

# Precise calculation of the $W$ boson pole mass beyond the Standard Model with FlexibleSUSY

Peter Athron<sup>1</sup>, Markus Bach<sup>2</sup>, Douglas H. J. Jacob<sup>3</sup>, Wojciech Kotlarski<sup>4</sup>, Dominik Stöckinger<sup>2</sup> and Alexander Voigt<sup>5</sup>

<sup>1</sup>Department of Physics and Institute of Theoretical Physics, Nanjing Normal University, Nanjing, Jiangsu 210023, China

<sup>2</sup>Institut für Kern- und Teilchenphysik, TU Dresden, Zellescher Weg 19, 01069 Dresden, Germany

<sup>3</sup>School of Physics and Astronomy, Monash University, Melbourne, Victoria 3800, Australia

<sup>4</sup>National Centre for Nuclear Research, Pasteura 7, 02-093 Warsaw, Poland

<sup>5</sup>Fachbereich Energie und Biotechnologie, Hochschule Flensburg, Kanzleistraße 91–93, 24943 Flensburg, Germany

We present an updated calculation of the  $W$  boson pole mass in models beyond the Standard Model with FlexibleSUSY. The calculation has a decoupling behavior and allows for a precise  $W$  pole mass prediction up to large new physics scales. We apply the calculation to several Standard Model extensions, including the MRSSM where we show that it can be compatible with large corrections to the  $W$  boson mass that would be needed to fit the recent 2022 CDF measurement.

## Contents

<b>1</b>	<b>Introduction</b>	<b>2</b>
<b>2</b>	<b>Non-decoupling problem in BSM calculations of the <math>W</math> boson pole mass</b>	<b>4</b>
<b>3</b>	<b>Calculation of the <math>W</math> boson pole mass</b>	<b>6</b>
3.1	Calculation of $M_W$ in FlexibleSUSY . . . . .	6
3.2	Uncertainty of $M_W$ . . . . .	8
<b>4</b>	<b>Applications</b>	<b>9</b>
4.1	MSSM . . . . .	9
4.2	Scalar Singlet Model . . . . .	11
4.3	MRSSM . . . . .	13
<b>5</b>	<b>Conclusions</b>	<b>15</b>
	<b>References</b>	<b>17</b>

# 1 Introduction

The pole mass of the  $W$  boson,  $M_W$ , is a very important precision test of the Standard Model (SM). The relationship between the muon decay and electroweak gauge boson masses and couplings leads to a prediction of  $M_W$  from the more precisely known values of the muon decay constant  $G_F$ , the  $Z$  boson pole mass  $M_Z$  and the electromagnetic coupling  $\alpha_{\text{em}}$ . Comparing this  $M_W$  prediction against the experimental measurement is a precision test of the SM. New physics can alter the prediction via quantum corrections from new particles or in some cases through a modified tree-level relationship between neutral and charged weak currents (i.e. tree level corrections to the so-called  $\rho$  parameter).

Updating the state-of-the-art SM calculation<sup>1</sup> in the on-shell (OS) scheme [1] with the latest data<sup>2</sup> we obtain  $M_W^{\text{SM,OS}} = 80.355 \text{ GeV}$  with the largest uncertainty being around 6 MeV (see Section 3.2 for more details). The state-of-the-art SM modified minimal subtraction ( $\overline{\text{MS}}$ ) calculation [2] performed at the same orders in perturbative couplings gives  $M_W^{\text{SM},\overline{\text{MS}}} = 80.351 \text{ GeV}$ , which is 4 MeV lower than the OS calculation. On-shell and  $\overline{\text{MS}}$  estimates of the uncertainties were 4 MeV in Ref. [1] and 3 MeV in Ref. [2], respectively, slightly lower than the renormalization scheme dependence difference, while Ref. [2] argues that the uncertainty of the  $\overline{\text{MS}}$  calculation should be smaller than the difference from the renormalization scheme dependence. However both predictions suffer from significant parametric uncertainties, the largest coming from the top quark mass. The value of  $M_W$  can also be predicted through global fits that do not include the direct measurements. Recent results give similar predictions to the OS calculation, see e.g.  $M_W^{\text{SM}} = (80.354 \pm 0.007) \text{ GeV}$  [3] and  $M_W^{\text{SM}} = (80.3591 \pm 0.0052) \text{ GeV}$  [4], where in these cases the parametric uncertainties are correctly combined in the fit.

There are many important precision measurements of the  $W$  mass that can influence the world average. Precision measurements of the  $W$  boson mass were performed by LEP experiments, ALEPH [5], DELPHI [6], L3 [7], OPAL [8] and these measurements have been taken into account in the LEP combination [9]. While  $W$  mass measurements are more challenging at hadron colliders, the 2012 D0 [10] and 2012 CDF [11] were able to improve the precision from an uncertainty of 33 MeV from the LEP combination to a 2013 Tevatron combination [12] that has an uncertainty of 16 MeV. Finally a 2017 ATLAS [13], and a recent 2021 LHCb [14] measurement have also been made. The most precise individual measurements are the ATLAS and CDF measurements (19 MeV), while a 2021 world average of  $M_W^{2021} = (80.379 \pm 0.012) \text{ GeV}$  was presented in the 2021 update to Ref. [15]. Most recently, however, CDF were able to use their full dataset (increasing from  $2.2 \text{ fb}^{-1}$  to  $8.8 \text{ fb}^{-1}$  of data) to significantly reduce the uncertainty with a result of  $M_W^{\text{CDF}} = (80.4335 \pm 0.0094) \text{ GeV}$  [16]. The uncertainty is now of a similar size to the theory uncertainty, but the central value has shifted so that it is now  $7\sigma$  away from the SM prediction. This however means that the new result is also in significant tension with previous measurements and the 2021 world average. Nonetheless we anticipate the new CDF measurement will only increase interest in precision predictions of  $M_W$ .

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<sup>1</sup>Both kinds of state-of-the-art SM calculations of  $M_W$  are organized as calculations of relationships between  $M_W$  and other precisely known electroweak quantities, which are numerically solved for  $M_W$ . The OS calculation is based on a relationship between  $M_W$  and  $M_Z$ ,  $G_F$  and the fine structure constant in the Thomson limit  $\alpha_{\text{em}}$  via the quantity  $\Delta r$ . The  $\overline{\text{MS}}$  calculation (which might also be called a mixed  $\overline{\text{MS}}$ /OS calculation) is based on a relationship between  $M_W$ ,  $G_F$ , the running  $\alpha_{\text{em}}$  and the running weak mixing angle  $\sin \theta_W$  via the quantity  $\Delta \hat{r}_W$ , where the running couplings are related to their respective OS counterparts via quantities  $\Delta \alpha_{\text{em}}$  and  $\Delta \hat{\rho}$ .

<sup>2</sup>We update the fit formulae from Ref. [1] with the data shown in Section 4.

Indeed very recently Ref. [17] demonstrated that new CDF  $M_W$  result can be explained in the two-Higgs doublet model. In Ref. [18], the inert 2HDM was found to be able to explain the 2022 CDF experimental result with a dark matter candidate with a mass between 54 GeV and 74 GeV. Ref. [19] has shown that it is possible to explain the muon  $g - 2$  anomaly [20] and the 2022 CDF measurement using a pair of scalar leptoquarks mixed together. Ref. [21] investigates the possibility of producing contributions which explain the 2022 value of  $M_W$  using axion-like particle, a dark photon, or chameleon dark energy. While they found that an axion-like particle and the dark photon had viable parameter regions which could explain the CDF value, chameleon dark energy was shown to be heavily constrained. Ref. [22] also found that it was possible to explain the CDF value of  $M_W$  through coupling a  $Z'$  boson directly to the Higgs. Ref. [23] finds low energy MSSM scenarios that can fit the large CDF  $M_W$ , while evading collider limits while Ref. [24] looks at SUSY explanations in the framework of extraordinary gauge mediation. Ref. [25] looked at explanations of this  $M_W$  anomaly and the GeV antiproton/gamma-ray excesses. The global electroweak fits were also updated in Refs. [17, 22, 26].

