

ARTICLE TEMPLATE

The time slot allocation problem in liberalised passenger railway markets: a multi-objective approach

Nikola Bešinović^a, Ricardo García-Ródenas^b, María Luz López-García^b, Julio Alberto López-Gómez^c †, José Ángel Martín-Baos^b

^aTechnische Universität Dresden, Dresden, Germany; ^bEscuela Superior de Informática, Universidad de Castilla-La Mancha, Ciudad Real, Spain; ^cEscuela de Ingeniería Minera e Industrial de Almadén, Universidad de Castilla-La Mancha, Almadén, Spain

ARTICLE HISTORY

Compiled January 23, 2024

ABSTRACT

The liberalisation of the European passenger railway markets through the European Directive EU 91/440/EEC states a new scenario where different Railway Undertakings compete with each other in a bidding process for time slots. The infrastructure resources are provided by the Infrastructure Manager, who analyses and assesses the bids received, allocating the resources to each Railway Undertaking. Time slot allocation is a fact that drastically influences the market equilibrium. In this paper, we address the time slot allocation problem within the context of a liberalized passenger railway market as a multi-objective model. The Infrastructure Manager is tasked with selecting a point from the Pareto front as the solution to the time slot allocation problem. We propose two criteria for making this selection: the first one allocates time slots to each company according to a set of priorities, while the second one introduces a criterion of fairness in the treatment of companies to incentive competition. The assessment of the impact of these rules on market equilibrium has been conducted on a liberalized high-speed corridor within the Spanish railway network.

KEYWORDS

Time Slot Allocation Problem; liberalised passenger railway markets; multi-objective optimisation

1. Introduction

In the last years, the European Directive EU 91/440/EEC has produced a significant change in the European passenger railway markets. This directive aims to facilitate the adoption of the Community railways to the Single Market and to increase their

†Corresponding Author

efficiency. To do that, it is intended to separate the infrastructure management from the provision of transport services and ensure access to the networks of the Member States for Railway Undertakings. The European so-called 4th railway package (EU Regulation 2016/2338) aims to remove the remaining barriers to the creation of a single European rail area. Additionally, the updated legislation includes the proposal to open up domestic passenger railways to new entrants and services from December 2019. This way, Railway Undertakings will be able either to offer competing services, such as a new train service on a particular route, or to bid for public service rail contracts through tendering.

The objectives of competition in the railway industry encompass several key aspects, including fare reduction (Vigren (2017)), enhanced service frequencies (as discussed in Tomeš et al. (2016)), and improved timetables (Li et al. (2019), Yang et al. (2023)), among other factors. These efforts collectively aim to stimulate higher demand for railway services.

Two common market integrations are present: vertical and horizontal. The former represents a single operator (the so-called *incumbent operator* or *national railway undertaking*) that operates in a monopoly situation and assumes a captive demand, which was customary in the member states of the European Union. The latter, also known as open passenger railway market, new railway operators (the so-called entrants) compete with each other for the time slot to which rail services are allocated. However, the expected transition from a vertical to a horizontal market has been very difficult (Nash (2008), Cantos-Sánchez, Pastor Monsálvez, and Serrano-Martínez (2021)). Some of the barriers that entrants face and that jeopardise fair competition among operators are: inferior resources with respect to the incumbent operator or the lack of regulations to ensure fair competition, among others (Ristić, Stojadinović, and Trifunović (2022)).

In a liberalised passenger railway system, there are three main interrelated actors: the Infrastructure Manager (IM), the Railway Undertakings (RUs) and the demand, which is represented by passengers. The IM is the owner and the manager of the railway infrastructure. The RUs, also known as train operating companies, are the companies that offer rail services to passengers using the infrastructure of the IM. Finally, the demand corresponds to passengers who want to make use of rail services to satisfy their transport needs for reaching from their origin to destination. The dynamics in a liberalised passenger railway system is the following: (i) the IM assigns capacities, which means a maximum number of time slot in which to operate, to each RU; (ii) the RUs, taking into account their assigned resources, place their bids to the IM in order to obtain a set of time slot in which to operate; (iii) the IM allocates time slot to the RUs, taking into account the bids received and the capacities assigned; and (iv) passengers choose one trip or another according to the final offer.

The dynamics described above define a complex system of relationships between the actors, in which the particular interests of each of them conflict with the optimal performance of the system. The IM has a central role in the whole system, as their decisions to allocate time slots to different RUs may affect the equilibrium of the system and even lead to a monopoly situation. It is a critical process in order to guarantee fair competition between the RUs, since an unfair allocation or one that benefits the incumbent operator may affect the profitability and performance of the

other RUs and the whole system (Broman and Eliasson (2019)).

In this context, there is a need to design optimal decision-making procedures in open passenger railway markets which guarantee equal treatment for all RUs involved in order to improve the socially efficient use of the railway system. It motivates the purpose of this paper: study how the decisions made by the IM when allocating time slots have an impact on the equilibrium state and determine the benefit of RUs.

Time slot allocation can be addressed using different approaches (Broman, Eliasson, and Aronsson (2022)): to carry it out, the IM can use an auction-based criterion or set different prices for each time slot in order to encourage certain frequencies. In this work, the latter situation is analysed with the aim of considering how the IM should act when all companies are willing to pay such an amount and the role of IM is that of an arbiter of bids rather than a selfish role of maximising its own profit.

This work proposes a new framework for Time Slot Allocation in Open railway markets (TSA-OPEN) problem in markets where multiple RUs operates under competition. For this purpose, TSA-OPEN problem is considered within an equilibrium model. Thus, for each of the RUs in the market, we obtain what their bid for operating rights in the time slots will be. This bid is so-called “strategy” and can be made up, in turn, of different requests or bids that the company can make with a given probability. The TSA-OPEN problem is formulated as a multi-objective optimisation problem as the IM has to deal with the conflicting interests of the different RUs. The canonical objective function of this problem is the profit function of the RUs. However, it remains unknown because its computation would require access to confidential data from them. In order to propose a criterion that can be shared by all participants, the distance between the time slot requested and the time slot allocated is transformed into a criterion that is transparent and known to all participants. The solution of this problem leads to multiple admissible solutions on what is known as the Pareto front. However, although any of these solutions is acceptable, the market equilibrium situation varies considerably. Two different criteria are proposed for the selection of such solutions by the IM: (1) the IM allocates time slots to each RU according to a set of priorities, (2) the allocation is made on the basis of a fairness criterion in order to treat all RUs equally. We conducted computational experiments to assess the real impact of the decisions made by the IM on the system. The scenario under analysis is the liberalized high-speed corridor in the Spanish railway market, which connects Madrid and Barcelona. We examined the criteria mentioned above while assuming that all features of RUs are equal, and our findings demonstrated significant differences based on the applied criterion.

The following are the main contributions of this article:

- A mathematical multi-objective optimisation formulation of the time slot allocation problem for multiple RUs is provided. This formulation places the role of the IM at the centre as the arbiter in the deployment of the competing market.
- Two selection criteria are introduced to assign the time slot: the use of priorities in the treatment of RUs and the application of an equity rule to treat all of them equally.
- An equilibrium model is proposed in order to integrate TSA-OPEN into the

- bidding process of RUs to achieve a market equilibrium.
- TSA-OPEN problem using the two proposed selection criteria are solved by using heuristic and exact approaches and their performance is compared.
 - The experiments conducted in this paper provide numerical evidence of how the criterion used by the IM significantly influences the equilibrium state, consequently affecting the economic performance of the RUs.
 - As a result of the above, the proposed framework constitutes a first fair approach to solve TSA-OPEN. The results underline the importance that the criteria employed by IM must be known by the RUs.

Finally, the rest of the paper is organised as follows. Section 2 reviews the literature related to the time slot allocation problem. Then, section 3 describes the methodology employed in this work, including the necessary background about liberalised passenger railway systems and the formulation of TSA-OPEN problem. Later, section 4 describes the heuristics to solve the TSA-OPEN problem. Section 5 shows the experiments carried out and the results obtained in a high-speed corridor in the Spanish railway network. Finally, section 6 summarises the conclusions and further works derived from this work.

2. Related work

In this section, the most recent methods for time slot allocation in classical/closed/monopolistic and open/liberalised railway markets are discussed. Then, the reviewed methods for liberalised passenger railway markets are compared to the proposed time slot allocation methods of this paper. Summarily, Table 2 shows this comparison. Finally, the scientific gaps are stated.

2.1. *Classical railway market*

Before the appearance of the EU regulations, which opened the door to the liberalisation of the European railways markets, the most common organisation of them was a completely integrated vertical and horizontal structure (Ait-Ali (2020)). It means there was a single actor which had a monopoly on the system, acting as IM and RU (Ait Ali and Eliasson (2019)). This way, the time slot allocation in this type of system was closed, opaque and carried out internally by the actor who monopolised the market (Ait-Ali (2020)).

The absence of competition in this market structure meant the aim of the time slot allocation problem was to build a feasible train timetable which maximised the profit of the company (Caprara et al. (2007)) and not violate track capacities and other operational constraints. In this kind of market, this problem is called the Track Allocation Problem (TAP) or Train Timetabling Problem (TTP). More formally, Schlechte (2012) formulated this problem in the following way: given a macroscopic railway model and a set of train slot requests, the solution of TTP will be the subset of train requests

which will be addressed as well as their exact arrival and departure times of the trains which will provide the service.

The most traditional approaches to address this problem in single-track railway networks are Integer Linear Programming (ILP) and Mixed-Integer Linear Programming (MILP) models (Carey and Lockwood (1995), Higgins, Kozan, and Ferreira (1996)). In order to solve the problem, a wide variety of methods have been applied. Due to the fact that TTP is an NP-hard problem, solution approaches based on heuristics have been widely applied, such as greedy heuristics (Cai, Goh, and Mees (1998)), tabu search and genetic algorithms (Higgins, Kozan, and Ferreira (1997)).

TTP problem in general railway networks has also been traditionally addressed by means of graph theoretic formulations (Caprara, Fischetti, and Toth (2002), Caprara et al. (2006)). Departing from these works, many other modifications have been proposed, leading to ILP models and Lagrangian relaxed ones which are also solved by means of heuristic algorithms (Cacchiani, Caprara, and Toth (2008)). Interested readers can consult Lusby et al. (2011) for more information on such models and solutions.

2.2. Liberalised railway market

Under the new regulation, state-owned companies (incumbents) have given rise to the IM and RUs. In the EU, two models have emerged. The first involves splitting the incumbent company into two independent entities —one focused on network management and the other on transportation. This scenario is exemplified by the Spanish and Italian railway markets. The second structure entails replacing the incumbent company with a holding company comprising two separate entities. France and Germany are instances whose railway markets follow this arrangement. The market structure’s impact is pivotal in fostering competition. Table 1 lists the names of the companies in these railway markets.

Country	Railway undertakings	Infrastructure Manager
Spain	RENFE, OUIGO, IRYO	ADIF
Italy	Trenitalia, Italo	RFI
France	SNFC VOYAGEURS, Trenitalia	SNFC Réseau
Germany	DB	DB Netz

Table 1. Examples of open markets

In liberalised or open-access railway systems, competition marks the development of models to represent this type of market. Thus, the models built must take into account the conflicting interests between the IM, the different RUs and also the interests of passengers in order to maximise the profit of all actors involved and contribute to social welfare. This way, the existing approaches are more focused on passenger’s behaviour and on the fair allocation of time slots to the RUs regarding the classic models, which are mainly focused on railway operations.

