

Integrated Sensing and Communications for Unsourced Random Access: Fundamental Limits

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Abstract—This work considers the problem of integrated sensing and communication (ISAC) with a massive number of unsourced and uncoordinated users. In the proposed model, known as the unsourced ISAC system (UNISAC), all active communication and sensing users share a short frame to transmit their signals, without requiring scheduling with the base station (BS). Hence, the signal received from each user is affected by significant interference from numerous interfering users, making it challenging to extract the transmitted signals. UNISAC aims to decode the transmitted message sequences from communication users while simultaneously detect active sensing users, regardless of the identity of the decoded and detected users. In this paper, we derive an achievable performance limit for UNISAC and demonstrate its superiority over conventional approaches such as ALOHA, time-division multiple access, treating interference as noise, and multiple signal classification. Through numerical simulations, we validate the UNISAC’s effectiveness in detecting and decoding a large number of users.

Index Terms—Integrated Sensing and Communications, Unsourced Random Access, Achievability Bound, Massive Random Access.

I. INTRODUCTION

In unsourced random access (URA), introduced by Polyanskiy in [1], a massive number of unidentified users perform sporadic transmission within a short period of time without engaging in any scheduling with the BS. Eliminating the need for scheduling significantly reduces the signaling overhead and latency of the URA system, thereby granting it the potential to support an unbounded number of users. In addition, the BS focuses solely on the information transmitted without considering the identity of the sender of the message. This approach is particularly suitable for the next generation of communication networks, where millions of

inexpensive devices are connected to the system, with only a fraction of them active during each transmission [2]–[11].

Existing research on URA has predominantly concentrated on extracting information transmitted by unsourced users (communication purpose). However, with the advent of sixth-generation (6G) mobile networks, additional functionalities such as computing, sensing, and security have become imperative [12], [13]. As hardware architectures, channel characteristics, and signal processing methods become increasingly similar across these different tasks, it is necessary to adapt the URA system to jointly accommodate these diverse purposes rather than focusing solely on information transmission [14].

In this paper, we consider a URA system that provides both sensing and communication functions concurrently, referred to as unsourced integrated sensing and communications (UNISAC). Specifically, we incorporate the object detection task alongside the communication task to accommodate a wide range of users. The UNISAC model serves two user categories: communication users and sensing users. Each category has distinct objectives for the receiver: decoding transmitted messages from communication users and detecting sensing users while estimating their angle of arrival (AOA). Due to the aggregation of signals from various sensing and communication sources at the receiver, the URA faces the complex task of analyzing each user’s signal among the interference from other users. In this paper, we investigate the achievable performance of the UNISAC model, taking into account the misdetection error and mean-squared error of AOA estimation (MSEAOA) for sensing users, in addition to the decoding error of communication users. Numerical results indicate that the proposed UNISAC model exhibits superior performance compared to conventional multiple access systems such as treat interference as noise (TIN), time division multiple access (TDMA), ALOHA, and multiple signal classification (MUSIC).

The rest of the paper is organized as follows: In Section II, we outline the system model. Then, the achievable bound of the UNISAC model is derived in Section III. Finally, before concluding the paper in Section V, we present numerical results in Section IV.

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Notation: Lower-case and upper-case boldface letters are used for denoting a vector and a matrix, respectively; $\text{diag}(\mathbf{A})$ generates a diagonal matrix by setting all off-diagonal elements of \mathbf{A} to zero; $\det(\mathbf{A})$ calculates the determinant of the matrix \mathbf{A} ; \mathbf{I}_N is an $N \times N$ identity matrix; $\text{Re}(t)$ gives the real part of t ; $\|\mathbf{A}\|^2$ calculates the squared Frobenius norm of the matrix \mathbf{A} ; the notation $\mathbf{A} \preceq \mathbf{B}$ indicates that the matrix $\mathbf{A} - \mathbf{B}$ is negative semidefinite; $\mathcal{CN}(0, b)$ denotes the zero-mean circularly symmetric complex white Gaussian noise with variance b ; $\mathbf{0}_{n,m}$ denotes a matrix of size $n \times m$ where all elements are zero; $F_{\chi^2}(t)$ is the cumulative distribution function of the chi-squared distribution with t degrees of freedom and $F_{\chi^2}^{-1}(t)$ is its inverse; χ_T^2 denotes a chi-squared random variable with T degrees of freedom; the function \mathbf{t}^R reverses the order of elements in the vector \mathbf{t} .

II. SYSTEM MODEL

Consider an ISAC system where K_{T_s} sensing users (objects) and K_{T_c} communication users are connected to a BS. As shown in Fig. 1, the BS is equipped with an M -element uniform linear array (ULA) with element spacing of $d = \lambda/2$, where λ represents the wavelength of the received signal. Due to sporadic behaviour, only a small fraction of sensing and communication users are active at each transmission, and a significant portion of users do not transmit any signal. All the active sensing and communication users simultaneously share the same length- n frame for transmitting signals. To generate its transmitted signal, each communication user maps its bit sequence $\mathbf{w}_j \in \mathbb{C}^{B_c}$ to the first 2^{B_c} rows of the codebook $\mathbf{A} \in \mathbb{C}^{2^{B_c+B_s} \times n}$, and each sensing user generates its transmitted signal by randomly choosing a row among the last 2^{B_s} rows of the codebook \mathbf{A} . Note that since there are no message sequences to be transmitted by the sensing users, the value of B_s lacks a predefined value and must be optimally chosen according to the system configuration.