Here we present a new generic precision calculation of the  $W$  boson pole mass that can be used in almost any Standard Model extension via `FlexibleSUSY` [27–29]. Previously in `FlexibleSUSY`, a  $\overline{\text{MS}}/\overline{\text{DR}}$  calculation was implemented [28], which included full one-loop and leading SM-like two-loop contributions. However the direct  $\overline{\text{MS}}/\overline{\text{DR}}$  calculation in BSM models can suffer from non-decoupling logarithms (see e.g. Figure 9 of Ref. [30]) generated from spurious incomplete higher-order corrections that are included. This can severely spoil the precision, rendering it ill equipped for resolving between the SM and new physics models, potentially even for a very large deviation, like the 2022 CDF measurement implies. The new calculation resolves this problem by implementing the calculation with a strict separation between the SM contributions and the BSM contributions. The SM calculation is performed at state-of-the-art precision, while the BSM part is a strict one-loop calculation with no spurious higher-order corrections inadvertently incorporated.

We apply this new calculation to several extensions of the Standard Model, namely the Minimal Supersymmetric Standard Model (MSSM), the Scalar Singlet Model (SSM) and the Minimal  $R$ -symmetric Supersymmetric Standard Model (MRSSM) [31]. We demonstrate the decoupling property of the models, and show scenarios where the models can predict the measured values of  $M_W$ , including the most recent measurement from CDF in the case of the MRSSM. In particular this means we provide a precise interpretation of the 2022 CDF measurement in the MRSSM, demonstrating that this very well motivated model can explain this dramatic new experimental result.

While the precision of our BSM calculation is not as high as the dedicated calculations in the SM [1, 2] and the MSSM [32, 33], it provides the most precise calculation of  $M_W$  in many SM extensions where no two-loop calculations have been performed and matches the SM precision in the decoupling limit. As such we believe the calculation that is distributed with `FlexibleSUSY` can be a very useful tool for studying the implications of the CDF measurement.

In the SM the precision of the state-of-the-art predictions in the OS and  $\overline{\text{MS}}$  schemes has been improved through many contributions that have been carefully evaluated over the years. They include full one-loop [34, 35] and full two-loop [36–53] corrections, as well as further important higher order corrections [54–63]. It is challenging to apply the same level of precision in extensions of the Standard Model but in this work we include precisely the same SM corrections as in the state-of-the-art SM  $\overline{\text{MS}}$  calculation of Ref. [2].

The MSSM is the most widely studied SM extension and  $M_W$  calculations have been per-

formed at the full one-loop [64–66], building on earlier work in Refs. [67–75], and two-loop [32, 76–80] level. The  $M_W$  calculation was also performed in the 2HDM in Ref. [81], who found an improvement compared to the SM prediction, similar to the MSSM.

A closely related calculation of the  $\rho$  parameter in the 2HDM was presented in Ref. [82]. The calculation of  $M_W$  has been performed in the Scalar Singlet Model in Ref. [83] in the on-shell scheme at the one-loop level. In the NMSSM an early calculation was performed in Ref. [84], while Ref. [85] generalised the  $\overline{\text{DR}}$  MSSM calculation to the NMSSM and a complete one-loop on-shell calculation has been performed for the BSM contributions [86] with state-of-the-art SM corrections and additional higher order SUSY corrections taken in account. The MRSSM predictions for  $M_W$  have been performed at the one-loop level [87] in the  $\overline{\text{DR}}$  scheme and in a recent on-shell calculation at the full one-loop level, which also takes into account all known SM contributions from higher orders [30].

However, dedicated calculations in many other models do not exist at all, and given the number of possible extensions of the SM it is hardly possible to carry out dedicated calculations that are rigorous and precise for each one. Simplistic estimations carried out for wider phenomenological work or rapid responses to new data run the risk of lacking sufficient precision to accurately resolve between new physics models and the Standard Model. Furthermore, a lack of understanding of the precision of simplistic estimates may result in faulty conclusions.

Auto-generated calculations from `FlexibleSUSY` [27–29] and `SARAH/SPheno` [88–92], on the other hand, aim to provide high precision calculations available for a very wide range of models. Unfortunately until now both of these codes implemented  $\overline{\text{MS}}/\overline{\text{DR}}$  calculations that suffer from non-decoupling logarithms that can spoil the precision if the the new physics particles are heavy. We hope that the calculation presented here and its availability within the `FlexibleSUSY` package will allow for more stable and precise calculations to be performed without requiring projects and papers focused on just developing dedicated calculations in individual models. While a proper quantitative assessment of the actual uncertainty is model dependent and we do not provide a thorough analysis of the numerical uncertainty in applications, we still hope that the existence of standard tools such as `SARAH/SPheno` and `FlexibleSUSY` implementing calculations like the one presented here can reduce the burden of this by generating the calculations for many different models in a uniform way.

This paper is structured as follows: In Section 2 we discuss the non-decoupling problem present in previous  $M_W$  calculations in `FlexibleSUSY` and other spectrum generators; in Section 3 we describe the improved calculation of  $M_W$  in `FlexibleSUSY`. In Section 4 we study the  $M_W$  prediction different BSM models with `FlexibleSUSY`. We conclude in Section 5.

## 2 Non-decoupling problem in BSM calculations of the $W$ boson pole mass

Since version 2.0 the  $W$  boson pole mass  $M_W$  can be predicted in `FlexibleSUSY`-generated BSM spectrum generators [28], given the experimental values for the  $Z$  boson pole mass  $M_Z$  and the Fermi constant  $G_F$  as input.<sup>3</sup> That calculation, like the ones in other public spectrum

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<sup>3</sup>The calculation of  $M_W$ , given  $G_F$  as input, is enabled by default in `FlexibleSUSY`, if the BSM model contains all SM particles and the BSM gauge group contains the SM gauge group as a factor. If these conditions are not fulfilled,  $M_W$  is used as input. To enforce the calculation of  $M_W$ , set `FSWeakMixingAngleInput = FSFermiConstant` in the corresponding `FlexibleSUSY` model file. To enforce the use of  $M_W$  as input, set `FSWeakMixingAngleInput = FSMassW`. See Ref. [28] for details.

generators, suffered from a non-decoupling problem which we describe here. Readers interested in the current calculation in `FlexibleSUSY` 2.7.0 may skip to Section 3, where the improved calculation is presented in a self-contained way.

The non-decoupling problem is present in all BSM calculations of  $M_W$  which are based on a straightforward generalization of the SM calculation in the  $\overline{\text{MS}}$  scheme of Ref. [93], such as in Ref. [66] and in the codes `SARAH/SPHeno` [88–92] and `FlexibleSUSY` 2.0.0–2.4.2 [28]. The issue was not a problem as long as the precision need for BSM calculations was lower and as long as only BSM masses around the electroweak scale were considered. The problem was first explicitly mentioned and discussed in Ref. [30], where the `SARAH/SPHeno` MRSSM implementation was compared with an on-shell scheme calculation with manifest decoupling.

In order to explain the problem and its origin we start from the basic relation for  $M_W$  put forward in Ref. [93], updated in Ref. [2],<sup>4</sup>

$$G_F = \frac{\pi \alpha_{\text{em}}(M_Z)}{\sqrt{2} M_W^2 \left(1 - \frac{M_W^2}{\hat{\rho} M_Z^2}\right)} (1 + \Delta\hat{r}_W) . \quad (1)$$

The building blocks are defined in the original references; in particular the `FlexibleSUSY` implementations are defined in Refs. [27, 28]. To illustrate the problem we focus on the influence of the fine structure constant

$$\alpha_{\text{em}}(M_Z) = \frac{\alpha_{\text{em,SM}}^{(5),\overline{\text{MS}}}(M_Z)}{1 - \Delta\alpha_{\text{em}}^{\text{SM}} - \Delta\alpha_{\text{em}}^{\text{BSM}}} , \quad (2)$$

where  $\Delta\alpha_{\text{em}}^{\text{BSM}}$  is a one-loop expression containing logarithms of the form  $\log(m_{\text{BSM}}/M_Z)$ .

In calculations using this approach, Eq. (1) is solved for  $M_W$  and evaluated exactly by numerical iteration, while its building blocks such as  $\Delta\alpha_{\text{em}}^{\text{BSM}}$  are evaluated at fixed order perturbation theory. In the SM it is known that this approach resums important higher-order contributions [93], similar to the resummation in the context of the on-shell scheme of Ref. [40].

