in g_{ij} and g_{00}) and corresponding Christoffel symbols Eq.(37):

$$g_{00} = -1 + \frac{2\omega}{c^2} + o(4), \quad (66)$$

$$g_{ij} = \delta_{ij} \left(1 + \frac{2\omega}{c^2} \right) + o(4). \quad (67)$$

Then Eqs.(64) and (65) become

$$c^2 - 2w - \left(1 + \frac{2w}{c^2} \right) \left| \frac{d\mathbf{x}}{dt} \right|^2 = 0, \quad (68)$$

$$\frac{d^2 x^i}{dt^2} = w_{,i} \left(1 + \frac{1}{c^2} \left| \frac{d\mathbf{x}}{dt} \right|^2 \right) - \frac{4}{c^2} \frac{dx^i}{dt} \left(\frac{d\mathbf{x}}{dt} \cdot \nabla w \right). \quad (69)$$

The solution of equations in 1PN level is

$$\mathbf{x} = \mathbf{x}_0 + c\mathbf{n}(t - t_0) + \mathbf{x}_{1P}, \quad (70)$$

where \mathbf{x}_{1P} is 1PN revision based on the Newtonian or the zeroth order solution ($\mathbf{x} = \mathbf{x}_0 + c\mathbf{n}(t - t_0)$) and \mathbf{n} is the directional unit vector which is given in the initial conditions. Finally, we have

$$c\mathbf{n} \cdot \frac{d\mathbf{x}_{1P}}{dt} = -2w, \quad (71)$$

$$\frac{d^2 \mathbf{x}_{1P}}{dt^2} = 2[\nabla w - 2\mathbf{n}(\mathbf{n} \cdot \nabla w)]. \quad (72)$$

Eqs.(71) and (72) agree to the parametrized 1PN equations of light ray deduced by will [7], when we put $\gamma = 1$ and replace Will’s U by w in DSX scheme. Based on 1PN solution of light ray (\mathbf{x}_{1P} as a known function), we deduce the 2PN equations of light ray in the extended DSX scheme by using the iterative method. Substituting the 2PN metric tensor in global coordinates (Eqs.(30-32) replaced $G_{\alpha\beta}$, W , W_a and Q_{ab} by $g_{\mu\nu}$, w , w_i and q_{ij})

into Eqs.(64) and (65), we have

$$0 = c^2 - 2w + \frac{2w^2}{c^2} + \frac{8w_i}{c^2} \frac{dx^i}{dt} - \left[1 + \frac{2w}{c^2} + \frac{2w^2}{c^4} \right] \left| \frac{d\mathbf{x}}{dt} \right|^2 - \frac{q_{ij}}{c^4} \frac{dx^i}{dt} \frac{dx^j}{dt}, \quad (73)$$

$$\begin{aligned} \frac{d^2 x^i}{dt^2} &= w_{,i} \left(1 - \frac{4w}{c^2} + \frac{1}{c^2} \left| \frac{d\mathbf{x}}{dt} \right|^2 \right) \\ &\quad - 2 \frac{dx^i}{dt} \left(\frac{3w_{,t}}{2c^2} + \frac{2w_{,j}}{c^2} \frac{dx^j}{dt} - \frac{2w_{(j,k)}}{c^4} \frac{dx^j}{dt} \frac{dx^k}{dt} - \frac{w_{,t}}{2c^4} \left| \frac{d\mathbf{x}}{dt} \right|^2 \right) \\ &\quad + \frac{4w_{i,t}}{c^2} + \frac{8w_{[i,j]}}{c^2} \frac{dx^j}{dt} - \frac{q_{ij,k}}{c^4} \frac{dx^j}{dt} \frac{dx^k}{dt} - \frac{q_{jk,i}}{2c^4} \frac{dx^j}{dt} \frac{dx^k}{dt}. \end{aligned} \quad (74)$$