Considering the synchronous transmission of all the users, the received signal at the BS is written as

$$\mathbf{Y} = \sum_{j \in \{\mathcal{A}_c, \mathcal{A}_s\}} \mathbf{b}_{\theta_j} \mathbf{a}_j + \mathbf{Z} \in \mathbb{C}^{M \times n}, \quad (1)$$

with

$$\mathbf{b}_{\theta_j} = \left[1, e^{-j\pi\theta_j}, e^{-j2\pi\theta_j}, \dots, e^{-j\pi(M-1)\theta_j} \right]^T, \quad (2)$$

where $\mathbf{a}_j \in \mathbb{C}^n$ denotes the j th row of the matrix \mathbf{A} , θ_j represents the AOA corresponding to the codeword \mathbf{a}_j , $\mathcal{A}_c \subset \{1, 2, \dots, 2^{B_c}\}$ and $\mathcal{A}_s \subset \{2^{B_c} + 1, \dots, 2^{B_c+B_s}\}$ are the sets of active row indices of the codebook \mathbf{A} associated with communication and sensing users, respectively (active row index refers to a row that is selected and transmitted by an active user), and \mathbf{Z} comprises symmetric complex Gaussian elements $\mathcal{CN}(0, \sigma_z^2)$ which serves as an additive Gaussian noise matrix. The objectives of the receiver are:

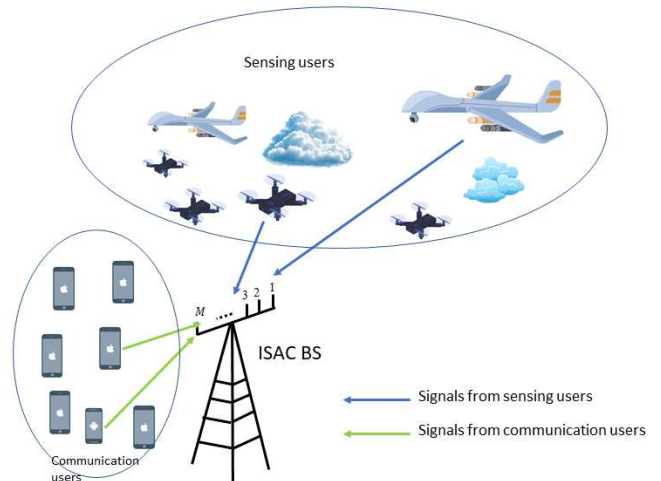


Fig. 1: Illustration of the UNISAC model. The system accommodates multiple users for sensing and communication purposes, with only a limited subset active at any given moment.

1) to detect the presence of active sensing users ($j \in \mathcal{A}_s$) by detecting their transmitted signals as well as estimating their corresponding AOAs, and 2) to decode the transmitted bit-sequences of active communication users, $\mathbf{w}_j, j \in \mathcal{A}_c$.

For measuring the detection and decoding performance of the UNISAC model, the per-user probability of error (PUPE) is adopted as

$$\epsilon = \frac{\mathbb{E} \{ |\mathcal{L}_{c,md}| + |\mathcal{L}_{s,md}| + |\mathcal{L}_{c,coll}| + |\mathcal{L}_{s,coll}| \}}{|\mathcal{A}_c| + |\mathcal{A}_s|} \quad (3)$$

where $\mathcal{L}_{c,md}$ and $\mathcal{L}_{s,md}$ denote the list of active communication and sensing user indices that are not detected, respectively, and $\mathcal{L}_{c,coll}$ and $\mathcal{L}_{s,coll}$ are the list of communication and sensing user indices that are in collision, respectively. Moreover, concerning the AOA estimation for the sensing users, the performance metric is the MSEAOA for the successfully detected sensing users, defined as follows:

$$\Delta = \frac{1}{|\mathcal{L}_{s,d}|} \sum_{i \in \mathcal{L}_{s,d}} \mathbb{E} \{ |\theta_i - \hat{\theta}_i|^2 \}, \quad (4)$$

where $\mathcal{L}_{s,d}$ represents the list of active sensing user indices that are successfully detected, and $\hat{\theta}_i$ is the estimate of θ_i .

III. ACHIEVABLE PERFORMANCE LIMITS

In this section, we derive achievable bounds for the PUPE and MSEAOA of the UNISAC model.