TSA-OPEN is a key problem to ensuring fair competition in competitive and open-access railway markets. It can be broadly defined as follows: given a set of RUs, a set of

time slots, the capacities of each RUs to operate them and the bids of the different RUs to operate the time slot, the result of TSA-OPEN will be an allocation of the time slot to the different RUs. According to Broman, Eliasson, and Aronsson (2022), fairness is one of the most important features that a TSA-OPEN method must meet. It is the task of IM to allocate time slots to different RUs taking into account the capacities of each one and solving the conflicts between them. A conflict or conflict state occurs when two or more RUs apply for access to the same track at the same time (Schlechte (2012)). In open-access competitive railway markets, conflict situations are frequent, since they are encouraged by the number of RUs in the market and the amount of traffic on the network among others (Broman, Eliasson, and Aronsson (2022)).

Traditionally, the time slot allocation problem has been solved through negotiations between the IM and the RUs, voluntary agreements and other simple criteria (Stojadinovic, Boskovic, and Bugarinovic (2019)). However, these methods are not very effective in competitive and open-access markets (Broman, Eliasson, and Aronsson (2022)). Currently, the main methods to solve the time slot allocation problem are grouped into three main approaches (Broman, Eliasson, and Aronsson (2022)): based on administrative and collaborative procedures (ACP), approaches based on social cost-benefit analysis (CBA) and allocation based on Willingness-to-pay (WTP).

The first approach for time slot allocation methods is based on ACP. This approach is used in all European Union (EU) countries and consists of applying a set of simple and pre-defined criteria to solve conflicts. Concerning this approach, Broman, Eliasson, and Aronsson (2022) describes the case of the Belgium railway market, where the IM classifies the train services according to priorities and when the conflict occurs, the least-priority class of service is adapted. Another recent example is Trafikverket (2020). In this document, the time slot allocation procedure of the Swedish railway market is described. In it, the different RUs submit their time slot request to the IM according to the national network statement. Then, the IM makes the allocation solving minor conflicts by means of small adjustments. Major conflicts are solved by means of informal discussions between the involved RUs. If an agreement is not reached, the IM solves the conflict using a set of rules of pre-defined criteria. The main advantages of administrative and collaborative procedures are usually that they are very clear and simple (Broman, Eliasson, and Aronsson (2022)). However, their main drawbacks are related to their problems of treating all companies fairly.

CBA is the second main approach for solving the time slot allocation problem. It is a very well-known methodology in the transportation field. However, the literature applying this approach to the time slot allocation problem is scarce. The fundamentals of CBA consist of defining all costs and benefits of the time slot allocation procedure in economical terms and summing up in order to know the extent to which the method designed contributes to welfare. Because of this, the main characteristic of a CBA time slot allocation procedure is fairness. However, the need to recover enough data from the system (RUs and passengers) to design it, could affect the achievement of the remaining good properties. Recently, Ait Ali, Warg, and Eliasson (2020) proposed a CBA based method for allocation and timetable comparison. For further reading, readers are referred to Broman, Eliasson, and Aronsson (2022), where their authors propose a mixed method which combines administrative procedures and CBA allocation.

Finally, WTP methods constitute the last approach to solving the time slot allocation problem. It is a very common approach, especially in competitive and open access markets, where multiple RUs compete between them in an auction or bidding process. This way, the RUs are sorted according to how much money they are willing to pay for a time slot. The main advantages of WTP methods are usually their simplicity and manageability. However, these methods normally suffer from a lack of transparency and above all from a lack of fairness. There are many efforts in literature to develop fairer WTP methods based on auctions. Some of these works are Kuo and Miller-Hooks (2015), where a combinatorial auction was employed in ongoing allocation schemes, Talebian, Zou, and Peivandi (2018), which proposes a negotiation approach in vertically integrated railway systems employing game-theoretic concepts and Stojadinovic et al. (2019), where an algorithm for iterative time slot allocation is proposed under a scheme of hybrid auctions. Finally, another related example is Stojadinovic, Boskovic, and Bugarinovic (2019), where their authors propose an allocation algorithm based on auctions to make the allocation.

Reference	Railway System		Framework				
	Country	Type of transport	Approach	Model	Method	Implementation	Fairness
Broman, Eliasson, and Aronsson (2022)	Belgium	Freight and passengers	ACP	Rule-based	Priorities	Theoretical	✗
Trafikverket (2020)	Sweeden	Freight and passengers	ACP	Rules and negotiation-based	Agreement and priorities	Theoretical	✗
Ait Ali, Warg, and Eliasson (2020)	Sweeden	Passengers (commuter-trains)	CBA	Mathematical modelling	Computing loss of societal benefits	Simulation	✓
Stojadinovic, Boskovic, and Bugarinovic (2019)	European	Passengers	WTP	Negotiation	Agreement and auction	Theoretical	✗
Kuo and Miller-Hooks (2015)	Synthetic	Freight	WTP	Auctions Modelling	Optimisation	Simulation	✗
Talebian, Zou, and Peivandi (2018)	Synthetic	Freight and Passengers	WTP	Game-Theory: Equilibrium problem	Optimisation	Simulation	✓
Stojadinovic et al. (2019)	European	Passengers	WTP	Mathematical modelling	Hybrid auctions	Simulation	✗
Broman, Eliasson, and Aronsson (2022)	Synthetic	Passengers	Hybrid	ACP and CBA	Welfare losses	Theoretical	✓
This paper: priority and equity rules	Spanish	Passengers	ACP	Mathematical modelling	Optimisation	Simulation	✓

Table 2. Comparison of state-of-the-art works related to time slot allocation.

Table 2 shows a comparison of the discussed works related to time slot allocation in liberalised passenger railway markets. Derived from this table, it is possible to conclude the following: first, there are many works which propose theoretically time slot allocation procedures, there is therefore a need to implement and validate the proposed allocation methods experimentally. Second, the references indicated in the table as fair do not always guarantee equal treatment among all companies. This is

because, when the allocation is based on WTP, the allocation may be unfair as not all RUs have the same resources. On the other hand, when the allocation is based on CBA, the study of these indicators is not always possible as it requires data that is often not available and it may favour the incumbent operator and make it more difficult for new entrants to compete. This way, there is a gap in designing allocation approaches that ensure equal treatment of all companies and that are simple and easy to implement.

In this paper, we focus on the liberalized Spanish railway market as our reference case. This market presents challenges for the application of WTP-based approaches due to the absence of auction mechanisms employed by the IM. Additionally, the CBA approach faces difficulties due to the requirement of reliable socio-economic data, limiting its transparency. Instead, we adopt an ACP-based approach.

Notably, an aspect that remains unexplored in the existing literature is the assessment of the impact of TSA-OPEN on competitive dynamics. To address this gap, we introduce two criteria that offer distinct treatments of the RUs. The priority criterion reflects the strategic dominance situation of the incumbent, while the fairness criterion advocates for equal treatment of all RUs. Both of these developed methods are characterized by their simplicity and practicality in terms of design and implementation. This study aims to shed light on the implications of these criteria for competition in the railway market.

3. The time slot allocation problem in open railway markets

In this section, a mathematical formalization of TSA-OPEN is provided. To achieve this, the foundation of a liberalized railway market is initially presented. Subsequently, the TSA-OPEN problem is formulated as a multi-objective optimization problem. The objective of the IM is to select one of the solutions from the Pareto front to be implemented in the system. In this article, two criteria are proposed for choosing a solution from the Pareto front: a priority-based rule and an equity rule. The first criterion can be viewed as administrative and collaborative procedures based on priorities, while the second is designed to promote equity in the treatment of the RU.

3.1. *Liberalised Railway Markets*

Figure 1 shows a scheme of a liberalised passenger railway market stated on the basis of the Spanish case. The railway system is composed of three main actors: a public company, the so-called IM that provides the infrastructure and a set of private companies. The second one is RUs, that operate on the railway network. A specific RU is denoted by o and the set of RUs by means of the set \mathcal{O} . Finally, the third main actor is the demand, which is represented by passengers. In a liberalised or open railway market, an equilibrium situation, in terms of the profit between RUs, is desirable in order to promote competition, enhance the service frequencies and reduce fares.

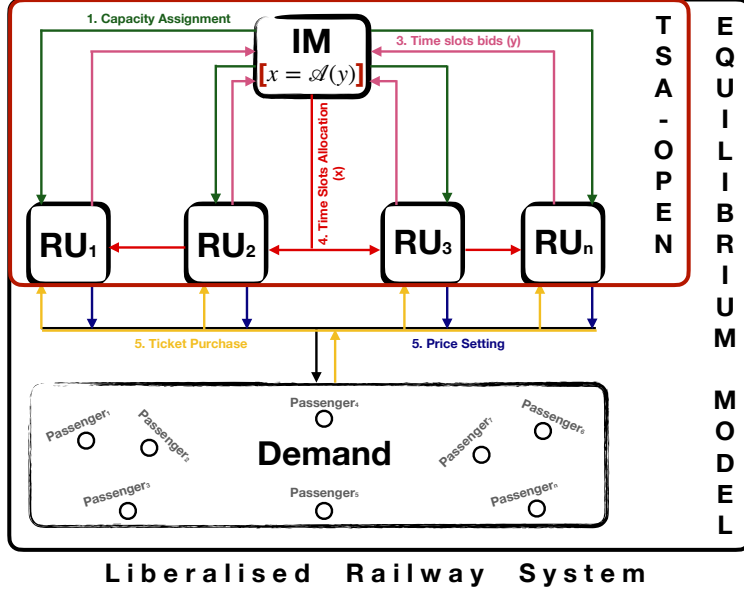


Figure 1. General scheme of a liberalised railway system and time slot allocation problem.

The IM selects one or several corridors for liberalization. The primary responsibility of the IM is to create a set of packages, each characterized by its capacity, which will be auctioned to a group of RUs. Based on the various offers received and technical considerations, the IM allocates these packages among a subset of RUs that have submitted bids. The capacity of the package assigned to RU o is represented as k_o . This corresponds to the fraction of the time slots that each RU can apply for in the auction process. Therefore, this company may operate in up to $k_o|\mathcal{R}|$ time slots, where $|\mathcal{R}|$ represents the total number of time slots in the corridor. These capacity assignments restrict the requests of each RU. Capacity allocation is the initial factor that influences whether fair competition exists among the RUs.

A time slot, represented as r , is linked to a train route, including its arrival and departure times at stations. To be more precise, it is a segment on a space-time diagram where a train operates without conflicts.

Once the capacity assignment is carried out, the RUs are ready to make their bids to the IM in order to obtain the operating rights in a time slot. To do that, the RUs need to take into account the capacities that have been previously assigned to them. The bids are modelled by a vector of binary variables, whose length is equal to the number of time slots. This way, a bid made by o company is denoted by y_o vector. The value of each component of the vector y_o is equal to 1 when the corresponding time slot is requested by RU o and 0 otherwise. Equation (1) shows this formalisation.

$$y_o = [y_{or_1}, y_{or_2}, \dots, y_{o|\mathcal{R}|}]. \quad (1)$$

Note that for the binary variable y_o to be feasible, it must satisfy the capacity restriction modelled by Equation (2) which prevents the RU o from bidding for more

time slots than it has capacity allocated to it.

$$\sum_{r \in \mathcal{R}} y_{or} \leq k_o |\mathcal{R}|. \quad (2)$$

After the bids placement, the IM allocates the time slots to each RU. To do that, the IM resolves conflicts present in the request y_o concerning other RUs, transforming it into a conflict-free request denoted as x_o . A conflict occurs when a time slot is requested by more than one RU. The result of the allocation process for RU o is the allocation vector x_o shown in Equation (3). It must be noted that the value 1 in an element of x_o means that the corresponding time slot has been assigned by the IM to the company o . Otherwise, the value in that element will be 0.

$$x_o = [x_{or_1}, x_{or_2}, \dots, x_{o|\mathcal{R}|}]. \quad (3)$$

In summary, the IM receives a feasible bid y of RUs, and then IM will provide an allocation of time slot x free of conflicts. Equation (4) abstracts this procedure as a mapping \mathcal{A} .

$$x = \mathcal{A}(y). \quad (4)$$

3.2. TSA-OPEN modelling

One of the most important objectives of IM is to provide a fair allocation between the RUs involved in the market. Fairness must be a key criterion if it is intended to promote the development of competition between RUs in the market, without excluding any of them, and thus offering passengers better travel alternatives.