We assume that the solution is $\mathbf{x} = \mathbf{x}_N + \mathbf{x}_{1P} + \mathbf{x}_{2P}$, where \mathbf{x}_N is Newtonian solution (straight line), \mathbf{x}_{1P} and \mathbf{x}_{2P} are 1PN and 2PN post Newtonian revision respectively. Then we have a series of relations:

$$x^i = x_0^i + cn^i(t - t_0) + x_{1P}^i + x_{2P}^i, \quad (75)$$

$$\frac{dx^i}{dt} = cn^i + \frac{dx_{1P}^i}{dt} + \frac{dx_{2P}^i}{dt}, \quad (76)$$

$$\frac{d^2 x^i}{dt^2} = \frac{d^2 x_{1P}^i}{dt^2} + \frac{d^2 x_{2P}^i}{dt^2}, \quad (77)$$

$$\left| \frac{d\mathbf{x}}{dt} \right|^2 = c^2 + 2c\mathbf{n} \cdot \frac{d\mathbf{x}_{1P}}{dt} + 2c\mathbf{n} \cdot \frac{d\mathbf{x}_{2P}}{dt} + \left| \frac{d\mathbf{x}_{1P}}{dt} \right|^2, \quad (78)$$

$$\frac{dx^i}{dt} \frac{dx^j}{dt} = c^2 n^i n^j + cn^i \frac{dx_{1P}^j}{dt} + cn^j \frac{dx_{1P}^i}{dt} + cn^i \frac{dx_{2P}^j}{dt} + cn^j \frac{dx_{2P}^i}{dt} + \frac{dx_{1P}^i}{dt} \frac{dx_{1P}^j}{dt}, \quad (79)$$

where we have neglected all of the terms higher than 2PN in Eqs.(78) and (79). Substituting Eqs.(76), (78) and (79) into Eq.(73), and considering Eq.(71), we obtain

$$\mathbf{n} \cdot \frac{d\mathbf{x}_{2P}}{dt} = \frac{4w_i}{c^2} n^i - \frac{1}{2c} \left| \frac{d\mathbf{x}_{1P}}{dt} \right|^2 + \frac{4w^2}{c^3} - \frac{q_{ij}}{2c^3} n^i n^j, \quad (80)$$

where $q_{ij} = q_{\langle ij \rangle} + \delta_{ij} q/3$, and $q_{\langle ij \rangle}$ is a trace-free symmetric tensor. In the global coordinates of the solar system, $q_{\langle ij \rangle} = 0$, therefore $q_{ij} = \delta_{ij} q/3$. In Eq.(80) \mathbf{x}_{1P} can be solved from Eqs.(71) and (72), then Eq.(80) becomes

$$\mathbf{n} \cdot \frac{d\mathbf{x}_{2P}}{dt} = \frac{4w_i}{c^2} n^i - \frac{1}{2c} \left| \frac{d\mathbf{x}_{1P}}{dt} \right|^2 + \frac{4w^2}{c^3} - \frac{q}{6c^3}. \quad (81)$$

Substituting Eqs.(71), (76), (77), (78) and (79) into Eq.(74), we have

$$\begin{aligned} \frac{d^2 x_{2P}^i}{dt^2} &= -\frac{2}{c} w_{,t} n^i + \frac{4}{c} w_{(j,k)} n^i n^j n^k + \frac{8}{c} w_{[j,k]} n^j - \frac{8}{c^2} w w_{,i} - \frac{4}{c} \left(\frac{d\mathbf{x}_{1P}}{dt} \cdot \nabla w \right) n^i \\ &\quad - \frac{4}{c} (\mathbf{n} \cdot \nabla w) \frac{dx_{1P}^i}{dt} - \frac{1}{c^2} q_{ij,k} n^j n^k + \frac{1}{2c^2} q_{jk,i} n^j n^k. \end{aligned} \quad (82)$$