Proposition 1. *For the UNISAC model, there exists a transceiver configuration where the communication and sensing users satisfy the power constraints \bar{P}_c and \bar{P}_s , respectively, and the performance metrics in (3) and (4)*

satisfy

$$\epsilon \leq P_{cons} + P_{coll} + P_{md}, \quad (5)$$

$$\Delta \leq \sum_{K_s=1}^{|\mathcal{A}_s|} \sum_{K_c=1}^{|\mathcal{A}_c|} P_{K_s, K_c} \mathbb{E}_{o_g} \left\{ \left| \operatorname{Re} \left(\frac{\log(o_g)}{j\pi(M-1)} \right) \right|^2 \right\}, \quad (6)$$

where P_{cons} represents the probability that at least one communication/sensing user surpasses the power constraint, $P_{coll} = \frac{\mathbb{E}\{|\mathcal{L}_{c,coll}| + |\mathcal{L}_{s,coll}|\}}{|\mathcal{A}_c| + |\mathcal{A}_s|}$,

$P_{md} = \frac{\mathbb{E}\{|\mathcal{L}_{c,md}| + |\mathcal{L}_{s,md}|\}}{|\mathcal{A}_c| + |\mathcal{A}_s|}$, and $P_{K_s, K_c} = \mathbb{P}(|\mathcal{L}_{c,md}| = K_c, |\mathcal{L}_{s,md}| = K_s)$. These values are obtained as:

$$P_{cons} = 1 - F_{\chi^2} \left(\frac{2n\bar{P}_s}{P'_s}, 2n \right)^{|\mathcal{A}_s|} F_{\chi^2} \left(\frac{2n\bar{P}_c}{P'_c}, 2n \right)^{|\mathcal{A}_c|}, \quad (7a)$$

$$P_{coll} \leq \frac{\sum_{i=2}^{\infty} \frac{i \binom{|\mathcal{A}_s|}{i}}{2^{B_s(i-1)}} + \sum_{j=2}^{\infty} \frac{j \binom{|\mathcal{A}_c|}{j}}{2^{B_c(j-1)}}}{|\mathcal{A}_c| + |\mathcal{A}_s|}, \quad (7b)$$

$$P_{md} = \sum_{K_s=1}^{|\mathcal{A}_s|} \sum_{K_c=1}^{|\mathcal{A}_c|} \frac{K_c + K_s}{|\mathcal{A}_c| + |\mathcal{A}_s|} P_{K_s, K_c}, \quad (7c)$$

$$P_{K_s, K_c} \leq \mathbb{P}(x_g > \eta) + e^{L_s + L_c + L_0}, \quad (7d)$$

$$L_0 = -\lambda_* g - nM \log(1 + \lambda_* \sigma_t^2 - \lambda_*^2 \sigma_z^2 \sigma_t^2), \quad (7e)$$

$$g = -M(\sigma_z^2 + \sigma_t^2)\eta, \quad (7f)$$

$$\sigma_t^2 = K_c P'_c + K_s P'_s, \quad (7g)$$

$$L_c = \sum_{i=0}^{K_c-1} \log \left(\frac{|\mathcal{A}_c| - i}{K_c - i} \right) + \log \left(\frac{2^{B_c} - i}{K_c - i} \right), \quad (7h)$$

$$L_s = \sum_{i=0}^{K_s-1} \log \left(\frac{|\mathcal{A}_s| - i}{K_s - i} \right) + \log \left(\frac{2^{B_s} - i}{K_s - i} \right), \quad (7i)$$

$$\lambda_* = \frac{-b_1 + \sqrt{b_1^2 - 4a_1c_1}}{2a_1}, \quad (7j)$$

$$a_1 = \sigma_t^2 \sigma_z^2 g, \quad b_1 = -g \sigma_t^2 + 2nM \sigma_z^2 \sigma_t^2, \quad (7k)$$

$$c_1 = -g - nM \sigma_t^2, \quad (7l)$$

$$o_g = 1 + (g_2 + \mathbf{g}^T \mathbf{g}^R)/M, \quad (7m)$$

where P'_c and P'_s denote the average transmitted powers of communication and sensing users, respectively, with $P'_c \leq P_c$ and $P'_s \leq P_s$, $x_g = \mathbf{a} \mathbf{A}_d^H (\mathbf{A}_d \mathbf{A}_d^H)^{-1} \mathbf{A}_d \mathbf{a}^H$, elements of $\mathbf{A}_d \in \mathbb{C}^{(|\mathcal{A}_c| + |\mathcal{A}_s|) \times n}$ and $\mathbf{a} \in \mathbb{C}^n$ are randomly chosen from $\mathcal{CN}(0, 1)$, η serves as a design parameter, each element of $\mathbf{g} \in \mathbb{C}^M$ follows $\mathcal{CN}(0, (\sigma_t^2 + \sigma_z^2)/(fP'_s))$, and $g_2 \sim \mathcal{CN}(0, 4M(\sigma_t^2 + \sigma_z^2)/(fP'_s))$, with f following the chi-squared distribution with $2n$ degrees of freedom.

Proof: In our analysis, we consider a random codebook $\mathbf{A} \in \mathbb{C}^{2^{B_c} + 2^{B_s} \times n}$, where every element in the first 2^{B_c} rows are drawn from $\mathcal{CN}(0, P'_c)$, and the last 2^{B_s} rows consist of

$\mathcal{CN}(0, P'_s)$. Each communication user randomly selects its transmitted signal from the first 2^{B_c} rows of the codebook \mathbf{A} , and each sensing user selects from the last 2^{B_s} rows. Note that in order to fulfill the achievability assumption, all the assumptions outlined in this section should result in an increase in errors.

