ANOMALOUS TIEN–GORDON SCALING IN A 1D TUNNEL JUNCTION

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ABSTRACT

We investigate the nonlinear ac transport through a quantum wire with an impurity in the presence of finite range electron–electron interactions. We discuss the influence of the spatial shape of the ac electric field onto transport properties of the system and find that the scaling behavior of the occupation probability of the sidebands depends on the range of the voltage drop.

1. INTRODUCTION

Time dependent quantum transport has attracted a lot of interest since the works of Tien and Gordon [1] and Tucker [2]; more recently, theoretical findings [3, 4] and experiments on quantum dots [5] and on superlattices [6] renewed the interest in photon–assisted transport in semiconductor nanostructures. In particular, the possibility to investigate experimentally time–dependent transport through mesoscopic systems has opened the way to a deeper understanding of new effects strongly relying on the spatiotemporal coherence of electronic states. Moreover, in most time–dependent experiments like electron pumps [7, 8], photon–assisted–tunneling [5, 9, 10], and lasers [11] require an analysis going beyond the linear response theory in the external frequency. Thus, many efforts have been devoted, in last years, to the theoretical investigation of nonlinearities in semiconductor nanostructures [12, 13], electronic correlations [14, 15], and screening of ac fields [16, 17].

The Tien–Gordon formula, according to which the dc component of the photo–induced current is given by a superposition of static currents I_0 (the currents without the ac field) weighted by integer order Bessel functions, is represented by the following formula

$$I_{\rm dc} = \sum_{n=-\infty}^{\infty} J_n^2\left(\frac{eV_1}{\hbar\Omega}\right) I_0\left(V_0 + n\hbar\Omega/e\right); \qquad (1)$$

the argument of the Bessel functions is linearly dependent on the ac voltage intensity V_1 and on the inverse of the driving frequency (or subharmonic) Ω. A selfconsistent theory, based on the scattering matrix approach, has shown that the side–band peaks depend on the screening properties of the system [17]; moreover theoretical investigations for superlattice microstructures showed an Ω^{-2} dependence of the transmission probability spectrum of the photonic sidebands (that is the argument of the Bessel functions), when a nonlocalized (a finite range) ac driving was taken into account [18, 19, 20].

In this paper, we investigate how 1d electron–electron interaction, in the framework of the Luttinger model [21, 22, 23], nonlinearities, due to the presence of an impurity, and a finite range ac electric field affect the photo–induced current. We will show that the TG formula is still valid, but the argument of the Bessel functions is not anymore linearly dependent on $1/\Omega$.

In the time dependent regime the nonlinearity of the system gives rise to frequency mixing and harmonic generation. Earlier treatments of the ac transport considered voltages, dropping only at the position of the barrier [24, 25], and zero range interactions between the electrons. Here, both of these are generalized to the more realistic situation of finite range of both, the electron–electron interaction and the electric field. As a matter of fact previous calculations [26] showed clearly that the spatial shape of the electric field does influence ac transport.

2. MODEL

The Hamiltonian for a Luttinger liquid of length L ($\to \infty$) with an impurity and subject to a time–dependent electric field is $H = H_0 + H_{\rm imp} + H_{\rm ac}$, where

$$H_0 = \sum_{k\neq 0} \hbar\omega(k)\; b_k^\dagger b_k. \qquad (2)$$

The dispersion relation of the collective excitations,

$$\omega_k = v_{\rm F}|k|\sqrt{1 + \hat{V}_{\rm ee}(k)/\hbar\pi v_{\rm F}},$$

depends on the Fourier transform of the finite range interaction potential [16]. We assume a 3d screened Coulomb potential of range α^{-1} projected onto a quantum wire of diameter $d \approx \alpha^{-1}$. The interaction decays exponentially

and one gets $V_{\rm ee}(x) = (V_{\rm L}\alpha/2){\rm e}^{-\alpha|x|}$, with interaction strength $V_{\rm L}$ [27]. For $\alpha \to \infty$, one obtains a zero–range interaction.
The tunneling barrier of height $U_{\rm imp}$ is localized at $x = 0$ [28, 29],

$$H_{\rm imp} = U_{\rm imp} \cos\left(2\sqrt{\pi}\vartheta(x=0)\right), \qquad (3)$$

with the phase variable of the Luttinger model

$$\vartheta(x) = {\rm i}\sum_{k\neq 0} {\rm sgn}(k)\sqrt{\frac{v_{\rm F}}{2L\omega(k)}}{\rm e}^{-{\rm i}kx}\left(b_k^\dagger + b_{-k}\right).$$