However, in a BSM context with heavy BSM masses the situation changes. E.g.  $\Delta\alpha_{\text{em}}^{\text{BSM}}$  schematically enters as

$$G_F = \left[ \frac{\pi \alpha_{\text{em}}(M_Z)}{\sqrt{2} M_W^2 \left(1 - \frac{M_W^2}{\hat{\rho} M_Z^2}\right)} (1 + \Delta\hat{r}_W) \right]_{\Delta\alpha_{\text{em}}^{\text{BSM}}=0} \times \left[ 1 + \Delta\alpha_{\text{em}}^{\text{BSM}} + (\Delta\alpha_{\text{em}}^{\text{BSM}})^2 + \dots \right] , \quad (3)$$

where the dots denote further terms involving  $\Delta\alpha_{\text{em}}^{\text{BSM}}$  of two-loop and higher order. For large BSM masses,  $\Delta\alpha_{\text{em}}^{\text{BSM}}$  increases logarithmically, i.e. it is non-decoupling. It is clear that in a complete calculation the prediction for  $M_W$  such non-decoupling effects must cancel order by order.

The calculations of Refs. [28, 66, 85, 88–92, 94] are complete at the one-loop BSM level. Hence the term  $\Delta\alpha_{\text{em}}^{\text{BSM}}$  in Eq. (3) combines with other one-loop terms such that the large one-loop logarithms cancel in the prediction for  $M_W$ . However the effectively generated two-loop

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<sup>4</sup>This equation is equivalent to Eqs. (57) and (67) of Ref. [28] if the appropriate relationship between the quantities  $\Delta\hat{r}$  and  $\Delta\hat{r}_W$  is used. Here and in all of this section we ignore the possibility for tree-level contributions to the  $\hat{\rho}$  parameter  $\Delta\hat{\rho}_{\text{tree}} \neq 0$ , since they are not relevant for the present discussion.

logarithms from the  $(\Delta\alpha_{\text{em}}^{\text{BSM}})^2$ -term cannot cancel since the calculation is not complete at the two-loop level.

For this reason the numerical evaluation of Eq. (1) with one-loop evaluation of all BSM building blocks leads to non-cancelling large logarithms. These are formally of two-loop or higher order and arise via  $(\Delta\alpha_{\text{em}}^{\text{BSM}})^2$  and similar terms involving the other building blocks  $\Delta\hat{\rho}$  and  $\Delta\hat{r}_W$ .

In typical applications such as the ones presented below or in Ref. [30], the numerical value of  $\Delta\alpha_{\text{em}}^{\text{BSM}}$  is around a few percent. Accordingly,  $(\Delta\alpha_{\text{em}}^{\text{BSM}})^2$  and other non-decoupling two-loop effects can be numerically estimated to be of the order permille and to shift  $M_W$  by  $\mathcal{O}(100 \text{ MeV})$ . This is exactly what has been observed in Ref. [30], see in particular their Figure 9. The behavior of the FlexibleSUSY 2.0 computation of  $M_W$  is essentially the same as the one of SARAH/SPheno, see Figure 10 of Ref. [28].

In order to avoid the non-decoupling problem, the computation must be explicitly truncated such that BSM effects are taken into account precisely at one-loop order. The resulting approach is described in the next section.

### 3 Calculation of the W boson pole mass

In FlexibleSUSY 2.5.0 the calculation of  $M_W$  has been modified to increase the precision and to avoid the non-decoupling behavior explained in the previous section (which used to be present e.g. in the calculation of  $M_W$  performed by FlexibleSUSY 2.0 [28]). The decoupling behavior is achieved by a strict one-loop calculation of BSM contributions, avoiding spurious incomplete two-loop contributions, such that in the limit of increasing BSM particle masses the predicted value for  $M_W$  converges to the SM prediction. In FlexibleSUSY 2.7.0 the calculation of  $M_W$  has been refined further to include the state-of-the-art SM contributions.

In the following we briefly describe the calculation of  $M_W$  in FlexibleSUSY 2.7.0.

#### 3.1 Calculation of $M_W$ in FlexibleSUSY

The calculation of  $M_W$  in FlexibleSUSY since version 2.5.0 is an adaptation of the procedure presented in Ref. [2] to BSM models, starting again effectively from Eq. (1). Its goals are to avoid the non-decoupling problem for heavy BSM masses but also to treat the SM contributions with the highest possible precision. Given the discussion of the previous section, the solution is to use Eq. (1), split all its building blocks into a sum of SM and BSM contributions, and solve for the ratio  $M_W^2/(M_W^{\text{SM}})^2$ , where  $M_W^{\text{SM}}$  is the SM prediction. This ratio is analytically evaluated as a strict fixed-order perturbative series, in our case truncated at one-loop order. In this way, decoupling is obtained, and the state-of-the-art SM prediction can be combined with a fixed-order BSM correction. As a result, the  $W$  boson pole mass can be expressed as

$$M_W^2 = (M_W^{\text{SM}})^2 (1 + \Delta_W). \quad (4)$$

In FlexibleSUSY 2.5.0–2.6.2 the value of  $M_W^{\text{SM}}$  has been fixed to  $M_W^{\text{SM}} = 80.385 \text{ GeV}$ , and since version 2.7.0 the value of  $M_W^{\text{SM}}$  is calculated using the fit formula of Eq. (45) from Ref. [2]. The term  $\Delta_W$  contains the tree-level and one-loop BSM contributions in the relation (4) between the SM prediction  $M_W^{\text{SM}}$  and the BSM prediction  $M_W$  and is given by

$$\Delta_W = \frac{s_W^2}{c_W^2 - s_W^2} \left[ \frac{c_W^2}{s_W^2} (\Delta\hat{\rho}_{\text{tree}} + \Delta\hat{\rho}_{\text{BSM}}) - \Delta\hat{r}_{W,\text{BSM}} - \Delta\alpha_{\text{em}}^{\text{BSM}} \right]. \quad (5)$$



All parameters entering this equation are consistently defined as BSM  $\overline{\text{MS}}/\overline{\text{DR}}$  parameters at the renormalization scale  $Q = M_Z$ .<sup>5</sup> Accordingly all appearing one-loop corrections are evaluated with the renormalization scale set to  $Q = M_Z$ . We have abbreviated  $s_W \equiv \sin(\theta_W)$  and  $c_W \equiv \cos(\theta_W)$ , where  $\theta_W$  is the BSM weak mixing angle in the  $\overline{\text{MS}}/\overline{\text{DR}}$  scheme at  $Q = M_Z$ . It is calculated from the relation [93]

$$s_W^2 c_W^2 = \frac{\pi \alpha_{\text{em}}(M_Z)}{\sqrt{2} M_Z^2 G_F \hat{\rho}_{\text{tree}} (1 - \Delta\hat{r})}. \quad (6)$$

With  $\alpha_{\text{em}}(M_Z)$  we denote the  $\overline{\text{MS}}/\overline{\text{DR}}$  electromagnetic coupling in the BSM model at the renormalization scale  $Q = M_Z$ . The calculation of the loop correction  $\Delta\hat{r}$  in Eq. (6) is described in Section 8 of Ref. [28].