If $q_{ij} = \delta_{ij} q/3$ is taken, we obtain

$$\begin{aligned} \frac{d^2 x_{2P}^i}{dt^2} &= -\frac{2}{c} w_{,t} n^i + \frac{4}{c} w_{(j,k)} n^i n^j n^k + \frac{8}{c} w_{[j,k]} n^j - \frac{8}{c^2} w w_{,i} - \frac{4}{c} \left(\frac{d\mathbf{x}_{1P}}{dt} \cdot \nabla w \right) n^i \\ &\quad - \frac{4}{c} (\mathbf{n} \cdot \nabla w) \frac{dx_{1P}^i}{dt} - \frac{1}{3c^2} (\mathbf{n} \cdot \nabla q) n^i + \frac{1}{6c^2} q_{,i}. \end{aligned} \quad (83)$$

Eqs.(80) and (82) are just the 2PN equations of light ray that we expected. By means of Eqs.(80) and (82), based on Eqs.(71) and (72), we can discuss light propagation at the 2PN level by means of the iterative method.

In the special case of the solar system, we may use a much simple formulae (Eqs.(81) and (83)), where Q (or q) is presented in Eq.(61) [24].

The 1PN term \mathbf{x}_{1P} in the equations can be obtained from the solutions of the 1PN equations of light ray, i.e. from Eqs.(71) and (72). Then the 2PN equations are solvable. Furthermore, if we knew ever higher order metric tensor (higher than 2PN), we also could get higher order equations of light ray by means of such iterative method. Besides, the iterative method can also be discussed on the parametrized 2PN equations of light ray, then we can discuss the 2PN equations of light ray in alternative gravitational theories which we will discuss in the following section.

4 Parametrized 2PN Equations of Light Ray

The extension may also be used in parametrized post Newtonian (PPN) formalism, which would be found a widely use for the relativistic experiments and observation [7]. In terms of more exact consideration [19], PPN extension for DSX scheme (1PN) is written as

$$g_{00} = -1 + \frac{2w}{c^2} - \frac{2}{c^4}\beta w^2 + O(6), \quad (84)$$

$$g_{0i} = -\frac{2(1+\gamma)}{c^3}w^i + O(5), \quad (85)$$

$$g_{ij} = \delta_{ij} \left(1 + \frac{2\gamma}{c^2}w \right) + O(4). \quad (86)$$

Comparing to a known parametrized second post Newtonian formalism [25] our extension for parametrized second post Newtonian formalism is taking a tentative form in the extended 2PN DSX scheme

$$g_{00} = -1 + \frac{2w}{c^2} - \frac{2}{c^4}\beta w^2 + O(6), \quad (87)$$

$$g_{0i} = -\frac{2(1+\gamma)}{c^3}w^i + O(5), \quad (88)$$

$$g_{ij} = \delta_{ij} \left(1 + \frac{2\gamma}{c^2}w \right) + \frac{\delta_{ij}\varepsilon}{c^4} \left(2w^2 + \frac{q}{3} \right) + \frac{\eta}{c^4}q_{\langle ij \rangle} + O(6), \quad (89)$$

When $\gamma = \beta = \varepsilon = \eta = 1$, then the metric tensor return to the case in general relativity. If we ignore all of $1/c^4$ terms in g_{ij} , our results agree with the one of Ref.[19]

From the parametrized 2PN metric tensor we can get its contravariant tensor as following

$$g^{00} = -1 - \frac{2w}{c^2} + \frac{2(\beta-2)w^2}{c^4} + O(6), \quad (90)$$

$$g^{0i} = -\frac{2(1+\gamma)}{c^3}w_i + O(5), \quad (91)$$

$$g^{ij} = \delta_{ij} \left(1 - \frac{2\gamma}{c^2}w \right) + \frac{\delta_{ij}}{c^4} \left[(4\gamma^2 - 2\varepsilon)w^2 - \frac{\varepsilon q}{3} \right] - \frac{\eta}{c^4}q_{\langle ij \rangle} + O(6). \quad (92)$$