For obtaining ϵ as shown in (5), we consider three sources of errors as P_{cons} , P_{coll} , and P_{md} . According to the definition, we have

$$P_{cons} = 1 - \prod_{j \in \{\mathcal{A}_c, \mathcal{A}_s\}} \mathbb{P}(\|\mathbf{a}_i\|^2/n < \bar{P}_l). \quad (8)$$

Considering $\frac{2}{P'_l} \|\mathbf{a}_i\|^2 \sim \chi_{2n}^2$ for $l \in \{c, s\}$, then we can derive P_{cons} as in (7a). We calculate P_{coll} in (7b) by considering that the probability that j users select the same signal is upper bounded by $\frac{\binom{|\mathcal{A}_l|}{j}}{2^{B_l(j-1)}}$, for $l \in \{c, s\}$. In the rest of this part, we calculate an upper bound for P_{md} .

To find active \mathbf{a}_i 's available in the received signal in (1), the decoder selects $|\mathcal{A}_c|$ signals from the the first 2^{B_c} rows of \mathbf{A} , and $|\mathcal{A}_s|$ signals from the last 2^{B_s} rows of it. Then, appending these signals together, the matrix $\mathbf{A}_d \in \mathbb{C}^{(|\mathcal{A}_c| + |\mathcal{A}_s|) \times n}$ is constructed. Note that there are $\binom{2^{B_s}}{|\mathcal{A}_s|} \binom{2^{B_c}}{|\mathcal{A}_c|}$ possible choices for the matrix \mathbf{A}_d . The receiver declares the matrix that satisfies the following strategy as the collection of detected signals from active communication and sensing users.

$$\hat{\mathbf{A}}_d = \arg \min_{\mathbf{A}_d} \|\mathbf{Y} f_p(\mathbf{A}_d)\|^2, \quad (9)$$

where

$$f_p(\mathbf{A}_d) = \mathbf{I}_n - \mathbf{A}_d^H (\mathbf{A}_d \mathbf{A}_d^H)^{-1} \mathbf{A}_d. \quad (10)$$

We can write the received signal in (1) in the following form

$$\mathbf{Y} = \mathbf{B}_a \mathbf{A}_a + \mathbf{Z}, \quad (11)$$

where \mathbf{b}_{θ_j} 's and \mathbf{a}_j 's for $j \in \{\mathcal{A}_c, \mathcal{A}_s\}$ construct the columns of $\mathbf{B}_a \in \mathbb{C}^{M \times (|\mathcal{A}_c| + |\mathcal{A}_s|)}$ and the rows of $\mathbf{A}_a \in \mathbb{C}^{(|\mathcal{A}_c| + |\mathcal{A}_s|) \times n}$, respectively. Let $\mathbf{A}_a = \begin{bmatrix} \mathbf{A}_{a,1} \\ \mathbf{A}_{a,2} \end{bmatrix}$ and $\mathbf{B}_a = [\mathbf{B}_{a,1}, \mathbf{B}_{a,2}]$, where $\mathbf{B}_{a,2}$ and $\mathbf{A}_{a,2}$ consist of $K_s + K_c$ columns and rows, respectively. We define $\mathbf{A}_e = \begin{bmatrix} \mathbf{A}_{e,1} \\ \mathbf{A}_{e,f} \end{bmatrix}$, where the rows of $\mathbf{A}_{e,f} \in \mathbb{C}^{(K_c + K_s) \times n}$ are K_s sensing signals and K_c communication signals that are not sent (false signals). Considering the definition of P_{K_s, K_c} in the statement of the Proposition 1 and the strategy in (9), we write

$$P_{K_s, K_c} \leq \mathbb{P}(\|\mathbf{Y} f_p(\mathbf{A}_e)\|^2 < \|\mathbf{Y} f_p(\mathbf{A}_a)\|^2) \quad (12)$$

$$\leq \mathbb{P}(x_g > \eta) + \tilde{P}_{K_s, K_c}, \quad (13)$$

where $\tilde{P}_{K_s, K_c} = \mathbb{P}(\|\mathbf{Y} f_p(\mathbf{A}_e)\|^2 < \|\mathbf{Y} f_p(\mathbf{A}_a)\|^2 | x_g \leq \eta)$, x_g and η are defined in the statement of the Proposition 1,

and in (12), we use the property

$$\mathbb{P}\left(\bigcap_{i \in \mathcal{S}} A_i\right) \leq \mathbb{P}(A_j), \quad j \in \mathcal{S}. \quad (14)$$

Since $\mathbf{A}_a f_p(\mathbf{A}_a) = \mathbf{0}_{(|\mathcal{A}_c|+|\mathcal{A}_s|),n}$, $f_p(\mathbf{A}_a) \preceq \mathbf{I}_n$, and $f_p(\mathbf{A}_a) f_p(\mathbf{A}_a)^H = f_p(\mathbf{A}_a)$, then

$$\|\mathbf{Y} f_p(\mathbf{A}_a)\|^2 \leq \|\mathbf{Z}\|^2. \quad (15)$$

The received signal in (11) can be written as

$$\mathbf{Y} = [\mathbf{B}_{a,1}, \mathbf{0}_{M,(K_s+K_c)}] \mathbf{A}_e + \mathbf{Z}' + \mathbf{Z}, \quad (16)$$