The IM acts as an arbitrator, who allocates the time slot considering the specific objectives of each RU. The profit or payoff of RU o is represented by the function $F_o(x_o, x_{-o})$, where x_o represents the allocation to RU o and x_{-o} represents the allocation to the rest of the RUs. Therefore, the IM should keep in mind the following multi-objective problem in the definition of allocation method \mathcal{A} .

$$\text{maximize } F = [F_1(x_1, x_{-1}), \dots, F_o(x_o, x_{-o}), \dots, F_{|\mathcal{O}|}(x_{|\mathcal{O}|}, x_{-|\mathcal{O}|})] \quad (5)$$

$$\text{subject to: } \sum_{r' \in \mathcal{R}} h_{or'}^r = y_{or} \quad (6)$$

$$\sum_{r' \in \mathcal{R}} h_{or'}^{r'} = x_{or} \quad (7)$$

$$h_{or'}^r \in \{0, 1\} \quad (8)$$

Constraints (6)-(8) represent the possibility of re-timing a time slot, introducing auxiliary variables h_{or}^r . These variables take the value 1 if the time slot r requested by RU o is re-timed to the time slot r' .

If \mathcal{P} represents the Pareto front of the problem (5)-(8), the IM should employ an allocation criterion that satisfies the condition:

$$x = \mathcal{A}(y) \in \mathcal{P}.$$

Otherwise, there would exist an allocation decision that would be more beneficial for all RUs. The IM is unaware of the multi-objective problem due to the unavailability of economic data from the RUs. To address this, a simplified and transparent version that can be understood by all involved parties is proposed. It is based on minimizing the discrepancies between the time slots requested by the RUs and the allocations ultimately made by the IM.

In this new formulation, the aim of each RU o is to minimise its total deviation D_o , that is, the sum of the deviations of all the requested time slots. Formally, it is noted by Equation (9).

$$D_o = \sum_{r \in \mathcal{R}} \delta_{or}, \quad (9)$$

where δ_{or} is the deviation of the rescheduled time slot r requested by operator o and is computed by the Equation (10).

$$\delta_{or} = \frac{1}{2} \sum_{r' \in \mathcal{R}} |y_{or} - h_{or'}^r| |t_r - t_{r'}|, \quad (10)$$

being t_r the time instant of time slot r and $t_{r'}$ is the time instant of the re-timed time slot r' .

The deviation-based formulation leads to the following multi-objective optimisation problem which will be addressed by the IM to solve TSA-OPEN problem:

$$\begin{aligned} & \text{maximize} && D = [D_1, \dots, D_o, \dots, D_{|\mathcal{O}|}] \\ & \text{subject to:} && (6) - (8) \end{aligned} \quad (11)$$

Solving TSA-OPEN problem allows to obtain a solution from the Pareto front. The following subsections describe two different criteria or approaches to choose Pareto optimum solutions: the first is based on a priority rule approach and the second is based on an equity rule.

3.3. TSA-OPEN modelling: priority rule

The first criterion proposed in this paper to choose a Pareto optimum is the priority rule. In this approach, the requests of the RUs are served according to a set of priorities that determine which RU is served first, which second and so on. Usually, priorities are related to the capacity allocated to the RUs. This way, the RU with the highest capacity will be served first while the RU with the lowest assigned capacity will be served last.

The priority rule means that an iterative process is carried out. In each iteration, an allocation problem is solved for a RU according to their priorities. This way, at the start of the resolution process, no time slot has been allocated to any RU. Then, according to the priorities of each of the RUs, the one with the highest priority will be selected and its requested time slot will be allocated, as none have been allocated previously. Later, the next RU in order of priority will be selected and the allocation procedure will be repeated. However, in this case, it is possible that conflicts may occur and that the second RU has requested time slots that have already been allocated to the first RU, so in this case the IM shall re-plan those slots trying to minimise the deviation from the requested time slot. This iterative process will be carried out for all the RUs. As the allocation is carried out for a new RUs, an increasing number of conflicts will arise. This is due to the additional slots allocated to previously served RUs, being necessary to re-plan the time slot allocated to each one.

Let's assume that the RUs have been prioritized in the order $o_1, \dots, o_{|\mathcal{O}|}$, and suppose that all companies $\{o_1, \dots, o' - 1\}$ have been analyzed, calculating their respective allocation variables x_{or} . Then, for the time slot allocation for company o' , the IM will minimize its deviation:

$$\text{Minimize } D_{o'} \quad (12)$$

$$\text{Subject to: } \sum_{r' \in \mathcal{R}} h_{o'r'}^r = y_{o'r}, \quad \forall r \in \mathcal{R} \quad (13)$$

$$\sum_{r' \in \mathcal{R}} h_{o'r'}^{r'} = x_{o'r}, \quad \forall r \in \mathcal{R} \quad (14)$$

$$\delta_{o'r} = \frac{1}{2} \sum_{r' \in \mathcal{R}} |y_{o'r} - h_{o'r'}^r| |t_r - t_{r'}|, \quad \forall r \in \mathcal{R} \quad (15)$$

$$D_{o'} = \sum_{r \in \mathcal{R}} \delta_{o'r} \quad (16)$$

$$x_{o'r} + \sum_{o \in \{o_1, \dots, o' - 1\}} x_{or} \leq 1, \quad \forall r \in \mathcal{R} \quad (17)$$

$$h_{o'r'}^r \in \{0, 1\}, \quad \forall r, r' \in \mathcal{R} \quad (18)$$

where the constraints (13)-(14) models replanning time slots, the constraints (15)-(16) define the computation of the deviation for the RU o' , and the constraint (17) prevents a time slot from being allocated to more than one RU. Note that the variables x_{or} in Equation (17) have been computed in prior iterations and, in this formulation,

they act as parameters. Finally, constraint (18) shows the binary nature of replanning variables.

Priority rule modelling is an ACP time slot allocation approach which, although simple and practical, represents the strategic dominance of the incumbent RU in the market. In the next section, an equity rule approach is proposed in order to ensure a fair allocation to all the RUs in the system.

3.4. *TSA-OPEN modelling: equity rule*

In contrast to the method introduced earlier, where the RUs are treated unequally, we introduce a method to ensure a fairer allocation. The proposed criterion, named equity rule, tries to balance the deviations suffered between RUs. In this case, the optimization model aims to minimise the total sum of deviations across all RUs within the market. However, it also enforces a constraint to limit the maximum difference that can be observed between the deviations experienced by the RUs.

The equity criterion is related to the capacity assigned to each RU. In this paper, the proposed approach imposes that the total displacement for the time slot of the RU o must be proportional to the size of its operating capacity, which is noted by $D_o = k_o \Delta$. However, due to the discrete nature of this constraint, it is not always possible to achieve exact equality in maintaining this proportionality. To address this limitation, a tolerance parameter ε is introduced to ensure its satisfaction.

Given the time slot requests y by the RUs, we calculate the total number of requested time slot as $n_y = \sum_{o \in \mathcal{O}} \sum_{r \in \mathcal{R}} y_{or}$, and the IM formulates the TSA-OPEN problem

under the equity rule as (19):

$$\text{Minimize } \sum_{o \in \mathcal{O}} D_o \quad (19)$$

$$\text{Subject to: } \sum_{r' \in \mathcal{R}} h_{or'}^r = y_{or}, \quad \forall o \in \mathcal{O}, \forall r \in \mathcal{R} \quad (20)$$

$$\sum_{r' \in \mathcal{R}} h_{or'}^r = x_{or}, \quad \forall o \in \mathcal{O}, \forall r \in \mathcal{R} \quad (21)$$

$$\delta_{or} = \frac{1}{2} \sum_{r' \in \mathcal{R}} |y_{or} - h_{or'}^r| |t_r - t_{r'}|, \quad \forall o \in \mathcal{O}, \forall r \in \mathcal{R} \quad (22)$$

$$D_o = \sum_{r \in \mathcal{R}} \delta_{or}, \quad \forall o \in \mathcal{O} \quad (23)$$

$$\sum_{o \in \mathcal{O}} x_{or} \leq 1, \quad \forall r \in \mathcal{R} \quad (24)$$

$$\Delta = \frac{1}{n_y} \sum_{o \in \mathcal{O}} \sum_{r \in \mathcal{R}} \delta_{or} \quad (25)$$

$$\Delta - \varepsilon \leq \frac{1}{k_o \cdot n_y} D_o \leq \Delta + \varepsilon, \quad \forall o \in \mathcal{O} \quad (26)$$

$$h_{or'}^r \in \{0, 1\} \quad \forall r, r' \in \mathcal{R}, \forall o \in \mathcal{O} \quad (27)$$

where the constraints (20)-(23) and (27) are similarly formulated as for the priority rule, but they apply to all RUs collectively. Constraint (24) ensure a time slot is only allocated to one RU. The constraint (25) computes the average deviation for the time slot and it is denoted by Δ and the constraint (26) imposes that the average deviation for all RUs is equal to Δ plus or minus an error ε .

4. Assessing TSA-OPEN in liberalized markets: an equilibrium game

The purpose of this article is twofold. First, it seeks to establish precise guidelines for \mathcal{A} to ensure transparent and equitable allocation of time slots. Since the conceptual method \mathcal{A} for assigning time slots affects the equilibrium of the open railway system, the second objective is to evaluate the impact of these guidelines on competition. To accomplish this, tools must be developed for assessing the rules \mathcal{A} . In this section, we introduce an equilibrium game approach as a model for the open railway market, in which the TSA-OPEN $x = \mathcal{A}(y)$ is a component of this equilibrium model.

The equilibrium problem at hand can be stated as a game in which the players are RUs and the strategies are the time slot requests y . The RUs need to determine which strategies to employ and with what probability to request them from the IM in order to maximise their payoff functions. For example, let's suppose a RU which has two request options: y_1 and y_2 . This RU is interested in knowing the probability values p_1 and p_2 , such as $p_1 + p_2 = 1$, where $p_1 \geq 0$ and $p_2 \geq 0$, for playing these requests to

maximize its profit.

The payoff function F corresponds to the economic revenue of the RU which is computed as the difference between the revenues from ticket sales and the costs of operating the time slot together with the costs of acquisition and depreciation of rolling stock.

The revenue obtained from ticket sales for the time slot r can be computed by $b_r = \sum_{\omega \in \mathcal{W}} g_{r\omega} z_{r\omega}$, where $g_{r\omega}$ is the demand of the time slot r in the origin-destination pair ω and $z_{r\omega}$ is the ticket price of for the origin-destination trip ω at time slot r . The revenue from ticket sales for RU o is computed as

$$J_o(x_o, x_{-o}) = \sum_{r \in \mathcal{R}} x_{or} b_r. \quad (28)$$

Note that the expression (28) is a simplification of the problem, in which it is assumed the number of passengers attended by a service depends only on the allocated time slot.