The term  $\Delta\hat{\rho}_{\text{tree}}$  in Eq. (5) denotes the tree-level BSM contribution to the  $\rho$ -parameter and is defined as

$$\Delta\hat{\rho}_{\text{tree}} = \hat{\rho}_{\text{tree}} - 1, \quad (7)$$

where  $\hat{\rho}_{\text{tree}}$  is the tree-level  $\rho$ -parameter in the BSM model.<sup>6</sup> The term  $\Delta\hat{\rho}_{\text{BSM}}$  in Eq. (5) contains the pure BSM one-loop contributions in the relation between the SM  $\rho$ -parameter,  $\hat{\rho}_{\text{SM}}$ , and the loop-level BSM  $\rho$ -parameter,  $\hat{\rho}_{\text{BSM}}$ , and is calculated as

$$\Delta\hat{\rho}_{\text{BSM}} = \frac{1}{m_Z^2} \left[ \Sigma_Z(m_Z^2) - \Sigma_Z^{\text{SM}}(m_Z^2) \right] - \frac{1}{m_W^2} \left[ \Sigma_W(m_W^2) - \Sigma_W^{\text{SM}}(m_W^2) \right], \quad (8)$$

where  $\Sigma_Z(p^2)$  and  $\Sigma_W(p^2)$  are the real parts of the transverse components of the momentum-dependent  $\overline{\text{MS}}/\overline{\text{DR}}$ -renormalized full one-loop BSM  $W$  and  $Z$  boson self-energies, evaluated at the squared momenta  $p^2 = m_W^2$  and  $p^2 = m_Z^2$ , respectively, where  $m_W$  and  $m_Z$  are the BSM  $\overline{\text{MS}}/\overline{\text{DR}}$   $W$  and  $Z$  boson masses. The symbols  $\Sigma_Z^{\text{SM}}(p^2)$  and  $\Sigma_W^{\text{SM}}(p^2)$  are the corresponding  $\overline{\text{MS}}$ -renormalized SM counterparts. The subtraction of the SM from the BSM  $W$  and  $Z$  self-energies is performed numerically so that mixing effects from new BSM particles are correctly taken into account. The one-loop contribution  $\Delta\hat{r}_{W,\text{BSM}}$  in Eq. (5) contains the pure BSM one-loop contributions to the relation (1) between  $M_W$  and  $G_F$  and is calculated as

$$\Delta\hat{r}_{W,\text{BSM}} = \frac{1}{m_W^2} \left[ \Sigma_W(0) - \Sigma_W^{\text{SM}}(0) - \Sigma_W(m_W^2) + \Sigma_W^{\text{SM}}(m_W^2) \right] + \delta_{\text{VB}}^{\text{BSM}}, \quad (9)$$

where  $\delta_{\text{VB}}^{\text{BSM}}$  contains the pure BSM vertex and box diagram contributions. The term  $\Delta\alpha_{\text{em}}^{\text{BSM}}$  in Eq. (5) denotes the one-loop threshold correction for the electromagnetic coupling between the SM and the BSM model, c.f. Section 5 in Ref. [28]. Since the r.h.s. of Eq. (4) depends on the value of  $M_W$ , the equation is solved iteratively using the experimentally measured value for  $M_W = 80.385$  GeV as initial value.

<sup>5</sup>The renormalization scale used to calculate  $\Delta_W$  with Eq. (5) is defined by the variable `LowScale` in the corresponding `FlexibleSUSY` model file. By default the variable is set to `LowScale = LowEnergyConstant[MZ]`, which corresponds to  $Q = M_Z$ .

<sup>6</sup>In this paper we always treat  $\Delta\hat{\rho}_{\text{tree}}$  as a small correction which is at most of the same numerical order as BSM one-loop corrections. Accordingly, the notion of “full BSM one-loop order” includes  $\Delta\hat{\rho}_{\text{tree}}$ , while products of  $\Delta\hat{\rho}_{\text{tree}}$  and one-loop quantities are neglected. Whenever we discuss decoupling and the limit of heavy BSM masses  $m_{\text{BSM}}$ , we implicitly assume that potential BSM vacuum expectation values contributing to  $\Delta\hat{\rho}_{\text{tree}}$  behave as  $1/m_{\text{BSM}}^2$ .

We remark that the prediction of  $M_W$  in Eq. (4) is complete at the one-loop level w.r.t. the BSM contributions. As described above this includes potential mixing effects between BSM and SM states. In this procedure mixing effects are accounted when the one-loop SM contributions in the fit formula for  $M_W^{\text{SM}}$  are cancelled through the numerical subtraction of the same contributions in Eqs. (8) and (9), such that the full one-loop pure BSM contributions remain. At the same time higher order SM contributions incorporated through the use of the SM fit formula do not include these mixing effects.

As stressed before, the calculation of  $M_W$  in Eq. (4) shows a decoupling property where  $\Delta_W$  in Eq. (5) behaves as  $1/m_{\text{BSM}}^2$  for increasing BSM particle masses. This is achieved by the strict treatment of the BSM loop corrections at one-loop level, where spurious two-loop and higher order contributions are avoided. It is particularly important that the expression for  $\Delta_W$  is evaluated in terms of a consistent set of parameters, perturbatively truncated at one-loop order. We have chosen the set of fundamental BSM  $\overline{\text{MS}}/\overline{\text{DR}}$  parameters at the renormalization scale  $Q = M_Z$ . One subtlety is that in this scheme, the running gauge couplings and  $\theta_W$  differ from the corresponding SM values, and the differences can contain non-decoupling logarithms of higher order. However, this does not spoil the decoupling property of the full expression  $\Delta_W$ , which is mainly governed by the  $1/m_{\text{BSM}}^2$  behavior, while any non-decoupling logarithms from  $\theta_W$  would only appear as a multiplicative factor to terms with this behavior. It would be possible to employ different schemes for the couplings entering Eq. (5), but studies of such alternatives are outside the scope of the present paper. We refer to Ref. [95], particularly Section 3.2, for a similar discussion of the role of different parametrizations and the danger of including fake logarithmically enhanced higher-order terms.

### 3.2 Uncertainty of $M_W$

The uncertainty  $\Delta M_W$  of the prediction of  $M_W$  with Eq. (4) can be divided into two contributions: The uncertainty  $\Delta M_W^{\text{SM}}$  from the calculation of  $M_W^{\text{SM}}$  from Ref. [2], which should be constructed from the parametric uncertainty and the missing higher order corrections, estimated at 9 MeV and 3 MeV respectively in Ref. [2]. Since we implement the SM prediction using the  $\overline{\text{MS}}$  fit formula there is also in principle an uncertainty from the fit itself, but this should always be less than 0.5 MeV. A second contribution is the uncertainty  $\Delta M_W^{\text{BSM}}$  from missing higher order BSM contributions. The latter of the two uncertainties is model-dependent and can be estimated for example by renormalization scale variation.

There exist simple parametrizations of  $M_W^{\text{SM}}$  from Eq. (4) from OS and  $\overline{\text{MS}}$  calculations. These parametrizations depend on the deviation of the masses  $M_h$ ,  $M_t$ ,  $M_Z$ , the hadronic contributions to the fine structure constant  $\Delta\alpha_{\text{had}}^{(5)}$ , and the strong coupling constant  $\alpha_s$  from predefined values. The parametrization of  $M_W^{\text{SM}}$  from the OS calculation is given in Eqs. (6)–(9) in Ref. [1]. The parametrization from the  $\overline{\text{MS}}$  calculation is given by Eq. (45) in Ref. [2]. Both are implemented in `FlexibleSUSY` 2.7.0, which takes `FlexibleSUSY`'s prediction of the Higgs boson pole mass and uses it to calculate the  $W$  boson pole mass. We show the variation that occurs in both OS and  $\overline{\text{MS}}$  predictions for  $M_W^{\text{SM}}$  in Table 1. As can be seen in the table we get a variation of up to  $\Delta M_W^{\text{SM}} = 6$  MeV when the top quark pole mass is varied over a 1 GeV range (c.f. the discussion of the uncertainty and the ambiguity of the definition of  $M_t$  in Section 60 of Ref. [15]).

Thanks to the decoupling property of the calculation, the prediction of  $M_W$  is precise even for heavy BSM spectra, where the uncertainty of the  $M_W$  prediction is only dominated by the SM prediction, i.e.  $\Delta M_W \approx \Delta M_W^{\text{SM}}$ . We study and illustrate this decoupling property in



	$\Delta M_W^{\text{SM}}$ (OS)	$\Delta M_W^{\text{SM}}$ ( $\overline{\text{MS}}$ )
$M_h \pm 1\sigma$	0.08 MeV	0.08 MeV
$M_t \pm 1\sigma$	1.8 MeV	1.8 MeV
$M_t \pm 1 \text{ GeV}$	6.0 MeV	6.1 MeV
$M_Z \pm 1\sigma$	2.6 MeV	—
$\Delta\alpha_{\text{had}}^{(5)} \pm 1\sigma$	1.3 MeV	1.3 MeV
$\alpha_s \pm 1\sigma$	0.59 MeV	0.62 MeV

**Table 1:** Variation of the  $M_W^{\text{SM}}$  pole mass, calculated in the on-shell and  $\overline{\text{MS}}$  scheme, respectively.  $M_h$ ,  $M_t$ ,  $M_Z$ ,  $\Delta\alpha_{\text{had}}^{(5)}$ ,  $\alpha_s$  are varied around their central values given in Eqs. (10) over their  $1\sigma$  range from the 2021 update of Ref. [15], and  $M_t$  is also varied by 1 GeV. Note that since the  $\overline{\text{MS}}$  formula from Ref. [2] is independent of  $M_Z$  we do not give a corresponding variation of  $M_W^{\text{SM}}$ .