where $\mathbf{Z}' = \mathbf{B}_{a,2} \mathbf{A}_{a,2}$. Assuming various components of the steering vector are mutually independent, each element of \mathbf{Z}' belongs to a $\mathcal{CN}(0, \sigma_t^2)$ distribution, where $\sigma_t^2 = K_c P_c' + K_s P_s'$. Using $\mathbf{A}_e f_p(\mathbf{A}_e) = \mathbf{0}_{(|\mathcal{A}_c|+|\mathcal{A}_s|),n}$ and $x_g \leq \eta$, we get

$$\|\mathbf{Y} f_p(\mathbf{A}_e)\|^2 \geq \|\mathbf{Z} + \mathbf{Z}'\|^2 + g, \quad (17)$$

where $g = -M(\sigma_z^2 + \sigma_t^2)\eta$. Substituting (15) and (17) into (13), and taking into account that the vectorized version of a matrix has the same Frobenius norm as the matrix itself, we have

$$\tilde{P}_{K_s, K_c} \leq \mathbb{P}(\|\mathbf{z} + \mathbf{z}'\|^2 + g < \|\mathbf{z}\|^2) \quad (18a)$$

$$\leq \mathbb{E}\left\{e^{-\lambda_1 \|\mathbf{z} + \mathbf{z}'\|^2 - \lambda_1 g + \lambda_1 \|\mathbf{z}\|^2}\right\} \quad (18b)$$

$$= \frac{e^{-\lambda_1 g + L_s + L_c}}{(1 + \lambda_1 \sigma_t^2)^{nM}} \mathbb{E}\left\{e^{\lambda_1 \|\mathbf{z}\|^2} e^{\frac{-\lambda_1 \|\mathbf{z}\|^2}{(1 + \lambda_1 \sigma_t^2)}}\right\} \quad (18c)$$

$$= \frac{e^{-\lambda_1 g + L_s + L_c}}{(1 + \lambda_1 \sigma_t^2 - \lambda_1^2 \sigma_z^2 \sigma_t^2)^{nM}} \quad (18d)$$

$$= e^{L_s + L_c - \lambda_* g - nM \log(1 + \lambda_* \sigma_t^2 - \lambda_*^2 \sigma_z^2 \sigma_t^2)}, \quad (18e)$$

where \mathbf{z} and \mathbf{z}' are the vectorized versions of \mathbf{Z} and \mathbf{Z}' , respectively, and L_s , L_c , and λ_* are defined in the statement of the Proposition 1. Note that λ_* represents the optimal value of λ_1 that minimizes the expression in (18d). To establish (18b), we utilize the Chernoff bound, which states that $\mathbb{P}(x > 0) \leq \mathbb{E}(e^{\lambda_1 x})$ for any $\lambda_1 > 0$, and to obtain (18c) and (18d), we use the identity

$$\mathbb{E}\left\{e^{x\|\mathbf{a} + \mathbf{b}\|^2}\right\} = \frac{1}{(1 - x\sigma_a^2)^n} e^{x\|\mathbf{b}\|^2/(1-x\sigma_a^2)}, \quad (19)$$

for $x\sigma_a^2 < 1$, $\mathbf{a} \sim \mathcal{CN}(\mathbf{0}_{n,1}, \sigma_a^2 \mathbf{I}_n)$, and $\mathbf{b} \in \mathbb{C}^n$. In (18c), we consider that there are $e^{L_s + L_c} = \binom{2^{B_s}}{K_s} \binom{2^{B_c}}{K_c} \binom{2^{B_c}}{K_c} \binom{2^{B_c}}{K_c}$ different realizations for the matrix \mathbf{z}' . By plugging (18e) into (13), an upper limit for P_{K_s, K_c} is obtained as in (7d), which directly leads to the result in (7c).

After detecting the sensing and communication signals, we can obtain a soft estimation of their corresponding steering vectors using least-squares (LS) estimator. Applying

LS on the received signal in (16), we have

$$\hat{\mathbf{B}}_{a,1} = \mathbf{Y} \mathbf{A}_e^H (\mathbf{A}_e \mathbf{A}_e^H)^{-1} \quad (20)$$

$$= \mathbf{B}_{a,1} + \tilde{\mathbf{Z}}, \quad (21)$$

where $\tilde{\mathbf{Z}} = (\mathbf{Z} + \mathbf{Z}') \mathbf{A}_e^H (\mathbf{A}_e \mathbf{A}_e^H)^{-1}$. It is clear that every row of $\tilde{\mathbf{Z}}$ follows $\mathcal{CN}(\mathbf{0}_{1,J}, (\sigma_z^2 + \sigma_t^2) (\mathbf{A}_a \mathbf{A}_a^H)^{-1})$, where $J = |\mathcal{A}_c| + |\mathcal{A}_s| - K_s - K_c$. For every $i \in \mathcal{A}_s$, the estimation of the steering vector is obtained by selecting the corresponding column of $\hat{\mathbf{B}}$, which yields

$$\hat{\mathbf{b}}_{\theta_i} = \mathbf{b}_{\theta_i} + \mathbf{z}_i, \quad (22)$$

where \mathbf{z}_i is the column of $\tilde{\mathbf{Z}}$ corresponding to the index i . Any positive definite matrix \mathbf{T} satisfies the condition $\mathbf{T}^{-1} \preceq \text{diag}(\mathbf{T})^{-1}$. Therefore, assuming every element of \mathbf{z}_i to follow $\mathcal{CN}\left(0, \frac{\sigma_z^2 + \sigma_t^2}{\|\mathbf{a}_i\|^2}\right)$ is a pessimistic assumption, leading to a higher MSEAOA due to increased noise variance. The receiver estimates θ_i as

$$\hat{\theta}_i = \text{Re}\left(\frac{1}{-j\pi(M-1)} \log\left(\frac{1}{M} \hat{\mathbf{b}}_{\theta_i}^T \hat{\mathbf{b}}_{\theta_i}^R\right)\right), \quad (23)$$