Concerning cost for RU o , it consists of two components. The first part is associated with the operative costs and the second with investment costs to provide the established services. The operational costs f_{or} take into account the network access variable cost fixed by IM and the staff and operational costs by ticketing, advertisement, passenger attendance, drivers, and so on. Regarding investment costs, RU o has to assume the cost of leasing rolling material to offer the service (C_o). Thus, the investment costs will depend on the number of units and the typology necessary to deploy the service. It is assumed all trains have the same typology and the rolling stock costs depend on the minimum number of trains needed to operate all accepted time slots. In investment costs, the maintenance costs are also included as well as the depreciation costs of the rolling stock. It is considered that the fixed cost to network access c_a has already been incurred by RUs. It does not affect the choice of time slot but is included to adjust the balance sheet of RU. The minimum number of trains needed to operate the service network (time slot) is denoted by $n_T(x_o)$. This introduces the investment needs of the operator and investment costs. Finally, the payoff function for a RU o , given the time slot allocated to it (x_o) and the time slot allocated to the rest of RUs (x_{-o}) is defined as

$$F_o(x_o, x_{-o}) = J_o(x_o, x_{-o}) - \sum_{r \in \mathcal{R}_o} f_{or} x_{or} - C_o n_T(x_o) - c_a. \quad (29)$$

Let's consider $\mathbf{p}_o = (\dots, p_{y_o}, \dots)$ as a probability distribution over the set of strategies that the RU o can employ ($y_o \in Y_o$). Then, for a game strategy $y = (y_{o_1} \dots, y_{o_{|O|}}) \in Y = Y_1 \times \dots \times Y_{|O|}$, its probability of being applied is given by: $p_y = \prod_{s=1}^{|O|} p_{y_{o_s}}$, and therefore, the expected benefit for RU o is:

$$u_o(\mathbf{p}_o, \mathbf{p}_{-o}) = \sum_{y \in Y} p_y F_o(x_o, x_{-o}), \quad (30)$$

$$x = \mathcal{A}(y). \quad (31)$$

In an equilibrium scenario, no RU can unilaterally enhance their expected economic outcomes. We will now mathematically formulate this situation:

Definition 4.1 (Nash equilibrium bid). A strategy profile \mathbf{p}_y^* for the bid is a Nash equilibrium if for all $o \in \mathcal{O}$, for all \mathbf{p}_{y_o} and for all $y_o \in Y_o$

$$u_o(\mathbf{p}_{y_o}, \mathbf{p}_{y_{-o}}^*) \leq u_o(\mathbf{p}_y^*). \quad (32)$$

It is worth noting that the allocation rule \mathcal{A} impacts the profit $u_o(\mathbf{p}) = \sum_{y \in Y} p_y F_o(\mathcal{A}(y))$ and, consequently, alters the equilibrium situation. This is the effect we aim to analyze.

Furthermore, it should be noted that calculating the equilibrium situation requires determining $x = \mathcal{A}(y)$ for all $y \in Y$. The proposed models are based on integer linear programming¹. While commercial solvers efficiently handle these models, the set Y presents a vast number of alternatives, necessitating the resolution of a substantial number of these problems. Consequently, to streamline the incorporation of \mathcal{A} into an equilibrium computation method, it is advisable to employ heuristic algorithms for its calculation. In the following two subsections, we introduce such algorithms for each of the rules.

4.1. *Priority rule*

In order to solve TSA-OPEN problem considering the priority rule modelled in section 3, a heuristic algorithm is proposed. In this method, the set of priorities that define the order in which the RUs will be attended is considered to be given. For example, priorities may be directly related to the capacities of the RUs in such a way that the more capacity a RU has, the more priority it has. Thus, the IM starts to allocate time slots for the company which has the highest priority and so on. The allocation is done assuming that the previous time slot has already been allocated and re-planning the conflicting time slot to the nearest available one. To illustrate this solution procedure, Algorithm 1 shows the pseudo-code of a heuristic algorithm for TSA-OPEN problem based on the priority rule.

This algorithm departs from a vector of priorities defined in line 3. It contains a priority value for each RU which will determine the order in which the requests of each one will be processed with respect to the other RUs. Then, in line 5, a for loop is used to select, in the first place, the RU with the highest priority and whose requests will be handled. Once the RU to be served is determined, a nested for loop in line 8 goes through the requests made by the RU, so that if the requested time slot is free,

¹Note that the absolute value of a magnitude appearing in constraints (15) and (22) can be modelled using linear programming through the following trick:

$$\begin{aligned} |x| &= x^+ + x^- \\ x &= x^+ - x^- \\ x^+ &\geq 0, x^- \geq 0 \end{aligned}$$

Algorithm 1 Pseudocode for solving heuristically TSA-OPEN problem using a priority rule

Require: $\mathcal{O}, \mathcal{R}, y_1, y_2, \dots, y_{|\mathcal{O}|}$

Ensure: x

```

1: function TSA-OPEN USING PRIORITY RULE
2:   – Priorities vector with the ordered indexes of the RUs to be served –
3:   Priorities =  $[o_1, o_2, o_3, \dots, o_{|\mathcal{O}|}]$ 
4:   – Solve an allocation problem for each RU –
5:   for  $i = 1 : |\mathcal{O}|$  do
6:     – Select RU by priority –
7:      $o' = \mathcal{O}[\text{Priorities}[i]]$ 
8:     for  $r$  requested time slot by  $o$  do
9:       if  $r$  is available then
10:        – Direct allocation of free time slot –
11:         $x_{o'r} = y_{o'r}$ 
12:       else
13:        – Re-timing of conflicting time slot –
14:         $h_{o'r'} = 1$  where  $r'$  is the closet available time slot of the requested  $r$ .
15:        – Allocate the re-timing time slot –
16:         $x_{o'r'} = 1$ 
17:       end if
18:     end for
19:   end for
20: end function

```

it is directly allocated to that RU (line 11) and if it has already been allocated, the nearest available time slot is assigned (line 16).

4.2. Equity rule

The heuristic method for solving TSA-OPEN problem using the equity rule modelled in section 3 is shown below. The objective of the TSA-OPEN problem is to minimize the overall deviation of the time slots requested by all the RUs in the system while simultaneously ensuring that the discrepancies between RUs do not differ excessively. The equity criterion is established on the basis of the assigned capacity to each RU. This way, the IM will allocate a time slot to the RU which, at a given time, has received the least number of time slots in relation to its capacity. The requests of the RUs are iteratively processed. In each iteration t , the relationship between the allocated time slots and the total capacity of a given RU (the value ϕ_o^t) is recalculated in order to determine which RU's request is to be served in that iteration. This process continues until all the requests made by the companies are fulfilled. The calculation of ϕ_o^t values is shown in Equation (33).

$$\phi_o^t = \frac{\sum_{r \in \mathcal{R}} x_{or}^t}{k_o}, \quad (33)$$

where x_{or}^t represents the value of the variable x at iteration t . After computing the ϕ_o^t values for each RU o then IM process the first request from the RU with the lowest value of ϕ_o^t . If the time slot requested is free, it is directly allocated to the RU. If not, it is re-timed. Algorithm 2 shows the pseudocode of the heuristic algorithm to solve time slot allocation based on equity rule criterion.

Algorithm 2 Pseudocode for solving heuristically TSA-OPEN problem using an equity rule

Require: $\mathcal{O}, \mathcal{R}, k, y_1, y_2, \dots, y_{|\mathcal{O}|}$

Ensure: x

```

1: function TSA-OPEN USING EQUITY RULE
2:   – Initialise  $\phi$  values to infinity –
3:    $\phi = []$ 
4:   for  $o = 1 : |\mathcal{O}|$  do
5:      $\phi[o] = +\infty$ 
6:   end for
7:   – Check RUs requests –
8:    $y = [y_1, y_2, \dots, y_{|\mathcal{O}|}]$ 
9:   – Computes the total number of requests received –
10:   $n_y = \sum_{o \in \mathcal{O}} \sum_{r \in \mathcal{R}} y_{or}$ 
11:  – Iterative request processing –
12:  for  $t = 1 : n_y$  do
13:    – Recalculate  $\phi_o$  values –
14:     $\phi_o = \frac{\sum_{r \in \mathcal{R}} x_{or}}{k_o}$ 
15:     $o' = \text{Arg min}_{o \in \mathcal{O}^*} \{\phi_o\}$ ; where  $\mathcal{O}^*$  is the set of RU's with time slots that need to
    be allocated
16:    Let  $r$  be the next time slot of RU  $o'$  to be processed.
17:    if  $r$  request can be satisfied then
18:       $x_{o'r} = 1$ 
19:    else
20:       $h_{o'r'} = 1$  where  $r'$  is the closet available time slot of the requested  $r$ .
21:      – Allocate the re-timing time slot –
22:       $x_{o'r'} = 1$ 
23:    end if
24:  end for
25: end function

```

In this algorithm, lines 3 – 6 defines a ϕ vector which contains the initial ϕ values for all the RUs in the market. Initially, these values are equals to ∞ . Then, lines 8 – 10 allow to obtain the total number of requests issued to the IM by all the RUs. A for loop in line 12 goes through all the requests, processing them one by one. In each iteration, ϕ values are computed for each RU. It will allow to determine the RU whose request is to be addressed in each iteration. Once the company to be served is identified, if the time slot requested is free it will be directly allocated (line 18), if not, the closet available time slot will be assigned (line 22).

5. Numerical Experiments

This section presents computational experiments using the proposed TSA-OPEN approach, applying both the priority and the equity rule criteria. The results provided in this section will allow us to answer the following three Research Questions (RQs):

- RQ1.** How does the use of the two rules proposed in the IM affect the equilibrium situation?
- RQ2.** How does the use of heuristic methods instead of exact methods impact the equilibrium situation?
- RQ3.** Are the conclusions still valid if we had used the exact methods to calculate the equilibrium situation?

For these purposes, Section 5.1 describes the considered Spanish railway corridor between Madrid and Barcelona. In section 5.2, the impact of the two allocation rules is analysed with respect to the profit of each RU (RQ1). Later, section 5.3 shows the performance of heuristic algorithms against the exact models (RQ2). Finally, section 5.4 discusses if the conclusions obtained will still be valid if the experiments were carried out using the exact algorithms (RQ3).

5.1. *Experimental settings: the Spanish railway market*

The experimental scenario corresponds to the Spanish passenger railway market. Specifically, the Madrid-Barcelona high-speed corridor has been considered. For this purpose, two origin-destination (O-D) pairs have been established: $\omega_1 = (\text{Madrid}, \text{Barcelona})$ and $\omega_2 = (\text{Barcelona}, \text{Madrid})$. The demand varies throughout the day according to the distribution given in Almodóvar and García-Ródenas (2013), which has been considered for the two pairs ω_1 and ω_2 . Figure 2 shows the number of passengers travelling in each time slot according to that distribution. We consider that if a company operates in a specific time slot r , it captures the entire demand assigned to that time slot.²

In the Spanish railway market, there are three RUs: RENFE (the incumbent operator), OUIGO and IRYO, and referred as to RU_1 , RU_2 and RU_3 and they are denoted respectively as o_1 , o_2 , and o_3 . Regarding planning time, it is considered one day of operation, starting at 06:15 and ending at 23:15, with a time slot every half hour. Therefore, a total of 35 time slots in 30-minute intervals are available for each of the origin-destination pairs. In order to study the impact of the allocation policy adopted by the IM on the competition between RUs, the following assumptions are made. First, all operators have allocated the same capacity, denoted as $k_o = 25\%$, allowing them a maximum capacity of operating 8 time slot per direction. This is to consider all RUs equally and the results obtained are solely the consequence of the actions of IM. Second, all RUs employ identical rolling stock and ticket prices. Otherwise, the results obtained could be the result of the policy price or rolling stock difference, rather than

²Given the levels of demand, this is a reasonable assumption.