Section 4 for concrete BSM models.

## 4 Applications

In the following we study the  $W$  boson pole mass prediction in different BSM models with `FlexibleSUSY`, as described in Section 3. We focus in particular on potential deviations from the SM prediction. If not stated otherwise, we use the following values for the SM parameters from Ref. [15]:

$$\begin{aligned}
\alpha_{\text{em}}(M_Z) &= 1/127.916, & G_F &= 1.1663787 \times 10^{-5} \text{ GeV}^{-2}, & \alpha_s(M_Z) &= 0.1179, \\
M_Z &= 91.1876 \text{ GeV}, & M_t &= 172.76 \text{ GeV}, & m_b(m_b) &= 4.18 \text{ GeV}, \\
\Delta\alpha_{\text{had}}^{(5)} &= 0.02766.
\end{aligned} \tag{10}$$

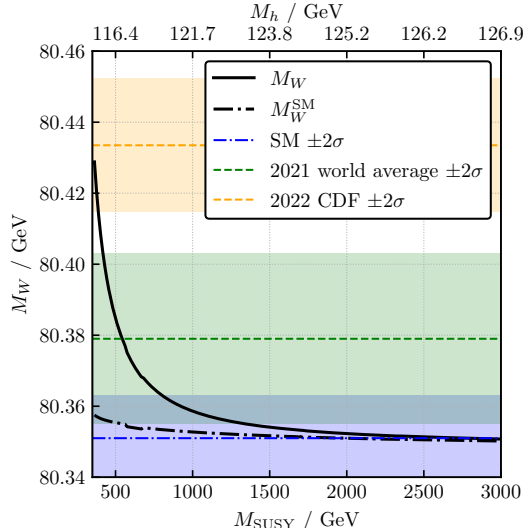
When the Higgs mass is also fixed to its measured value,  $M_h = 125.25 \text{ GeV}$ , in addition to using the parameters in Eqs. (10) for the other inputs, the SM prediction for the  $W$  boson pole mass in the  $\overline{\text{MS}}$  scheme is  $M_W^{\text{SM}} = 80.351 \text{ GeV}$  [2]. This is the SM prediction we use in the following sections.<sup>7</sup>

### 4.1 MSSM

The MSSM is one of the most widely studied and best motivated extensions of the Standard Model. Therefore we demonstrate our new calculation in this model. We use the following parameter scenario with large stop and sbottom mass splitting and a common supersymmetry mass scale  $M_{\text{SUSY}}$ :

$$\begin{aligned}
\tan \beta &= 20, & \mu &= m_A = M_i = M_{\text{SUSY}} \quad (i = 1, 2, 3), \\
A_t &= X_t + \mu / \tan \beta, & A_f &= 0 \quad (f = u, d, c, s, b, e, \mu, \tau), \\
m_{\tilde{q}}^2 &= m_{\tilde{u}}^2 = m_{\tilde{d}}^2 = M_{\text{SUSY}}^2 \mathbf{1}, & m_{\tilde{l}}^2 &= m_{\tilde{e}}^2 = (M_{\text{SUSY}}/2)^2 \mathbf{1},
\end{aligned} \tag{11}$$

<sup>7</sup>The on-shell calculation from Ref. [1] yields  $M_W^{\text{SM}} = 80.355 \text{ GeV}$  for the parameter set given in Eqs. (10), which is 4 MeV larger than the  $\overline{\text{MS}}$  calculation. A more recent on-shell calculation with `FeynHiggs` gave  $M_W^{\text{SM}} = (80.353 \pm 0.004) \text{ GeV}$  [33].



**Figure 1:** Prediction of  $M_W$  in the MSSM as a function of the common supersymmetry scale  $M_{\text{SUSY}}$  (black solid line). The black dash-dotted line shows the SM prediction using the  $\overline{\text{MS}}$  fit formula from Ref. [2] for the corresponding value of the Higgs pole mass  $M_h$  (top axis). The blue dash-dotted line shows the SM prediction  $M_W^{\text{SM}} = (80.351 \pm 0.006)$  GeV for the fixed value of the Higgs pole mass  $M_h = 125.25$  GeV, including its uncertainty. The green dashed line shows the 2021 world average experimental value  $M_W^{2021} = (80.379 \pm 0.012)$  GeV including its uncertainty. The orange dashed line shows the CDF value  $M_W^{\text{CDF}} = (80.4335 \pm 0.0094)$  GeV including its uncertainty.

with  $X_t = -\sqrt{6}M_{\text{SUSY}}$ . The slepton masses are chosen to be smaller than the squark masses in order to increase the loop corrections to  $M_W$ . The SM-like Higgs boson pole mass  $M_h$  has been calculated using a fixed-order  $\overline{\text{DR}}$  calculation at the full one-loop level, including dominant two-loop [96–100] and three-loop contributions via the `Himalaya` package [101–104].

The value of  $M_W$  predicted in the scenario (11) is shown in Figure 1 as black solid line as a function of the common supersymmetry mass scale  $M_{\text{SUSY}}$ . For increasing  $M_{\text{SUSY}}$  the prediction for  $M_W$  converges to the SM value  $M_W^{\text{SM}}$  (black dash-dotted line), which nicely illustrates the decoupling behavior of the calculation. In these scenarios the experimental value for  $M_h = 125.25$  GeV is correctly predicted by the MSSM for  $M_{\text{SUSY}} \approx 2023$  GeV. For this value of  $M_{\text{SUSY}}$  the MSSM predicts  $M_W \approx 80.352$  GeV, which deviates by 1 MeV from the SM prediction of  $M_W^{\text{SM}} \approx 80.351$  GeV. However in the simple scenario shown here we only see significant BSM corrections for light SUSY masses that are excluded by experimental searches and where the SM-like Higgs mass is also too small to match the measured value. Therefore the CDF value  $M_W^{\text{CDF}}$  (orange band in Figure 1) cannot be explained in this simple MSSM scenario we have used to demonstrate the decoupling property of our new calculation.

It is certainly possible to move away from this simplified scenario with common SUSY masses to try to obtain larger values of  $M_W$  when the Higgs boson pole mass is  $M_h = 125.25$  GeV. However we find it is rather challenging to fit a very large value while consistently evading experimental constraints on the sparticles and fitting the Higgs mass measurement. This is consistent with the findings in the literature [32], and a very recent paper [33] also showed that if one requires, in addition, an explanation of muon  $g - 2$  data, the maximum  $M_W$  in the

MSSM is 80.376 GeV. Therefore while we do not entirely exclude the possibility of an MSSM explanation we leave that for dedicated studies.<sup>8</sup> However we expect such scenarios will be rare and conclude that the prospects for explaining the 2022 CDF measurement are not great, and if it is confirmed this measurement motivates other extensions where larger  $M_W$  corrections are easier to obtain.