Plugging (22) into (23) and using the structure in (2), we get

$$\hat{\theta}_i = \text{Re}\left(\frac{1}{-j\pi(M-1)} \log\left(e^{-j\pi(M-1)\theta_i} + z'_i\right)\right) \quad (24)$$

$$= \theta_i + z''_i, \quad (25)$$

where $z'_i = \frac{2}{M} \mathbf{b}_{\theta_i}^T \mathbf{z}_i^R + \frac{1}{M} \mathbf{z}_i^T \mathbf{z}_i^R$, and $z''_i = \text{Re}\left(\frac{1}{-j\pi(M-1)} \log(1 + e^{j\pi(M-1)\theta_i} z'_i)\right)$. Considering that randomly shifting of \mathbf{z}_i does not change its distribution, the random variable o_g in the statement of the Proposition 1 has the same probability distribution function as $1 + e^{j\pi(M-1)\theta_i} z'_i$. Hence, the MSEAOA can be upper bounded as in (6). ■

Remark 1. Finding $\mathbb{E}_{o_g}\{\cdot\}$ and $\mathbb{P}(x_g > \eta)$ in (6) and (7d) becomes challenging because of the complicated probability distribution functions of o_g and x_g . Hence, we employ the Monte Carlo method to estimate them by generating multiple realizations.

IV. NUMERICAL RESULTS

To assess the performance of the proposed UNISAC model, we compare its achievability result derived in Proposition 1 with that of traditional multiple-access models. We evaluate various configurations under the following conditions: The bit-sequence length of communication users is $B_c = 100$, the antenna array consists of $M = 5$ elements, and the number of channel-users is $n = 5000$. Other parameters are selected to reach the target PUPE and the target MSEAOA of $\epsilon_o = 0.1$ and $\Delta_o = 5 \times 10^{-4}$, respectively.

Definition 1. (Approximate channel coding rate in the finite blocklength regime [15]): For any $P_{er} > 0$, there exists a $(2^B, n, P_{er})$ code in an additive white Gaussian noise channel, where

$$R \approx \log(1 + S) - \sqrt{\frac{1}{n} \frac{S(S+2) \log_2^2 e}{2(S+1)^2}} Q^{-1}(P_{er}), \quad (26)$$

where R and S denote channel coding rate and the signal-to-noise ratio (SNR), respectively, P_{er} is the block error rate, $Q(\cdot)$ denotes the standard Q -function, and e is the Euler's number.

Definition 2. (Theorem 4.1 [16]): For the channel model in (11) with K active users, the lower bound for the MSEAOA is obtained as

$$\Delta = \frac{\sigma_z^2}{2} \left(\sum_{t=1}^n \text{Re} \left(\mathbf{X}_t \mathbf{D}^H f_p(\mathbf{B}_a^H) \mathbf{D} \mathbf{X}_t^H \right) \right)^{-1}, \quad (27)$$

where $f_p(\mathbf{B}) = \mathbf{I}_n - \mathbf{B}^H (\mathbf{B} \mathbf{B}^H)^{-1} \mathbf{B}$, $\mathbf{X}_t \in \mathbb{C}^{K \times K}$ is a diagonal matrix with the t th column of the matrix \mathbf{A}_a on its diagonal, and $\mathbf{D} \in \mathbb{C}^{M \times K}$ represents the element-wise partial derivative of \mathbf{B}_a with respect to θ , $\mathbf{D} = \partial \mathbf{B}_a / \partial \theta$.

Definition 3. (Chapter 8 [17]): Let \mathcal{H}_0 and \mathcal{H}_1 be the null and alternative hypotheses, respectively, with corresponding channel models represented as follows: $\mathcal{H}_0 : \mathbf{y} = \mathbf{z}$ and $\mathcal{H}_1 : \mathbf{y} = \mathbf{s} + \mathbf{z}$, where $\mathbf{z} \sim \mathcal{CN}(\mathbf{0}_{T,1}, \delta \mathbf{I}_T)$ represents the additive noise term and $\mathbf{s} \in \mathbb{C}^T$ is the desired signal. The misdetection probability for detecting the signal \mathbf{s} can be computed as follows:

$$p_{miss} = Q \left(\sqrt{\frac{\|\mathbf{s}\|^2}{2\delta}} \right). \quad (28)$$