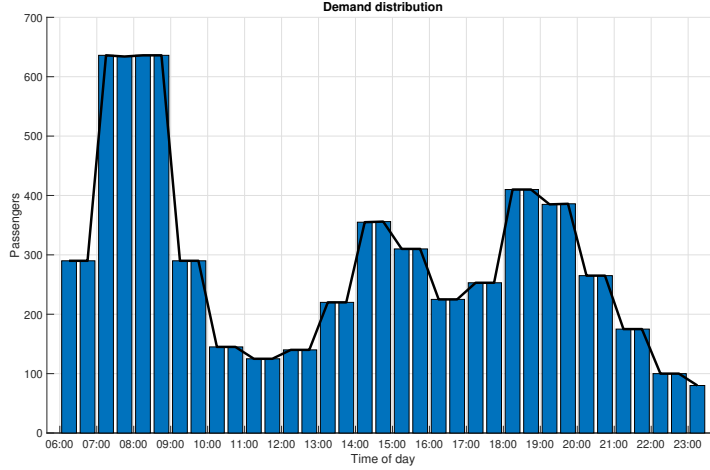


Figure 2. Demand distribution

the result of the IM's policy.

A comprehensive cost framework has been adopted for this analysis. Thus, the ticket price has been set to 70 euros. The same rolling stock unit can operate several time slots if the time constraints allow it. The model calculates the minimum number of rolling stock units that a RU needs to operate all allocated time slots. Finally, a daily amortisation cost of 11,490 euros has been included for each unit of rolling stock that the RU needs to have available to serve its time slots. This cost reflects the depreciation and maintenance expenses associated with the rolling stock units. Furthermore, the direct operational cost of each time slot amounts to 2,950 euros per slot. This cost factor encompasses various components, such as power consumption and personnel expenses. By considering these cost factors in our analysis, it is intended to provide an economic evaluation that accounts for both the revenue generation potential through ticket sales and the necessary expenditure required to operate the services efficiently. Table 3 summarises the parameters employed in the computational experiments.

The heuristic algorithms have been coded using the MATLAB programming language and run on a Windows 10 at 4 GHz AMD FX-8370 eight-core processor with 32 GB of RAM. Regarding exact algorithms, CPLEX has been used to solve TSA-OPEN problem. The equilibrium model shown in (32) has been solved in two stages: (1) the computation of the set of feasible strategies has been carried out according to García Rodenas et al. (2023), (2) the equilibrium problem is solved in this set by using the algorithm described and implemented in Chatterjee (2009).

5.2. RQ1. *How does the use of the two rules proposed in the IM affect the equilibrium situation?*

In this subsection, the equilibrium problem defined in the equation (32) considering both priority and equity rules has been solved. The equilibrium situation is composed

EXPERIMENTAL SETTINGS			
Name	Notation	Formulation	Value
O-D pairs	\mathcal{W}	$\mathcal{W} = \{\omega_1, \omega_2\}$	$\mathcal{W} = \{(\text{Madrid, Barcelona}), (\text{Barcelona, Madrid})\}$
RUs	\mathcal{O}	$\mathcal{O} = \{o_1, o_2, o_3\}$	$\mathcal{O} = \{RENFE, OUIGO, IRYO\}$
Time slot	\mathcal{R}	$\mathcal{R} = \{r_1, r_2, r_3, \dots, r_{35}\}$	$\mathcal{R} = \{06 : 15, 06 : 45, 07 : 15, \dots, 23 : 30\}$
Capacities	k	$k = [k_1, k_2, k_3]$	$k = [25\%, 25\%, 25\%]$
Ticket price	z_{rw}		70€
Daily amortization cost	C_o		11490€
Cost operational slot	f_{or}		2950€

Table 3. Experimental settings set in the numerical experiments

by a set of strategies for each RU. A pure strategy is defined as a single time slot request (bid). Furthermore, a combined strategy is a set of pure strategies which are played by the RUs with a probability value. Table 4 shows the obtained equilibrium state for both rules using the heuristic algorithms designed. The column one is associated to the kind of rule, the column two represents the probability to use a combined strategy. Then, columns three and four details the RU and the corresponding origin-destination pair. Finally, column y_o shows the request of each RU while column x_o represents the time slot finally allocated by the IM.

According to Table 4, the solution of the TSA-OPEN problem with priority rule consists of a combined strategy for RU_2 (composed by two pure strategies, which are played with probabilities $p_{y1} = 0.5628$ and $p_{y2} = 0.4372$) and one pure strategy for RU_1 and RU_3 which are employed with probability one.

Similarly, Table 4 also shows the equilibrium situation when solving the TSA-OPEN problem considering the equity rule. The equilibrium situation in this case also leads to a combined strategy composed of two pure strategies for RU_2 . However, while the probabilities in the case of the priority rule for each of the simple strategies were similar, in this case, the probability of RU_2 playing the first simple strategy is $p_{y1} = 0.0415$ while the probability of playing the second simple strategy is $p_{y2} = 0.9585$.

In order to answer RQ1, a two-fold analysis is going to be carried out. It will take into account the time slot allocation and the economic revenue associated with each of the RUs. This way, the impact of the two rules in the equilibrium situation will be assessed.

On the one hand, the allocation results will be discussed. Figures 3 and 4 show the time slot allocated to the different RUs and the demand captured in each one. Concretely, the abscissa axis represents each of the time slots while the ordinate axis represents the number of passengers travelling in each time slot, according to the optimal strategies of the RUs³. Finally, the colour of the bar represents the RU to which the time slot has been assigned and on which the passengers are travelling. In addition, each sub-figure displays this information for an origin-destination pair.

Concerning the allocation using priority rule, Figures 3(a) y 3(b), the time slots with the highest demand are allocated to RU_1 , i.e. the one that in the priority rule is served first. Similarly, the time slots with the lowest demand are assigned by the

³It is worth noting that the optimal strategy for RUs consists of employing two simple strategies with different probabilities. Here, only one is represented for simplicity.

Rule	Probability (p_y)	RU	ω	y_o	x_o
Priority Rule	$p_{y1} = 0.5628$	RU1	ω_1	[7:45, 8:15, 8:45, 9:45, 14:45, 15:15, 18:15, 19:15]	[7:45, 8:15, 8:45, 9:45, 14:45, 15:15, 18:15, 19:15]
			ω_2	[7:15, 7:45, 8:45, 13:45, 15:45, 18:15, 18:45, 20:15]	[7:15, 7:45, 8:45, 13:45, 15:45, 18:15, 18:45, 20:15]
		RU2	ω_1	[7:45, 8:15, 9:45, 14:45, 15:15, 18:15, 19:15, 20:45]	[7:15, 9:15, 10:15, 14:15, 15:45, 18:45, 19:45, 20:45]
			ω_2	[7:15, 7:45, 8:45, 14:45, 17:45, 18:15, 18:45, 20:15]	[6:45, 8:15, 9:15, 14:45, 17:45, 19:15, 19:45, 20:45]
		RU3	ω_1	[6:45, 7:45, 13:15, 14:45, 15:15, 18:15, 19:15, 20:15]	[6:15, 6:45, 13:15, 13:45, 16:15, 17:45, 20:15, 21:15]
			ω_2	[7:15, 7:45, 13:15, 13:45, 15:45, 18:45, 20:15, 20:45]	[6:15, 9:45, 13:15, 14:15, 16:15, 17:15, 21:15, 21:45]
	$p_{y2} = 0.4372$	RU1	ω_1	[7:45, 8:15, 8:45, 9:45, 14:45, 15:15, 18:15, 19:15]	[7:45, 8:15, 8:45, 9:45, 14:45, 15:15, 18:15, 19:15]
			ω_2	[7:15, 7:45, 8:45, 13:45, 15:45, 18:15, 18:45, 20:15]	[7:15, 7:45, 8:45, 13:45, 15:45, 18:15, 18:45, 20:15]
		RU2	ω_1	[7:45, 8:45, 9:45, 14:45, 15:15, 16:45, 18:15, 19:15]	[7:15, 9:15, 10:15, 14:15, 15:45, 16:45, 18:45, 19:45]
			ω_2	[7:15, 7:45, 13:15, 13:45, 15:45, 19:45, 20:15, 20:45]	[6:45, 8:15, 13:15, 14:15, 16:15, 19:45, 20:45, 21:15]
		RU3	ω_1	[6:45, 7:45, 13:15, 14:45, 15:15, 18:15, 19:15, 20:15]	[6:15, 6:45, 13:15, 13:45, 16:15, 17:45, 20:15, 20:45]
			ω_2	[7:15, 7:45, 13:15, 13:45, 15:45, 18:45, 20:15, 20:45]	[6:15, 9:15, 12:45, 14:45, 15:15, 19:15, 21:45, 22:15]
Equity Rule	$p_{y1} = 0.0415$	RU1	ω_1	[7:45, 8:15, 13:15, 14:45, 15:15, 18:15, 19:15, 19:45]	[8:15, 9:15, 12:45, 14:15, 14:45, 19:15, 20:45, 21:45]
			ω_2	[7:15, 7:45, 8:45, 13:45, 15:45, 16:45, 18:15, 18:45]	[7:45, 8:45, 9:45, 13:15, 16:45, 17:15, 19:45, 20:15]
		RU2	ω_1	[6:45, 7:45, 8:15, 15:15, 18:15, 19:15, 19:45, 20:45]	[7:15, 8:45, 9:45, 15:45, 18:45, 19:45, 20:15, 22:15]
			ω_2	[7:15, 7:45, 8:45, 13:45, 15:45, 18:15, 18:45, 20:15]	[8:15, 9:15, 10:15, 14:45, 16:15, 18:45, 19:15, 20:45]
		RU3	ω_1	[8:15, 9:45, 13:15, 14:45, 16:45, 18:15, 19:15, 19:45]	[7:45, 10:15, 13:45, 15:15, 17:15, 17:45, 18:15, 21:15]
			ω_2	[7:15, 7:45, 13:45, 15:15, 15:45, 18:15, 18:45, 19:45]	[6:45, 7:15, 14:15, 15:15, 15:45, 17:45, 18:15, 21:15]
	$p_{y2} = 0.9585$	RU1	ω_1	[7:45, 8:15, 13:15, 14:45, 15:15, 18:15, 19:15, 19:45]	[8:15, 9:15, 12:45, 15:45, 16:15, 19:15, 20:45, 21:45]
			ω_2	[7:15, 7:45, 8:45, 13:45, 15:45, 16:45, 18:15, 18:45]	[7:45, 8:45, 9:45, 13:15, 16:45, 17:15, 19:15, 20:15]
		RU2	ω_1	[6:45, 8:15, 9:15, 13:15, 15:15, 16:45, 18:15, 19:15]	[7:15, 8:45, 9:45, 14:15, 14:45, 17:15, 19:45, 20:15]
			ω_2	[7:15, 8:45, 13:15, 15:45, 16:45, 18:15, 20:15, 20:45]	[8:15, 9:15, 13:45, 16:15, 17:45, 19:45, 20:45, 21:15]
		RU3	ω_1	[8:15, 9:45, 13:15, 14:45, 16:45, 18:15, 19:15, 19:45]	[7:45, 10:15, 13:45, 15:15, 17:45, 18:15, 18:45, 21:15]
			ω_2	[7:15, 7:45, 13:45, 15:15, 15:45, 18:15, 18:45, 19:45]	[6:45, 7:15, 14:15, 15:15, 15:45, 18:15, 18:45, 21:45]

Table 4. Equilibrium solution using the heuristic algorithms and considering the priority and equity rules

algorithm to the RU with the lowest priority. Graphically, both figures show how the IM's actions cause the players' actions to be dominated, i.e. RU_1 imposes its criterion on RU_2 and RU_3 and, in turn, RU_2 imposes its criterion on RU_3 . The priority rule ensures a preference for the RU_1 that can lead to a market imbalance and ultimately to a monopoly situation.