## 4.2 Scalar Singlet Model

Before we consider a model which can give larger corrections to the  $W$  boson pole mass we first briefly demonstrate the calculation in one of the most popular non-SUSY models, the Scalar Singlet Model (SSM). The SSM is a simple extension of the SM which couples a real scalar to the Higgs doublet. Here we look at the  $\mathbb{Z}_2$  symmetric SSM, which restricts the allowed interactions between the Higgs and scalar singlet, giving an extended Higgs potential of

$$V = -\mu^2|H|^2 + \frac{\lambda}{2}|H|^4 + \frac{\mu_S^2}{2}S^2 + \frac{\lambda_S}{2}S^4 + \frac{\lambda_{HS}}{2}H^\dagger HS^2. \quad (12)$$

The coupling  $\lambda_{HS}$  allows for mixing between the neutral component of the Higgs doublet  $H$  and the neutral scalar singlet when both have vacuum expectation values (VEVs)  $v$  and  $v_S$ , respectively,

$$H = \begin{pmatrix} \sigma^\pm \\ \frac{1}{\sqrt{2}}(\phi^0 + v + i\sigma^0) \end{pmatrix} \quad \text{and} \quad S = s + v_S, \quad (13)$$

which we assume here. Thus, after electroweak symmetry breaking (EWSB), the neutral scalar fields  $\phi_0$  and  $s$  are rotated into the Higgs boson mass eigenstates  $(h_1, h_2)^T$  via the mixing matrix  $R(\alpha)$ ,

$$\begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = R(\alpha) \begin{pmatrix} \phi^0 \\ s \end{pmatrix}, \quad \text{where} \quad R(\alpha) = \begin{pmatrix} \cos(\alpha) & -\sin(\alpha) \\ \sin(\alpha) & \cos(\alpha) \end{pmatrix}. \quad (14)$$

The SM-like Higgs mass is determined via the mixing angle  $\alpha$ . If  $\sin^2(\alpha) < 0.5$  the lighter Higgs is doublet-dominated and thus is associated with the SM-like Higgs. If instead  $\sin^2(\alpha) > 0.5$ , then the lighter Higgs is singlet-dominated.<sup>9</sup> In this model when the singlet-dominated state is heavier than the SM-like Higgs, we expect negative corrections to the  $W$  boson pole mass [83].

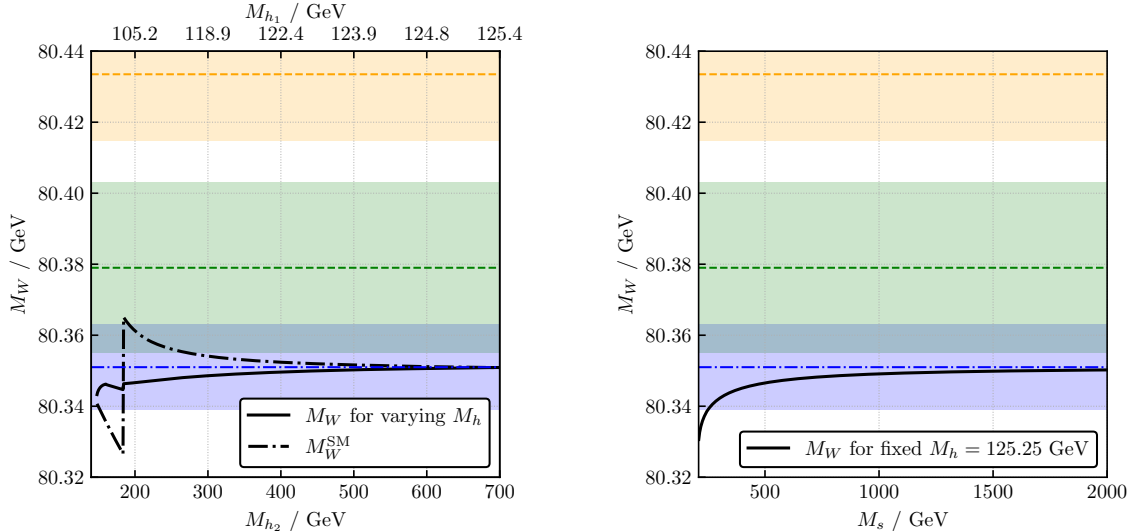
Following the Lagrangian of the SSM we examine two sets of parameter choices,

$$\lambda = 0.36, \quad \lambda_S = 1.4, \quad \lambda_{HS} = 0.8, \quad (15a)$$

$$M_h = 125.25 \text{ GeV}, \quad \lambda_S = 2.122, \quad \lambda_{HS} = 0.9917, \quad (15b)$$

<sup>8</sup>In fact while this paper was in preparation a Ref. [23] presented MSSM scenarios with large  $M_W$  corrections fitting the 2022 CDF measurement.

<sup>9</sup>Note that this selection is not done automatically in `FlexibleSUSY` 2.7.0. Instead the user must specify which index of the Higgs mass eigenstate multiplet corresponds to the SM-like Higgs. For example, this can be set in entry 22 of the `FlexibleSUSY` block in a Les Houches input file. In this model we fixed entry `FlexibleSUSY[22]` to 0 if the point has  $\text{ZH}[1,2] < \sqrt{0.5}$  (i.e. the entry of the block `ZH` corresponding to an off-diagonal element of  $R(\alpha)$  or  $\sin^2(\alpha)$  is less than 0.5), and `FlexibleSUSY[22]` fixed to 1 if  $\text{ZH}[1,2] > \sqrt{0.5}$ .



**Figure 2:** Prediction of  $M_W$  in the the Scalar Singlet Model with a  $\mathbb{Z}_2$  symmetry. The blue, green and orange lines and regions are as in Figure 1. In the left panel, the parameter set (15a) is used. As in Figure 1, the solid black lines indicate the prediction of  $M_W$  where the Higgs mass has been allowed to vary, and the black dash-dotted line shows the SM prediction using Eq. (45) from Ref. [2]. The light and heavier Higgs masses are shown on the top and bottom axes, respectively. The right panel uses the parameter set from (15b) and shows the prediction of  $M_W$  for a Higgs boson pole mass fixed to  $M_h = 125.25$  GeV, as a function of the scalar mass  $M_s$ , which is shown on the bottom axis.

while  $\mu_S^2$  and  $\mu^2$  are fixed to fulfill the one-loop electroweak symmetry breaking conditions. We scan over  $v_S$ , and get the results shown in Figure 2. The left panel is included to show how mixing effects between the SM Higgs and the scalar singlet are handled and subtle issues related to this. Here the input parameters are fixed as in Eqs. (15a), which includes the Higgs quadratic coupling  $\lambda$  and leaves both Higgs masses  $M_{h_i}$  varying (rather than fixing one to 125.25 GeV). The left panel shows two distinct regions, separated by a discontinuity in the SM contribution to  $M_W$  shown as the dash-dotted black line. The boundary of the two regions is at approximately  $v_S \approx 64$  GeV and  $M_{h_2} \approx 184$  GeV. On the left side of this boundary the singlet-dominated pole mass  $M_s$  is smaller than the SM-like Higgs mass  $M_h$ , while on the right side it is larger. Note that the SM contribution to  $M_W^{\text{SM}}$  is varying because it depends on the SM-like Higgs mass, which is not fixed to the measured value in this plot, but is instead varying. The mixing angle  $\alpha$  is used to determine which state is the SM-like one. As  $\sin(\alpha)$  passes through  $\sqrt{0.5}$ , the mass of the state we treat as SM-like changes from  $M_h = M_{h_1} \approx 98$  GeV to  $M_h = M_{h_2} \approx 184$  GeV, leading to the discontinuity in the SM contribution  $M_W^{\text{SM}}$ .

However, as described in Section 3, this mixing effect is correctly accounted for at the one-loop level in our calculation of  $M_W$  in Eq. (4). This is achieved by the inclusion of the full BSM one-loop contributions and the numerical subtraction of the pure SM one-loop part in  $\Delta_W$ . In this way the discontinuity is avoided in our SSM calculation of  $M_W$  at the one-loop level (black solid line in the left panel). However, the SM Higgs mass also enters the calculation of  $M_W^{\text{SM}}$  at the two-loop level, which is incorporated in the  $\overline{\text{MS}}$  fit formula we use. This two-loop pure SM-Higgs part is not cancelled in our calculation, because we aim to correctly include all known

higher order corrections to the SM contribution (via the fit formula) and do not include BSM contributions beyond one-loop.

Nonetheless, in the case of mixing between BSM and SM states, such as the Higgs as we see here, there is no clear definition of which state is the SM one. Therefore when we fix the SM Higgs to be the state which has the most doublet content, as we do here, we find a small discontinuity in the prediction of  $M_W$  in the BSM model, which is of two-loop order. This effect can be seen in the SSM studied here in the small discontinuity in the solid black line in the left panel in Figure 2. The impact of this is of the order 1–2 MeV and is thus encouragingly small, suggesting that the impact from this mixing issue is not large.