Then, the Parametrized 2PN Christoffel symbols:

$$\Gamma_{00}^0 = -\frac{w_{,t}}{c^3} + O(5), \quad (93)$$

$$\Gamma_{0i}^0 = -\frac{w_{,i}}{c^2} + \frac{2(\beta-1)ww_{,i}}{c^4} + O(6), \quad (94)$$

$$\Gamma_{00}^i = -\frac{w_{,i}}{c^2} - \frac{2(1+\gamma)w_{i,t}}{c^4} + \frac{2(\beta+\gamma)ww_{,i}}{c^4} + O(6), \quad (95)$$

$$\Gamma_{ij}^0 = \frac{2(1+\gamma)}{c^3}w_{(i,j)} + \delta_{ij} \frac{\gamma w_{,t}}{c^3} + O(5), \quad (96)$$

$$\Gamma_{0j}^i = -\frac{2(1+\gamma)}{c^3}w_{[i,j]} + \delta_{ij}\frac{\gamma w_{,t}}{c^3} + O(5), \quad (97)$$

$$\begin{aligned} \Gamma_{jk}^i &= \left(\frac{\gamma}{c^2} + \frac{2(\varepsilon - \gamma^2)w}{c^4}\right)(\delta_{ij}w_{,k} + \delta_{ik}w_{,j} - \delta_{jk}w_{,i}) + \frac{\varepsilon}{6c^4}(\delta_{ij}q_{,k} + \delta_{ik}q_{,j} - \delta_{jk}q_{,i}) \\ &\quad + \frac{\eta}{2c^4}(q_{<ij>,k} + q_{<ik>,j} - q_{<jk>,i}) + O(6). \end{aligned} \quad (98)$$

Substituting the 1PN part of Eqs.(87)-(98) into Eqs.(64) and (65), we get the parametrized 1PN equations of light ray:

$$\mathbf{cn} \cdot \frac{d\mathbf{x}_{1P}}{dt} = -(1+\gamma)w, \quad (99)$$

$$\frac{d^2\mathbf{x}_{1P}}{dt^2} = (1+\gamma)[\nabla w - 2\mathbf{n}(\mathbf{n} \cdot \nabla w)], \quad (100)$$

which have been shown in Ref.[7] (see Eq.(6.14) and (6.15) of [7]).

Considering $\mathbf{x} = \mathbf{x}_0 + \mathbf{n}(t - t_0) + \mathbf{x}_{1P} + \mathbf{x}_{2P}$, Eqs.(99), (100) and substituting whole Eqs.(87)-(98) into Eqs.(64) and (65) we get the parametrized 2PN equations of light ray:

$$\mathbf{n} \cdot \frac{d\mathbf{x}_{2P}}{dt} = \frac{2(1+\gamma)w_i}{c^2}n^i - \frac{1}{2c}\left|\frac{d\mathbf{x}_{1P}}{dt}\right|^2 + \frac{(\beta + 2\gamma + 2\gamma^2 - \varepsilon)w^2}{c^3} - \frac{\varepsilon q}{6c^3} - \frac{\eta q_{(ij)}}{2c^3}n^i n^j, \quad (101)$$

$$\begin{aligned} \frac{d^2x_{2P}^i}{dt^2} &= -\frac{(1+\gamma)}{c}w_{,t}n^i + \frac{2(1+\gamma)}{c}w_{(j,k)}n^i n^j n^k + \frac{4(1+\gamma)}{c}w_{[i,j]}n^j \\ &\quad - \frac{2(\beta + 2\gamma + 2\gamma^2 - \varepsilon)}{c^2}ww_{,i} - \frac{2(1+\gamma)}{c}\left(\frac{d\mathbf{x}_{1P}}{dt} \cdot \nabla w\right)n^i - \frac{2(1+\gamma)}{c}(\mathbf{n} \cdot \nabla w)\frac{dx_{1P}^i}{dt} \\ &\quad - \frac{4(\varepsilon - \gamma^2 - \beta + 1)}{c^2}w(\mathbf{n} \cdot \nabla w)n^i - \frac{\eta}{c^2}q_{(ij),k}n^j n^k - \frac{\varepsilon}{3c^2}q_{,k}n^i n^k \\ &\quad + \frac{\eta}{2c^2}q_{(jk),i}n^j n^k + \frac{\varepsilon}{6c^2}q_{,i}. \end{aligned} \quad (102)$$