Concerning the allocation using equity rule, Figures 4(a) y 4(b), the main difference with respect to the obtained results from the priority rule is that the allocation of the time slots capturing the highest demand are distributed equally among all RUs. Thus,

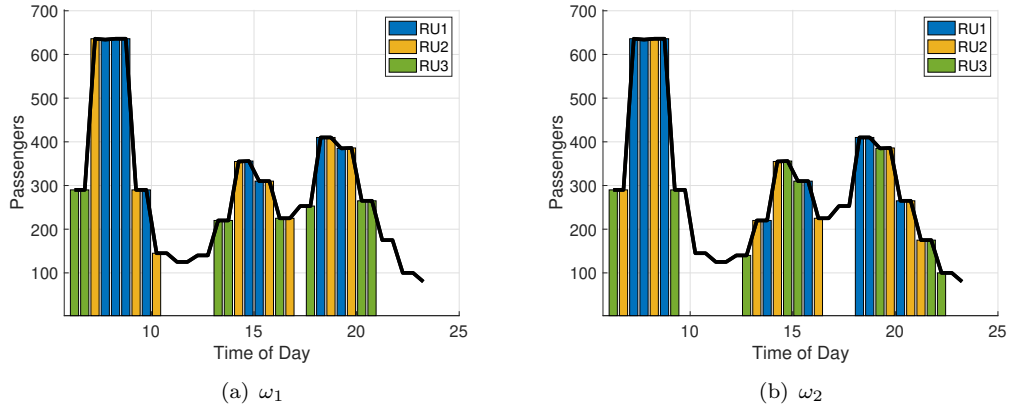


Figure 3. Time slot allocation under Priority Rule: Heuristic solution

it can be seen that for the two origin-destination pairs, the RUs in the market manage to obtain the operating rights of slots that capture high demand, just as they also receive the rights of other slots where the number of passengers is not as large.

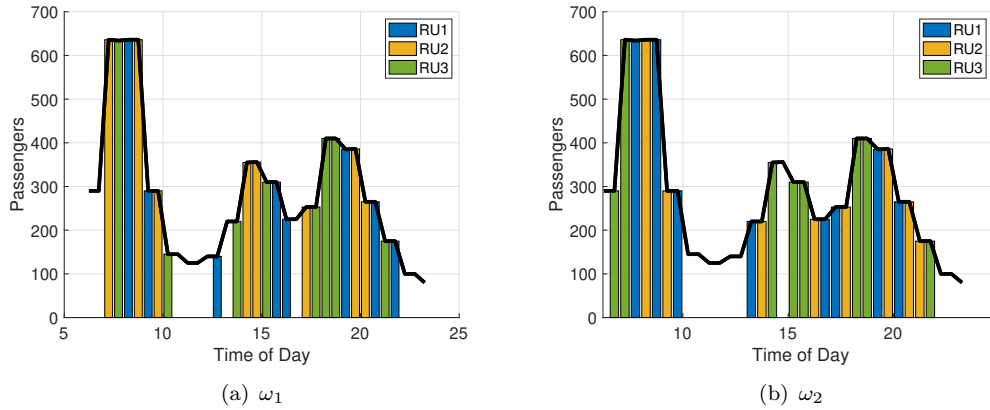


Figure 4. Time slot allocation under Equity Rule: Heuristic solution

On the other hand, the economic revenue of the RUs is going to be analysed in order to evaluate the equilibrium situation provided by the solution of TSA-OPEN using priority and equity rule. Table 5 shows, for each of the RUs in the market, the number of time slots that have been assigned in each origin-destination pair. In this case, as the capacity of each RU is 25%, each of them receives a total of 16 time slots, eight in each origin-destination pair. In addition, the table also shows the total number of passengers served by the RUs in the assigned time slots in each pair, as well as the total number of passengers served. The fact that decimal numbers appear in these data is because the results in the table are aggregated over the combined strategy, so the total number of passengers is the number of passengers served in each simple strategy multiplied by the probability of playing each simple strategy. Finally, the table also shows the number of rolling stock units that each RU needs to serve the assigned services as well as the total benefit obtained calculated through the equation (29). These values are also computed over the combined strategy, so decimal numbers may appear. These values are obtained after applying an algorithm to compute the

minimum number of trains which are required to meet planned schedules.

\mathcal{A}	RUs	OD ₁ Pass.	$ \mathcal{R}_1 $	OD ₂ Pass.	$ \mathcal{R}_2 $	Total Passengers	Rolling Stocks	Revenue
Priority rule	o_1	3657	8	3521	8	7178	6	330320
	o_2	2779.5	8	2725.9	8	5505.4	5.6	218262
	o_3	1977.4	8	2010.5	8	3987.9	4.9	119945
Equity rule	o_1	2433.3	8	2908	8	5341.4	5	213244.8
	o_2	3171	8	2460.9	8	5631.9	5.1	232628.4
	o_3	2550.5	8	2889.5	8	5440	5	220146.8

Table 5. Economical results of solving TSA-OPEN problem using heuristic algorithms

According to the results of Table 5, it is possible to check and contrast the results provided by Figures 3 and 4. In the first row, which corresponds to the results of the priority rule, there is an imbalance in the market in favour of the RU_1 , since it captures many more passengers than the rest of the RUs and, therefore, has much greater economic benefits than the rest. Therefore, the impact of priority rule over the equilibrium situation is negative, since it leads to a situation of dominance of the RU_1 which produces an imbalance in the market, causing some RUs to go out of business and, ultimately, leading to a monopoly situation in the market.

Concerning the results of the equity rule which are shown in the second row of Table 5, it is possible to observe a completely different market situation. In this case, the number of passengers captured by the RUs does not differ as significantly as in the previous case, which is also reflected in the number of rolling stock units that each RU needs (the same in this case) and in the economic benefits of all of them, which is much more balanced. Therefore, it is possible to verify that the equity rule impacts the equilibrium situation positively, guaranteeing that all RUs involved obtain similar benefits, which enables competition within the market.

5.3. RQ2. *How does the use of heuristic methods instead of exact methods impact the equilibrium situation?*

With the purpose of studying the performance of the two heuristic algorithms proposed to solve TSA-OPEN problem using priority and equity rule, exact algorithms have also been used to solve both problems. Concretely, TSA-OPEN problem was solved using CPLEX. The results obtained by CPLEX in solving the problem using the priority and the equity rule are shown below.

On the one hand, Figure 5 shows the allocation results provided by heuristic and exact algorithms when TSA-OPEN problem with priority rule is solved. In this figure, an image matrix composed of three rows and two columns can be observed. Each row corresponds to a different RU in the market, while each column corresponds to a different origin-destination pair. Thus, Figures 5(a) and 5(b) correspond to RU_1 in both directions when applying the priority rule. For each figure in the matrix, a so-called ring diagram shows the requested and allocated slots, where the outer ring with the thickest line represents the requested slots by the RUs, the middle ring shows the allocation provided by the heuristic algorithm while the inner ring with the thinnest line corresponds to the allocation obtained by the exact algorithm.

Analysing Figure 5 it is possible to contrast that, for RU_1 , the allocation provided

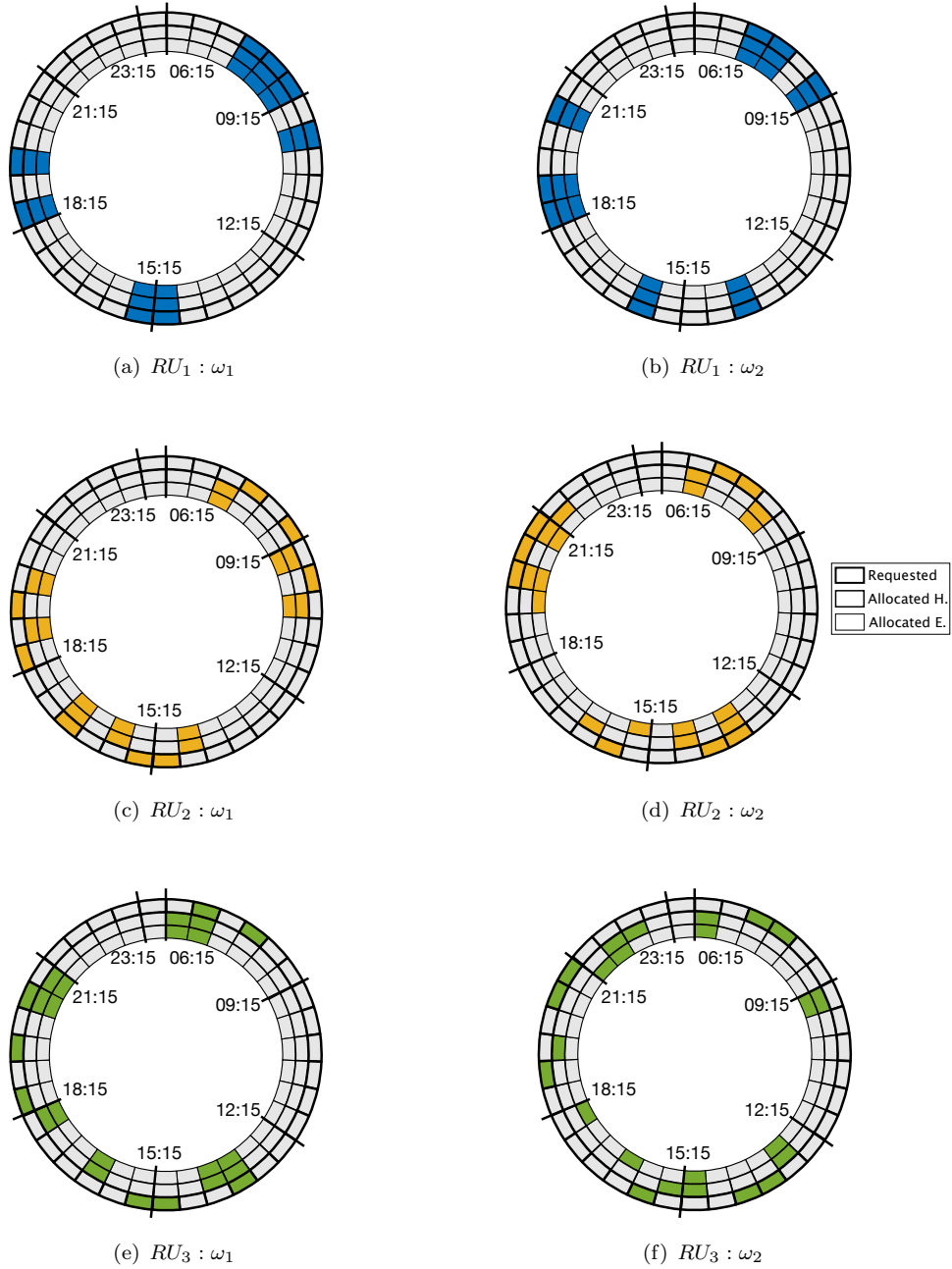


Figure 5. TSA-OPEN solution under priority rule. Each row represents a RU in the market and each column an origin-destination pair. The graphs compare the requested time slot (outer ring) to those one allocated by using heuristic (middle ring) and exact (inner ring) algorithms.

for both heuristic and exact algorithms is the same in the two origin-destination pairs. Regarding RU_2 , the allocation for the first pair obtained by both algorithms is also the same. However, the allocation provided for the second pair ω_2 is slightly different in both algorithms, allocating six of the eight time slots equally by both algorithms. Finally, something very similar happens with RU_3 , where the allocation for ω_1 origin-destination pair is the same using both exact and heuristic algorithms, but in the

case of the second pair, only five of the eight time slots are allocated equally by both algorithms. In this test scenario in which all companies have the same capacity, the priorities were assigned to RU_1 , RU_2 and RU_3 respectively (note that if priorities were assigned according to the capacity of the RUs, the incumbent operator will be favoured). As can be seen in this figure, the coincidence of both algorithms in the solution is an indicator of the quality of the proposed heuristic algorithm.