When the singlet-dominated state is the lighter one, we find positive contributions to  $M_W$  compared to the SM contributions, though for this unrealistic choice of the Higgs mass the prediction is actually smaller than that of the SM with  $M_h = 125.25$  GeV, as indicated by the blue dash-dotted line in left panel of Figure 2. When  $v_S > 64$  GeV, the singlet state is heavier than the SM-like Higgs state so the contributions are negative. As  $v_S$  increases towards the right of the plot the scalar singlet effectively decouples from the SM, and we see that the SSM and SM values of  $M_W$  approach each other. In the right panel of Figure 2, where the Higgs mass has been fixed to  $M_h = 125.25$  GeV, we examine the case  $M_s > M_h$ . This shows that while in the SSM one can get non-negligible negative contributions when the two Higgses are close together in mass, as the mass scale of the scalar increases it eventually decouples leaving behind the SM value of  $M_W$ .

We do not expect that the SSM can explain the value of  $M_W$  from the 2022 CDF measurement, because large positive corrections require a singlet-dominated state that is lighter than the SM-like Higgs, with very large singlet-doublet mixing. However it is clear that  $M_W$  is a relevant constraint in this model and the precision calculation presented here can be used in precision tests of the Scalar Singlet Model.

### 4.3 MRSSM

The MRSSM is a non-minimal supersymmetric extension of the Standard Model which has BSM contributions to the  $W$  boson mass at tree-level that can increase it above the SM prediction. Due to its  $R$ -symmetry all gauginos are Dirac particles in the MRSSM, and unwanted sources of flavor violation from supersymmetry-breaking trilinear couplings are forbidden. A detailed description of the model and the precise definitions for all parameters we refer to here can be found in Section 2 of Ref. [87].

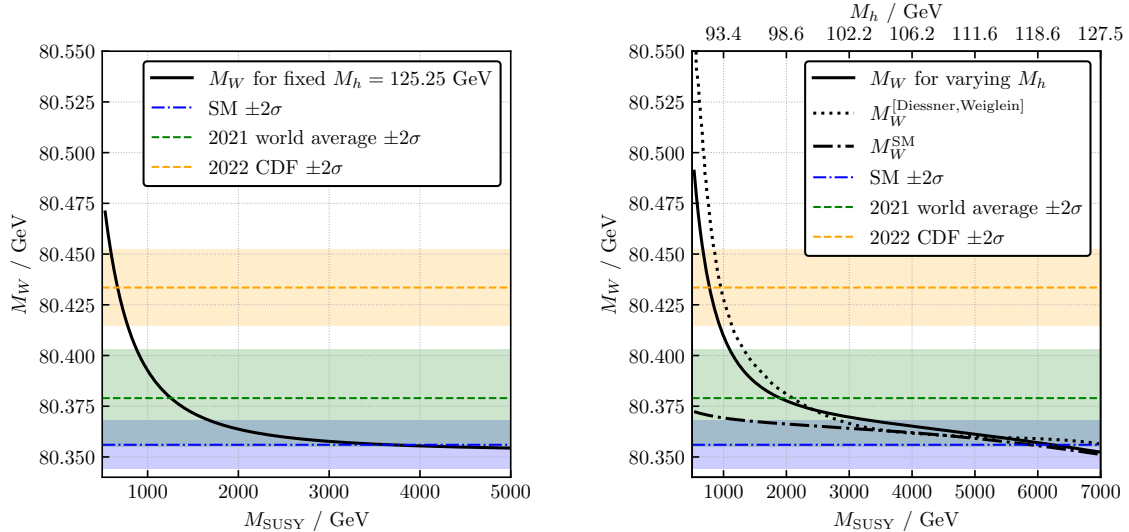
In addition we highlight here that the MRSSM contains two specific mechanisms which can increase the  $W$  boson mass, which distinguish it from e.g. the MSSM. The first mechanism is the appearance of new Yukawa-like parameters in the superpotential  $\Lambda_{u,d}$ ,  $\lambda_{u,d}$  which contribute to the  $W$  boson pole mass similarly as the top-Yukawa coupling [87]. Secondly, the MRSSM contains a Hypercharge  $Y = 0$ ,  $SU(2)_L$  Higgs triplet  $T$  with vacuum expectation value  $v_T$ , which is responsible for the positive tree-level contribution to the  $W$  boson mass [30, 31, 87].

At tree-level the relation between the  $\overline{\text{DR}}$   $W$  and  $Z$  boson masses  $m_W$  and  $m_Z$  reads,

$$m_W^2 = m_Z^2 \cos^2 \theta_W + g_2^2 v_T^2, \quad (16)$$

where  $\tan \theta_W \equiv g_1/g_2$ , which corresponds to a tree-level  $\rho$ -parameter of

$$\hat{\rho}_{\text{tree}} = 1 + \frac{4v_T^2}{v_d^2 + v_u^2} \quad (17)$$



**Figure 3:** Prediction of  $M_W$  in the MRSSM as a function of the common supersymmetry scale  $M_{\text{SUSY}}$  (black solid line), for parameter values as given in Eq. (18) and below. In the left panel the Higgs pole mass that is used to calculate  $M_W^{\text{SM}}$  is fixed to  $M_h = 125.25$  GeV. In the right panel the Higgs pole mass as predicted by the MRSSM is used to calculate  $M_W^{\text{SM}}$ . The black dash-dotted line shows the SM prediction using the  $\overline{\text{MS}}$  fit formula from Ref. [2] for the corresponding value of the Higgs pole mass  $M_h$  (top axis). The black dotted line shows the prediction from Ref. [30]. The blue dash-dotted line shows the SM prediction  $M_W^{\text{SM}} = (80.354 \pm 0.006)$  GeV for the fixed value of the Higgs pole mass  $M_h = 125.25$  GeV, including its uncertainty. The green dashed line shows the 2021 world average experimental value  $M_W^{2021} = (80.379 \pm 0.012)$  GeV including its uncertainty. The orange dashed line shows the CDF value  $M_W^{\text{CDF}} = (80.4335 \pm 0.0094)$  GeV including its uncertainty.

with  $g_1$  and  $g_2$  being the  $U(1)_Y$  and  $SU(2)_L$   $\overline{\text{DR}}$  gauge couplings, respectively. In Figure 3 we show the  $W$  boson pole mass as predicted by our improved calculation in the MRSSM as a function of a common supersymmetry mass scale  $M_{\text{SUSY}}$ , where the dimensionful superpotential and soft-breaking MRSSM parameters are set to [30]

$$\begin{aligned}
m_{R_u}^2 &= m_{R_d}^2 = m_S^2 = m_T^2 = m_O^2 = \frac{2B_\mu}{\sin 2\beta} = M_{\text{SUSY}}^2, \\
m_{\bar{q}}^2 &= m_{\bar{l}}^2 = m_{\bar{e}}^2 = m_u^2 = m_d^2 = M_{\text{SUSY}}^2 \mathbf{1}, \\
M_B^D &= M_W^D = M_O^D = \mu_d = \mu_u = \frac{M_{\text{SUSY}}}{2}.
\end{aligned} \tag{18}$$

The dimensionless superpotential couplings are set to  $\Lambda_d = -1$ ,  $\Lambda_u = -1.03$ ,  $\lambda_d = 1.0$  and  $\lambda_u = -0.8$ . The ratio of the Higgs doublet vacuum expectation values has been set to  $\tan \beta \equiv v_u/v_d = 3$ . The Higgs triplet vacuum expectation value  $v_T$  is fixed by the one-loop electroweak symmetry breaking conditions. The tree-level expressions for these can be found in Eqs. (2.15)–(2.16) in Ref. [87]. The top quark pole mass and the strong coupling have been set to  $M_t = 173.0$  GeV and  $\alpha_s(M_Z) = 0.1181$ , respectively, for comparison with Ref. [30].