In special case of the solar system, we have $q_{(ij)} = 0$, therefore the equations become

$$\mathbf{n} \cdot \frac{d\mathbf{x}_{2P}}{dt} = \frac{2(1+\gamma)w_i}{c^2}n^i - \frac{1}{2c}\left|\frac{d\mathbf{x}_{1P}}{dt}\right|^2 + \frac{(\beta + 2\gamma + 2\gamma^2 - \varepsilon)w^2}{c^3} - \frac{\varepsilon q}{6c^3}, \quad (103)$$

$$\begin{aligned} \frac{d^2x_{2P}^i}{dt^2} &= -\frac{(1+\gamma)}{c}w_{,t}n^i + \frac{2(1+\gamma)}{c}w_{(j,k)}n^i n^j n^k + \frac{4(1+\gamma)}{c}w_{[i,j]}n^j \\ &\quad - \frac{2(\beta + 2\gamma + 2\gamma^2 - \varepsilon)}{c^2}ww_{,i} - \frac{2(1+\gamma)}{c}\left(\frac{d\mathbf{x}_{1P}}{dt} \cdot \nabla w\right)n^i - \frac{2(1+\gamma)}{c}(\mathbf{n} \cdot \nabla w)\frac{dx_{1P}^i}{dt} \\ &\quad - \frac{4(\varepsilon - \gamma^2 - \beta + 1)}{c^2}w(\mathbf{n} \cdot \nabla w)n^i - \frac{\varepsilon}{3c^2}(\mathbf{n} \cdot \nabla q)n^i + \frac{\varepsilon}{6c^2}q_{,i}. \end{aligned} \quad (104)$$

When $\beta = \gamma = \varepsilon = \eta = 1$, Eqs.(101), (102), (103) and (104) return to Eqs.(80), (82), (81) and (83) respectively. Maybe in our parametrized 2PN equations of light ray, four parameters are not sufficient. But if we compare with Ref.[25], these four parameters are main parameters in the 2PN equations of light ray.

5 Conclusion Remarks

1. We have extended DSX scheme to 2PN level approximation, by means of which we can discuss the 2PN contribution on the trajectory of light ray. Our extension keep the main advantage in DSX scheme (the linear equations of the potential and vector potential, and the linear relationship between w , w_i and W , W_a). In the case of the solar system, we obtain a special solution under the PN gauge. But we should point out that if we want to discuss the 2PN contribution on the motion of bodies (not light ray), we have to extend the metric

further more ($q_{00} \sim O(8)$ and $q_{0i} \sim O(7)$) which someone might finish in future. Considering the precision of ASTROD, they will need the theoretical precision up to the 2PN level for the motion of bodies.

2. We have discussed 2PN equations of light ray based on the extended DSX scheme by means of the iterative method. If we have the higher order metric, we also can obtain the higher order equations of light ray with a similar iterative method in this paper. Therefore in some meaning the way shown in the section 3 is general. The special case for equations of light ray propagating in the solar system($q_{ij} = \frac{1}{3}q$) is also considered.

3. We also use a parametrized 2PN metric tensor (with four parameters) and its corresponding Christoffel symbols to substitute into the null geodetic equations and obtain parametrized 2PN equations of light ray. Although we have not induced four parameters strictly, but it agree with the main parameter in [25]. Also if 2PN part of g_{ij} is neglected, the metric tensor return to the one in [19] which has been deduced strictly.

Anyway, we hope our work to be useful for the measurement of future space missions.

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