On the other hand, Figure 6 shows the results of heuristic and exact algorithms when TSA-OPEN is solved by means of equity rule. Here, the results are quite different. However, these results are reasonable since, under this approach, the time slots are proportionally distributed according to the RUs's capacities. In this case, the solutions of exact and heuristic approaches are different. Thus, it is possible to check that, for RU_1 , only four and three time slots respectively are equally assigned by both algorithms for the two origin-destinations pairs. For RU_2 in the ω_1 pair, two time slots are equally assigned by heuristic and exact algorithms while in ω_2 , there are three time slots. Finally, regarding RU_3 , only one time slot is assigned by both heuristic and exact algorithms for the first pair, while in the second origin-destination pair, three time slots are equally assigned by both algorithms.

In order to summarize the results of Figures 5 and 6, Table 6 shows a study of the deviation of each of the RUs. The deviation for a RU is the sum of the time differences between the time slot it requested and the time slot it was finally allocated. Thus, the smaller the deviation, the more the allocation of time slot was in line with the RU's request, while the larger the deviation, the greater the discrepancy between the time slot requested and those finally allocated to the RU. This way, Table 6 shows this comparison. It is possible to check the performance of heuristic and exact algorithms is similar when the priority rule is employed, since the deviation for RU_1 and RU_2 is the same, while the difference in the deviation of RU_3 is only half an hour, it means, only one time slot deviation. Thus, the total deviation using the priority rule is around twenty hours. However, in the case of equity rule, the differences are greater. The heuristic approach exhibits an average deviation of approximately ten hours, whereas the exact approach yields half the deviations for all the RUs.

A	Algorithm	D_{o1}	D_{o2}	D_{o3}	Total Deviation
Priority Rule	Heuristic	0	6h 30m	13h 30m	20h
	Exact	0	6h 30m	13h	19h 30m
Equity Rule	Heuristic	15h 30m	12h	8h 30m	36h
	Exact	6h 30m	5h	5h	16h 30m

Table 6. Deviation analysis of TSA-OPEN solutions

The results provided by Table 6 seem to indicate that, when solving the TSA-OPEN problem with priority rule, it is indifferent to use heuristic and exact algorithms. However, when attempting to address the TSA-OPEN problem using the equity rule, exact algorithms ensure an optimal allocation of time slots with respect to the disparity between the requests made by the different carriers. To validate this observation from the table and considering that temporal disparity differs from the economic outcomes each of the RUs would achieve, we will reevaluate the equilibrium strategy using exact methods. The outcomes are presented in Table 7.

From Table 7, it can be seen that in terms of the allocation results with priority rule,

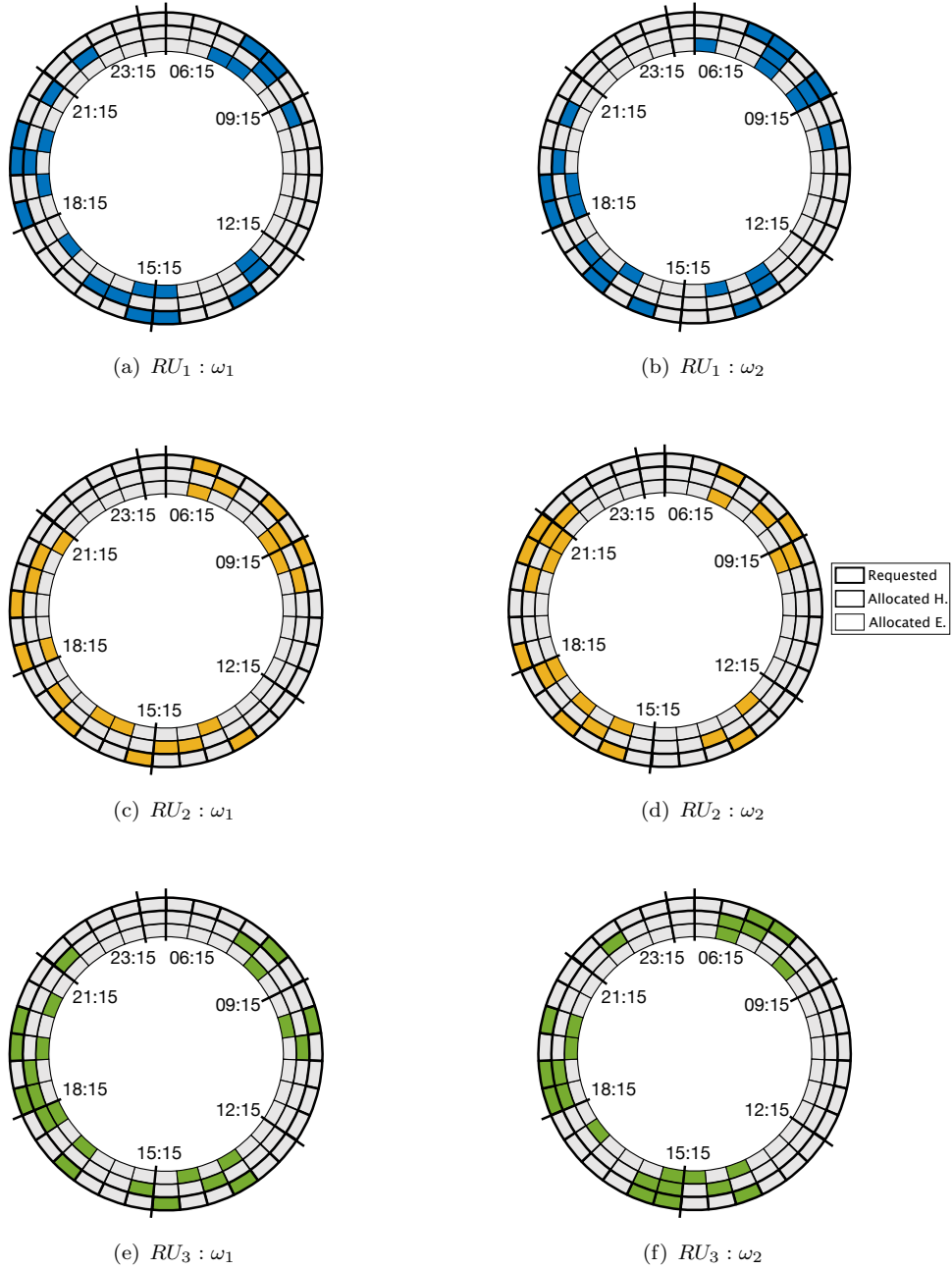


Figure 6. TSA-OPEN solution under equity rule. Each row represents a RU in the market and each column is an origin-destination pair. The graphs compare the requested time slot (outer ring) to those allocated by using heuristic (middle ring) and exact (inner ring) algorithms.

the economic benefits obtained are very similar (slightly higher) than those obtained by the heuristic algorithms, both in terms of the number of passengers travelling on each company and the benefit obtained by each of them. These results, therefore, are in line with those obtained in Table 6, which showed how the discrepancy obtained for each of the companies by the heuristic and exact algorithms was similar, although slightly better than that obtained by the exact algorithms.

\mathcal{A}	TOCs	OD ₁	$ \mathcal{R}_1 $	OD ₂	$ \mathcal{R}_2 $	Passengers	Rolling Stocks	Revenue
Priority rule	o_1	3657	8	3521	8	7178	6	330320
	o_2	2779.5	8	2780	8	5559.5	5	228517.7
	o_3	2021.2	8	2071.2	8	4092.4	5.1	124374.7
Equity rule	o_1	3134.8	8	3178.2	8	6313	5	281260.6
	o_2	2649.9	8	2411.8	8	5061.7	5	193192.5
	o_3	2614.7	8	2816.2	8	5430.9	5	219512.1

Table 7. Economical results of solving TSA-OPEN problem using exact algorithms

On the other hand, the results obtained when the exact algorithms are used to solve the TSA-OPEN problem with the equity rule are striking. In this particular case, both the number of passengers and the economic benefit obtained by each of the RUs differ significantly from those obtained by the heuristic algorithms. These economic differences are due to the discrepancy obtained by the heuristic algorithms and the exact algorithms (see Table 6). However, the results obtained by the exact algorithms, despite guaranteeing a smaller discrepancy between the RUs' requests and the time slots finally allocated, maintain a significant imbalance in terms of the number of passengers travelling on each RU and the profits of each RU, showing a non-equilibrium solution with respect to the exact algorithms.

Despite this negative outcome regarding the quality of the heuristic using the equity rule, there is a significant highlight: when addressing the TSA-OPEN problem with the equity rule, it becomes evident that the RUs can learn how the IM operates and devise a strategy that leads to an equilibrium ensuring equal or similar profits for all of them. Therefore, if the IM sets a rule that is transparent and does not lead to a dominant strategy of one over the others, the RUs will come to play a strategy that allows them to reach an equilibrium situation and perceive similar profits. In other words, what is important is not only that the discrepancy between requested and allocated time slots is small, but that all RUs in the market are treated equally by the IM.

5.4. RQ3. *Are the conclusions still valid if we had used the exact methods to calculate the equilibrium situation?*

To answer this question, it is necessary to start from the results obtained in the previous section. As indicated in Table 7 compared to Table 5, the results obtained by the heuristic and exact algorithms for the resolution of the TSA-OPEN with priority rule are similar in terms of discrepancy, number of passengers captured and economic benefits obtained by the RUs. In both cases, it can be observed that the priority rule imposes a dominance of the RU₁ over the rest in the market, which jeopardises the situation of liberalisation and competition. Therefore, in this case, the conclusions are the same whether exact or heuristic algorithms are used to solve the TSA-OPEN problem.

However, when addressing the TSA-OPEN problem with an equity rule, the conclusions differ based on whether heuristic or exact algorithms are employed for problem-solving. As demonstrated in the previous section, the use of exact algorithms ensures an optimal alignment between the time slots requested by the RUs and those ultimately assigned. However, this criterion has not been employed as the criterion $x = \mathcal{A}(y)$ in

the equilibrium calculation (see Equations (30)-(31)), and as a result, it does not lead to a balance in the benefits received by the companies. To obtain a definitive answer for **R3** in the case of the equity rule, equilibrium calculation using exact methods for the allocation is required. The challenge lies in the necessity to solve a large number of linear programming problems, which is the motivation behind utilizing heuristic methods.

One can postulate the response to **R3** for the equity rule using exact methods based on the insights gained from heuristic approaches. In alignment with the results presented in the previous section, where all companies achieve comparable revenues despite a substantial divergence between the requested and allocated time slots, it becomes apparent that these companies familiarize themselves with the operations of the IM and formulate strategies to attain an equilibrium in which they all receive equal benefits.

The conjecture to assert that the RUs will reach an *equitable equilibrium*, i.e. indistinguishable companies achieve the same revenue, is as follows: If the method utilized, denoted as $\mathcal{A}(y)$, were universally known among all RUs, and furthermore, if \mathcal{A} possessed the property that swapping the requests y_o of two RUs results in the IM reciprocally swapping the time slot assignments, then the equilibrium would indeed be equitable, and the role of the IM would be neutral in this context.

6. Conclusions and further works

The European passenger railway markets are progressing steadily towards the creation of a common and unified railway area, where multiple RUs compete with each other for operating rights in time slots, which means a greater supply for passengers and an opportunity to improve rail services. This evolution, which is marked by the competitive aspect of the liberalised rail markets, is leading to changes in railway planning, operation and management, which require new models and algorithms for their efficient resolution.

This paper is a novel research work about the modelling of open passenger railway systems. The first contribution is the modelling of TSA-OPEN problem, which is mathematically formulated in order to capture the conflicting objectives of the RUs in a liberalised passenger railway market like the Spanish one.