In the left panel of Figure 3 the SM-like Higgs pole mass  $M_h$  that is used to calculate  $M_W^{\text{SM}}$  in Eq. (4) is fixed to the constant value  $M_h = 125.25$  GeV. The value of  $M_W$  predicted by the



MRSSM in this scenario (black solid line) shows the described decoupling behavior for increasing  $M_{\text{SUSY}}$  and converges to the SM prediction with a constant offset due to  $\hat{\rho}^{\text{tree}} \neq 1$ . However, the SM-like Higgs pole mass predicted by the MRSSM (using the one-loop fixed order calculation of `FlexibleSUSY`) actually depends on  $M_{\text{SUSY}}$  and thus cannot be fixed to a constant value. To illustrate the effect of the variation of  $M_h$  with  $M_{\text{SUSY}}$ , we show in the right panel the prediction of  $M_W$ , where the value of  $M_h$  predicted by the MRSSM is used to calculate  $M_W^{\text{SM}}$ . Since  $M_h$  is increasing with increasing  $M_{\text{SUSY}}$  (see second horizontal axis on the top of the right panel), the value for  $M_W^{\text{SM}}$  is no longer constant and decreases for increasing  $M_{\text{SUSY}}$  (black dash-dotted line). Due to the decoupling behavior, the value of  $M_W$  still converges to  $M_W^{\text{SM}}$ .

In the shown scenario (18) the experimental value for  $M_h = 125.25$  GeV is correctly predicted by the MRSSM for  $M_{\text{SUSY}} \approx 6767$  GeV. For this value of  $M_{\text{SUSY}}$  the MRSSM predicts  $M_W \approx 80.353$  GeV, which deviates by 1 MeV from the SM prediction of  $M_W^{\text{SM}} \approx 80.352$  GeV, nicely illustrating the decoupling behavior. However, in the shown scenario the value of  $M_W^{\text{CDF}}$  (orange band in Figure 3) cannot be reproduced for realistic scenarios where the SM like Higgs pole mass is close to 125.25 GeV.<sup>10</sup>

Moving away from the scenario (18) we find that both  $M_h$  and  $M_W^{\text{CDF}}$  can be accommodated in a scenario with  $\tan\beta \approx 24.7$ ,  $\Lambda_d \approx -1.59$ ,  $\Lambda_u \approx -1.70$ ,  $\lambda_d \approx 2.18$ ,  $\lambda_u \approx -0.293$  and  $M_{\text{SUSY}} \approx 1763$  GeV.<sup>11</sup> In Figure 4 we show 2-dimensional parameter scans around this point (marked with a green star symbol  $\star$ ) together with contours showing the value of the Higgs boson pole mass  $M_h$ . As shown in Figure 4a, the crucial parameters to obtain a correct  $W$  boson pole mass are the aforementioned top-Yukawa-like parameters  $\lambda_u$  and  $\Lambda_u$ . This is in agreement with previous findings in Ref. [87]. The shape also suggests a quadratic dependence of  $M_W$  on the parameters, which fits with the analytic expressions that have been obtained for simplified cases, see Eq. (4.16) of Ref. [87]. The dependence of  $M_W$  on  $\lambda_d$  and  $\Lambda_d$  is very weak and therefore not shown. Focusing now on  $\Lambda_u$  and  $M_{\text{SUSY}}$  we can see that while the CDF  $W$  mass measurement can be explained for almost any  $M_{\text{SUSY}}$  value by adjusting  $\Lambda_u$ , combining this with the Higgs mass measurement selects a rather narrow  $M_{\text{SUSY}}$  range, of about 1.7–1.8 TeV (within  $1\sigma$ ) from amongst the scenarios we consider in this scan. In both panels the white regions at the edges are unphysical. The color contours for  $M_W$  are fixed by deviations around the CDF measurement, the 2021 world average and the SM on-shell prediction. As such the colors do not represent even shifts in the  $W$  boson mass and care should be taken to avoid confusion from this. Black contour lines show different values of the SM-like Higgs  $M_h$ , in particular these include the measured value  $M_h = 125.25$  GeV.<sup>12</sup>

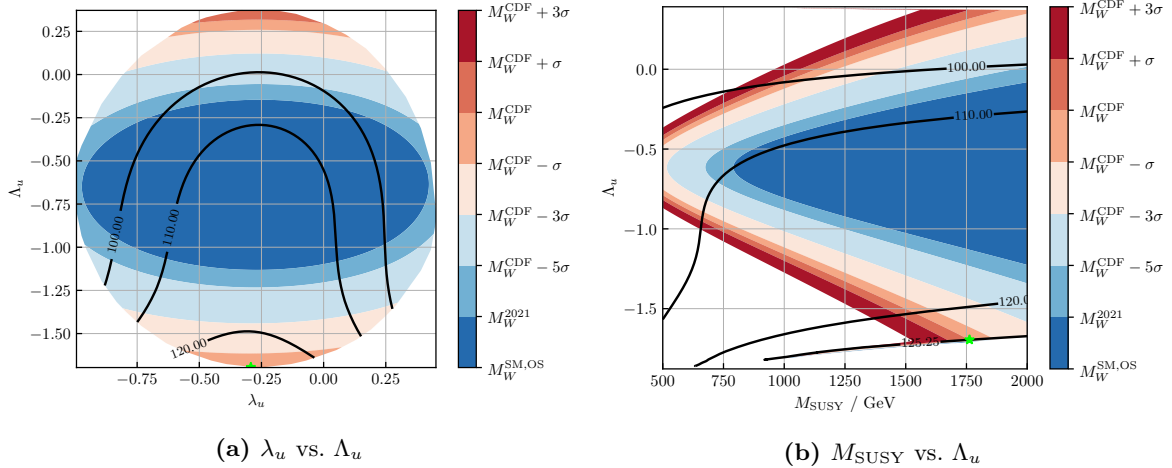
## 5 Conclusions

In this short paper we have presented the new `FlexibleSUSY` calculation of  $M_W$  in a modified  $\overline{\text{MS}}/\overline{\text{DR}}$ -renormalization scheme. Our new calculation ensures decoupling with the precise  $\overline{\text{MS}}$  Standard Model prediction being recovered in the limit where BSM masses are much larger than

<sup>10</sup>It is well known that the fixed-order calculation of the Higgs mass we use here has a substantial uncertainty at large  $M_{\text{SUSY}}$ , which is around  $\Delta M_h \approx 8$  GeV when the SUSY scale is 7 TeV [105]. However even taking this sizeable uncertainty into account it is clear that in this particular scenario it is impossible to explain both measurements simultaneously.

<sup>11</sup>The `FlexibleSUSY` input and the SLHA [106, 107] spectrum files for this point are attached to the `arXiv` submission of this publication.

<sup>12</sup>For these smaller  $M_{\text{SUSY}}$  values we estimate the uncertainty of our fixed order one-loop calculation to be at most  $\Delta M_h \approx 4$  GeV [105].



**Figure 4:** Prediction of the  $W$  boson pole mass  $M_W$  and contours for the Higgs boson pole mass  $M_h$  in the MRSSM around the point  $\tan\beta \approx 24.7$ ,  $\Lambda_d \approx -1.59$ ,  $\Lambda_u = -1.70$ ,  $\lambda_d \approx 2.18$ ,  $\lambda_u \approx -0.293$  and  $M_{\text{SUSY}} \approx 1763$  GeV (marked with a  $\star$ ). The color contours label landmark  $M_W$  values in terms of the number of standard deviations from  $M_W^{\text{CDF}}$ , or in the case of medium and dark blue bands, the 2021 world average and the SM on-shell prediction, respectively. As such the value for  $M_W$  varies a lot among the colored regions.

the Standard Model masses. The Standard Model contribution includes all currently known contributions, obtained through the fit formula from the state-of-the-art  $\overline{\text{MS}}$  computation in Ref. [2]. The BSM contributions are calculated at strict one-loop level avoiding the introduction of spurious higher order terms.

We have briefly illustrated the calculation in the MSSM, SSM and the MRSSM. In the MRSSM we have also shown that this model can explain the large deviation observed in the recent 2022 CDF measurement of the  $W$  boson mass, which is the most precise measurement to date. We believe that this updated calculation is very timely and will be very useful for testing proposed explanations of this intriguing new result, as well as a general constraint on Standard Model extensions.

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