The benchmark strategies are described below. The first model is termed the ‘‘optimistic’’ model, where we adopt optimistic assumptions leading to lower bounds on PUPE and MSEAOA. The lower bound for $\mathbb{E}\{|\mathcal{L}_{s,coll}| + |\mathcal{L}_{c,coll}|\}$ is obtained as $\sum_{l \in \{c,s\}} \frac{\binom{|A_l|}{2}}{2^{B_l}} \left(\frac{2^{B_l} - 1}{2^{B_l}} \right)^{|A_l| - 2}$. To ensure that $\mathbb{E}\{|\mathcal{L}_{s,md}| + |\mathcal{L}_{c,md}|\} / (|\mathcal{A}_c| + |\mathcal{A}_s|) \rightarrow 0$, we employ Shannon limit as

$$\frac{B_T}{n} \rightarrow \mathbb{E} \left\{ \log_2 \left(\det \left(\mathbf{I}_M + \frac{P_{cl}}{\sigma_z^2} \mathbf{B}_a \mathbf{B}_a^H \right) \right) \right\}, \quad (29)$$

where $B_T = |\mathcal{A}_c| B_c + |\mathcal{A}_s| B_s$ and P_{cl} denote the total number of transmitted bits and the transmitted power by each sensing and communication user, respectively. To obtain the lower bound of the MSEAOA, we use Definition 2 for a single user to obtain

$$\Delta = \frac{0.5\sigma_z^2}{\pi^2 n \bar{P}_s \sum_{i=1}^{M-1} i^2}. \quad (30)$$

In the second model, which is called ‘‘TDMA’’ model, we divide the frame into $|\mathcal{A}_c| + |\mathcal{A}_s|$ subframes, and every active

sensing and communication user transmits its signal through an individual subframe. In this case, we can obtain an approximation of the $\mathbb{E}\{|\mathcal{L}_{c,md}|\} / |\mathcal{A}_c|$ by adopting Definition 1 with $S = M \bar{P}_c / \sigma_z^2$ and $R = B_c / N_{\text{TDMA}}$, where $N_{\text{TDMA}} = n / (|\mathcal{A}_c| + |\mathcal{A}_s|)$. Using (28), we obtain $\mathbb{E}\{|\mathcal{L}_{s,md}|\} / |\mathcal{A}_s| = Q \left(\sqrt{\frac{\bar{P}_s N_{\text{TDMA}} M}{2\sigma_z^2}} \right)$. For MSEAOA, we employ the result in Definition 2 for a single user, which gives $\Delta = 0.5\sigma_z^2 / (\pi^2 N_{\text{TDMA}} \bar{P}_s \sum_{i=1}^{M-1} i^2)$. The third model is known as ‘‘TIN’’ model, where all users transmit their signals simultaneously, and the receiver treats signals of all users except a desired user as noise. In this case, $\mathbb{E}\{|\mathcal{L}_{c,md}|\} / |\mathcal{A}_c|$ is approximated by inserting $S = M \bar{P}_c / \sigma_n^2$ and $R = B_c / n$ into (26), and $\mathbb{E}\{|\mathcal{L}_{s,md}|\} / |\mathcal{A}_s| \approx Q \left(\sqrt{\frac{\bar{P}_s n M}{2\sigma_n^2}} \right)$, where

$\sigma_n^2 = \bar{P}_c |\mathcal{A}_c| + \bar{P}_s |\mathcal{A}_s| + \sigma_z^2$. The MSEAOA is calculated using Definition 2 by employing the single-user scenario as $\Delta = 0.5\sigma_n^2 / (\pi^2 n \bar{P}_s \sum_{i=1}^{M-1} i^2)$. The ‘‘TDMA-MUSIC’’ model divides the length- n frame into two subframes with lengths $n/2$. In the first subframe, the TDMA strategy is employed for decoding the communication users, and in the second subframe, $|\mathcal{A}_s|$ sensing users transmit their signals simultaneously. The AOs of the sensing users in the second subframe are determined using MUSIC estimator.

For the communication part, $\mathbb{E}\{|\mathcal{L}_{c,md}|\} / |\mathcal{A}_c|$ is approximated by putting $S = M \bar{P}_c / \sigma_z^2$ and $R = B_c / N_{\text{TDMA}2}$ into (26), where $N_{\text{TDMA}2} = 0.5n / |\mathcal{A}_c|$. The MSEAOA is obtained by (27), considering $|\mathcal{A}_s|$ users’ transmission through $n/2$ channel uses. In the ‘‘ALOHA’’ model, we divide the frame into T subframes, and each user randomly selects a subframe to transmit its signal. The receiver only identifies signals from users that are not involved in collisions. The block error of the sensing and communication users is approximated by $\mathbb{E}\{|\mathcal{L}_{l,md}|\} / |\mathcal{A}_l| \approx 1 - (1 - P_{md,l}) (1 - f_{po}(1; \alpha_{po}) / \sum_{i=1}^{\infty} f_{po}(i; \alpha_{po}))$, where $\alpha_{po} = (|\mathcal{A}_s| + |\mathcal{A}_c|) / T$, $l \in \{c, s\}$, $f_{po}(i; a)$ denotes the probability distribution function of the Poisson distribution with the parameter a , $P_{md,c}$ is approximated by plugging $S = M \bar{P}_c / \sigma_z^2$ and $R = B_c T / N_{\text{ALOHA}}$ in (26), and $P_{md,s}$ is calculated as $P_{md,s} = Q \left(\sqrt{\frac{\bar{P}_s N_{\text{ALOHA}} M}{2\sigma_z^2}} \right)$, with $N_{\text{ALOHA}} = n / T$. In these computations, we use the approximation that the number of users engaged in an i -collision follows a Poisson distribution with a parameter of α_{po} [18]. Also, the MSEAOA is obtained using Definition 2, which gives $\Delta = 0.5\sigma_z^2 / (\pi^2 N_{\text{ALOHA}} \bar{P}_s \sum_{i=1}^{M-1} i^2)$.