Efficiently solving the TSA-OPEN problem is a pivotal concern for fostering competition in the passenger railway market. In this study, we demonstrate that any criterion \mathcal{A} employed by the IM must select points from the Pareto front. We propose two solution approaches with these characteristics: the priority rule and the equity rule. Both criteria are straightforward to implement using linear programming and entail low computational costs. Additionally, we have introduced heuristic versions of these criteria in this study, driven by the necessity to compute the equilibrium state to study these rules, although these heuristics are not required for practical implementation.

Additionally, this paper presents a case study designed to evaluate the performance of the two proposed approaches. The study focuses on the Madrid-Barcelona corridor within the Spanish railway market. The objective of this experiment is to investigate the function of the IM in time slot allocation with the aim of promoting competition between the RUs and preventing the emergence of a monopolistic situation. Heuristic and exact algorithms have been applied to solve TSA-OPEN problem using both priority and equity rules.

The comparison between heuristic and exact algorithms has led to the conclusion that both approaches obtain similar solutions when TSA-OPEN problem is solved using the priority rule. However, this approach tends to benefit the operator with the highest priority, exhibiting relevant differences between the revenues of the different RUs.

Furthermore, we have found that the heuristic equity rule achieves a fair equilibrium, wherein indistinguishable players obtain identical revenue. This demonstrates that the use of this rule by the IM assigns it a neutral role in the market. Additionally, we have observed that the heuristic rule yields results that deviate significantly from the optimal solutions obtained through the exact method.

The discussion of question **R3** leads us to speculate that the exact equity rule also attains a fair equilibrium. However, the fact that the exact method produces deviations closer to the initial requests of the various companies makes it more practical. It allows the RUs to anticipate the outcomes of their actions and reach the equilibrium state more quickly.

The dynamics in a liberalised railway passenger system are complex and require novel models to capture. For future works, it is intended to extend this model in order to represent the complete dynamics in a liberalised railway market. It will lead to a more complex game-theoretic model which represents competition in liberalised passenger railway markets, taking into account optimal strategies, demand modelling and equilibrium prices.

Acknowledgements

This work was supported by grant PID2020-112967GB-C32 funded by MCIN/AEI/10.13039/501100011033 and by *ERDF A way of making Europe*.

References

- Ait-Ali, Abderrahman. 2020. "Methods for Capacity Allocation in Deregulated Railway Markets." PhD diss.
- Ait Ali, Abderrahman, and Jonas Eliasson. 2019. *Railway capacity allocation: a survey of market organizations, allocation processes and track access charges*. Technical Report 2019:1.
- Ait Ali, Abderrahman, Jennifer Warg, and Jonas Eliasson. 2020. "Pricing commercial train

- path requests based on societal costs.” *Transportation Research Part A: Policy and Practice* 132: 452–464. <https://doi.org/https://doi.org/10.1016/j.tra.2019.12.005>, <https://www.sciencedirect.com/science/article/pii/S0965856419303829>.
- Almodóvar, M, and R García-Ródenas. 2013. “On-line reschedule optimization for passenger railways in case of emergencies.” *Computers & Operations Research* 40 (3): 725–736. <https://doi.org/https://doi.org/10.1016/j.cor.2011.01.013>, <https://www.sciencedirect.com/science/article/pii/S0305054811000256>.
- Broman, Emanuel, and Jonas Eliasson. 2019. “Welfare effects of open access competition on railway markets.” *Transportation Research Part A: Policy and Practice* 129: 72–91. <https://doi.org/https://doi.org/10.1016/j.tra.2019.07.005>, <https://www.sciencedirect.com/science/article/pii/S0965856418305615>.
- Broman, Emanuel, Jonas Eliasson, and Martin Aronsson. 2022. “Efficient capacity allocation on deregulated railway markets.” *Journal of Rail Transport Planning & Management* 21: 100294. <https://doi.org/https://doi.org/10.1016/j.jrtpm.2021.100294>, <https://www.sciencedirect.com/science/article/pii/S2210970621000597>.
- Cacchiani, Valentina, Alberto Caprara, and Paolo Toth. 2008. “A column generation approach to train timetabling on a corridor.” *4OR* 6 (2): 125–142. <https://doi.org/10.1007/s10288-007-0037-5>, <https://doi.org/10.1007/s10288-007-0037-5>.
- Cai, X, C J Goh, and ALISTAIR I Mees. 1998. “Greedy heuristics for rapid scheduling of trains on a single track.” *IIE Transactions* 30 (5): 481–493. <https://doi.org/10.1023/A:1007551424010>, <https://doi.org/10.1023/A:1007551424010>.
- Cantos-Sánchez, Pedro, José Manuel Pastor Monsálvez, and Lorenzo Serrano-Martínez. 2021. “Vertical and Horizontal Separation in the European Railway Sector and its Effects on Productivity.” *International Encyclopedia of Transportation: Volume 1-7* 1 (2): 503–507. <https://doi.org/10.1016/B978-0-08-102671-7.10092-2>, <http://www.jstor.org/stable/40600020>.
- Caprara, Alberto, Matteo Fischetti, and Paolo Toth. 2002. “Modeling and Solving the Train Timetabling Problem.” *Operations Research* 50: 851–861. <https://doi.org/10.1287/opre.50.5.851.362>.
- Caprara, Alberto, Leo Kroon, Michele Monaci, Marc Peeters, and Paolo Toth. 2007. “Chapter 3 Passenger Railway Optimization.” In *Transportation*, edited by Cynthia Barnhart, Gilbert B T Handbooks in Operations Research Laporte, and Management Science, Vol. 14, 129–187. Elsevier. <https://www.sciencedirect.com/science/article/pii/S0927050706140037>.
- Caprara, Alberto, Michele Monaci, Paolo Toth, and Pier Luigi Guida. 2006. “A Lagrangian heuristic algorithm for a real-world train timetabling problem.” *Discrete Applied Mathematics* 154 (5): 738–753. <https://doi.org/https://doi.org/10.1016/j.dam.2005.05.026>, <https://www.sciencedirect.com/science/article/pii/S0166218X05003045>.
- Carey, Malachy, and David Lockwood. 1995. “A Model, Algorithms and Strategy for Train Pathing.” *Journal of the Operational Research Society* 46 (8): 988–1005. <https://doi.org/10.1057/jors.1995.136>, <https://doi.org/10.1057/jors.1995.136>.
- Chatterjee, Bapi. 2009. “An optimization formulation to compute Nash equilibrium in finite games.” *Proceedings of International Conference on Methods and Models in Computer Science, ICM2CS09* 1–5. <https://doi.org/10.1109/icm2cs.2009.5397970>.
- García Rodenas, Ricardo, Nikola Besinovic, María Luz López-García, Julio Alberto López-Gómez, and José Ángel Martín-Baos. 2023. *Game-Theoretic Modeling of Competition in the Liberalized Passenger Railway Market*. Technical Report.
- Higgins, A, E Kozan, and L Ferreira. 1996. “Optimal scheduling of trains on a single line track.” *Transportation Research Part B: Methodological* 30 (2): 147–161. [https://doi.org/https://doi.org/10.1016/0191-2615\(95\)00022-4](https://doi.org/https://doi.org/10.1016/0191-2615(95)00022-4), <https://www.sciencedirect.com/science/article/pii/0191261595000224>.
- Higgins, A, E Kozan, and L Ferreira. 1997. “Heuristic Techniques for Single Line Train Scheduling.” *Journal of Heuristics* 3 (1): 43–62. <https://doi.org/10.1023/A:1009672832658>, <https://doi.org/10.1023/A:1009672832658>.
- Kuo, April, and Elise Miller-Hooks. 2015. “Combinatorial auctions of railway track capac-

- ity in vertically separated freight transport markets.” *Journal of Rail Transport Planning & Management* 5 (1): 1–11. <https://doi.org/https://doi.org/10.1016/j.jrtpm.2014.12.001>, <https://www.sciencedirect.com/science/article/pii/S2210970614000675>.
- Li, Dewei, Tianyu Zhang, Xinlei Dong, Yonghao Yin, and Jinming Cao. 2019. “Trade-off between efficiency and fairness in timetabling on a single urban rail transit line under time-dependent demand condition.” *Transportmetrica B: Transport Dynamics* 7 (1): 1203–1231. <https://doi.org/10.1080/21680566.2019.1589598>, <https://doi.org/10.1080/21680566.2019.1589598>.
- Lusby, Richard M, Jesper Larsen, Matthias Ehrgott, and David Ryan. 2011. “Railway track allocation: models and methods.” *OR Spectrum* 33 (4): 843–883. <https://doi.org/10.1007/s00291-009-0189-0>, <https://doi.org/10.1007/s00291-009-0189-0>.
- Nash, Chris. 2008. “Passenger railway reform in the last 20 years – European experience reconsidered.” *Research in Transportation Economics* 22 (1): 61–70. <https://doi.org/https://doi.org/10.1016/j.retrec.2008.05.020>, <https://www.sciencedirect.com/science/article/pii/S0739885908000152>.
- Ristić, Bojan, Nikola Stojadinović, and Dejan Trifunović. 2022. “Conditions for effective on-track competition in the European passenger railway market: A yardstick for regulations.” *Transport Policy* 119: 1–15. <https://doi.org/https://doi.org/10.1016/j.tranpol.2022.02.006>, <https://www.sciencedirect.com/science/article/pii/S0967070X22000452>.
- Schlechte, Thomas. 2012. “Railway Track Allocation: Models and Algorithms.” PhD diss., Saarbrücken, Germany.
- Stojadinovic, Nikola, Branislav Boskovic, and Mirjana Bugarinovic. 2019. “Bridging the gap between infrastructure capacity allocation and market-oriented railway: An algorithmic approach.” *Transport* 34: 1–14. <https://doi.org/10.3846/transport.2019.11035>.
- Stojadinovic, Nikola, Branislav Boskovic, Dejan Trifunovic, and Sladana Jankovic. 2019. “Train path congestion management: Using hybrid auctions for decentralized railway capacity allocation.” *Transportation Research Part A: Policy and Practice* 129: 123–139. <https://doi.org/https://doi.org/10.1016/j.tra.2019.08.013>.
- Talebian, Ahmadreza, Bo Zou, and Ahmad Peivandi. 2018. “Capacity allocation in vertically integrated rail systems: A bargaining approach.” *Transportation Research Part B: Methodological* 107: 167–191. <https://doi.org/https://doi.org/10.1016/j.trb.2017.12.001>, <https://www.sciencedirect.com/science/article/pii/S0191261517301042>.
- Tomeš, Zdeněk, Martin Kvizda, Monika Jandová, and Václav Rederer. 2016. “Open access passenger rail competition in the Czech Republic.” *Transport Policy* 47: 203–211. <https://doi.org/https://doi.org/10.1016/j.tranpol.2016.02.003>, <https://www.sciencedirect.com/science/article/pii/S0967070X16300397>.
- Trafikverket. 2020. *Network Statment 2020*. Technical Report. Swedish Transport Administration.
- Vigren, Andreas. 2017. “Competition in Swedish passenger railway: Entry in an open access market and its effect on prices.” *Economics of Transportation* 11–12: 49–59. <https://doi.org/https://doi.org/10.1016/j.ecotra.2017.10.005>, <https://www.sciencedirect.com/science/article/pii/S2212012217300977>.
- Yang, Ruixia, Baoming Han, Qi Zhang, Zhenyu Han, and Yuxuan Long. 2023. “Integrated optimization of train route plan and timetable with dynamic demand for the urban rail transit line.” *Transportmetrica B: Transport Dynamics* 11 (1): 93–126. <https://doi.org/10.1080/21680566.2022.2040064>, <https://doi.org/10.1080/21680566.2022.2040064>.