In Fig. 2, we compare the achievability performance of the proposed UNISAC with the existing multiple access models by plotting the required E/N_0 to reach the target PUPE and MSEAOA, where E/N_0 is the average per-user energy of

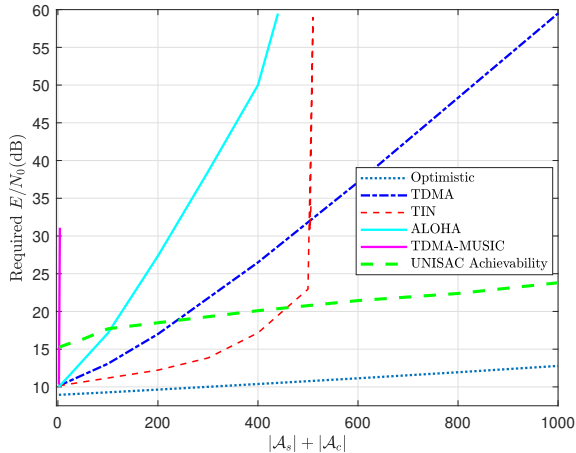


Fig. 2: The required E/N_0 as a function of the number of active sensing and communication users (with $|\mathcal{A}_c| = |\mathcal{A}_s|$), comparing UNISAC’s achievable result against existing multiple access models to achieve a target PUPE of $\epsilon_0 = 0.1$ and a target MSEAOA of $\Delta_0 = 5 \times 10^{-4}$.

the system which is defined as

$$\frac{E}{N_0} = \frac{(\bar{P}_c |\mathcal{A}_c| n_c + \bar{P}_s |\mathcal{A}_s| n_s)}{(|\mathcal{A}_c| + |\mathcal{A}_s|) \sigma_z^2}, \quad (31)$$

where n_c and n_s denote the signal lengths of each communication and sensing user, respectively. The x -axis in this figure is $|\mathcal{A}_s| + |\mathcal{A}_c|$ with $|\mathcal{A}_s| = |\mathcal{A}_c|$. Observing Fig. 2, it is clear that the UNISAC model demonstrates significantly better performance than other models under conditions with a high number of active users. The TDMA-MUSIC model’s underperformance stems from its inability to handle cases where the number of sensing users surpasses the number of antenna elements. The poor performance of ALOHA, TIN, and TDMA with a large number of users are attributed to dense collisions, high interference levels, and short signal lengths, respectively. Conversely, the UNISAC model has potential to achieve superior performance by mitigating collisions, reducing interference, and assigning longer signal lengths.

In order to evaluate the effectiveness of the achievable result for the MSEAOA, Fig. 3 compares the MSEAOA presented in (6) with the optimistic outcome illustrated in (30), considering various values of M . To obtain this result, we assume that $P_{K_s, K_c} = 1$ when $K_s = 0$ and $K_c = 0$, and $P_{K_s, K_c} = 0$ otherwise. This figure illustrates the enhancement in the AOA estimation achieved by the UNISAC model as the number of antenna elements is increased. Additionally, the minimal difference between the achievability and optimistic results indicates a tight capacity region for the AOA estimation provided by the UNISAC model.

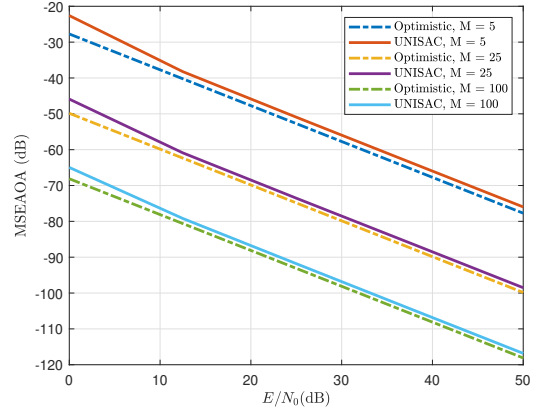


Fig. 3: Achievable and optimistic results of MSEAOA for different numbers of antenna elements M .

V. CONCLUSIONS

In this paper, we have addressed the problem of ISAC in scenarios with a massive number of unsourced and uncoordinated users. The proposed system allows active communication and sensing users to share a short transmission frame without coordination with the base station. We introduced an achievable performance bound for the proposed ISAC system and compared its effectiveness against traditional models like ALOHA, TDMA, TIN, and MUSIC. Through computer simulations, we have verified the effective performance of the proposed model in detecting and decoding a large number of users. This research contributes to advancing the understanding and practical implementation of ISAC systems in real-world scenarios with massive user counts.

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