The coupling to the external driving voltage yields

$$H_{\rm ac} = e\int_{-\infty}^{\infty} dx \varrho(x) V(x,t).$$

The electric field is related to the voltage drop by differentiation, $E(x,t) = -\partial_x V(x,t)$, and the charge density is $\varrho(x) = k_{\rm F}/\pi + \partial_x\vartheta(x)/\sqrt{\pi}$. The space–time dependent electric field, $E(x,t) = E_{\rm dc}(x) + E_a(x)\cos(\Omega t)$, such that $E_a(x) = E_1{\rm e}^{-|x|/a}$, gives a voltage drop $V_1 \equiv \int_{-\infty}^{\infty} dx E_a(x) = 2E_1 a$. The spatial dependence of the dc part of the electric field does not need to be specified, as only the overall voltage drop, $V_0 \equiv \int_{-\infty}^{\infty} dx E_{\rm dc}(x)$, is of importance in dc transport [26].

3. METHODS AND RESULTS

The current at the barrier is given by the expectation value $I(x=0,t) = \langle j(x=0,t)\rangle$, where the current operator is defined via the continuity equation, $\partial_x j(x,t) = -e\partial_t\rho(x,t)$. For a high barrier, the tunneling contribution to the current can be expressed in terms of forward and backward scattering rates which are proportional to the tunneling probability Δ^2. The latter may be obtained in terms of the barrier height $U_{\rm t}$ by using the instanton approximation [30]. The result can be written in terms of the one-electron propagator $S + {\rm i}R$ [25],

$$\begin{aligned} I(x=0,t) = e\Delta^2 \int_0^\infty {\rm d}\tau\, {\rm e}^{-S(\tau)} \sin R(\tau) \\ \times \sin\left[\frac{e}{\hbar}\int_{t-\tau}^{t} {\rm d}t' V_{\rm eff}(t')\right], \end{aligned} \qquad (4)$$

with

$$\begin{aligned} S(\tau) + {\rm i}R(\tau) = \frac{e^2}{\pi\hbar}\int_0^{\omega_{\rm max}} \frac{{\rm d}\omega}{\omega}\mathcal{R}e\left\{\sigma^{-1}(x=0,\omega)\right\} \\ \times\left[(1-\cos\omega\tau)\coth\frac{\beta\omega}{2} + {\rm i}\sin\omega\tau\right], \end{aligned}$$

where $\beta = 1/k_{\rm B}T$, $\omega_{\rm max}$ the usual frequency cutoff that corresponds roughly to the Fermi energy [31], and the ac conductivity of the system without impurity is [26]

$$\sigma(x,\omega) = \frac{-{\rm i}v_{\rm F}e^2\omega}{\hbar\pi^2} \int_0^\infty\!\!\!\!\!\!- {\rm d}k \frac{\cos kx}{\omega^2(k) - (\omega + {\rm i}0^+)^2}. \qquad (5)$$

Furthermore, the effective driving voltage is related to the electric field by [26]

$$\begin{aligned} V_{\rm eff}(t) &= \int_{-\infty}^{\infty} {\rm d}x \int_{-\infty}^{t} {\rm d}t' E(x,t') r(x,t-t') \\ &= V_0 + \frac{\hbar\Omega}{e}|z|\cos(\Omega t - \varphi_z), \end{aligned} \qquad (6)$$

where $r(x,\omega) = \sigma(x,\omega)/\sigma(x,\omega)$, $|z|$ and φ_z are, respectively, modulus and argument of

$$z = \frac{e}{\hbar\Omega}\int_{-\infty}^{\infty} {\rm d}x E_a(x) r(x,\Omega). \qquad (7)$$

With the above assumptions about the shapes of the driving field and the interaction potential one obtains

$$|z| = \frac{eV_1}{\hbar\Omega}\frac{1}{\sqrt{1 + a^2k^2(\Omega)}} A\left(\frac{\Omega}{v_{\rm F}\alpha}, \frac{k(\Omega)}{\alpha}, \alpha a\right), \qquad (8)$$

where $k(\Omega)$ is the inverse of the dispersion relation and

$$A^2(u,v,w) = \frac{1}{1+u^2}\left[1 + v^2\frac{(u+wv)^2}{(uw+v)^2}\right]. \qquad (9)$$

In the following, we concentrate on the results for the dc component of the current which does not depend on x and is directly given by the current at the barrier, for which we only need to know only $|z|$,

$$I_{\rm dc} = \sum_{n=-\infty}^{\infty} J_n^2(|z|)\, I_0\left(V_0 + n\frac{\hbar\Omega}{e}\right). \qquad (10)$$

The important point here is that the driven dc current is completely given in terms of $I_0(V_0)$, the nonlinear dc current-voltage characteristic of the tunnel barrier,

$$I_0(V_0) = e\Delta^2\int_0^\infty {\rm d}\tau {\rm e}^{-S(\tau)}\sin R(\tau)\sin\left(\frac{eV_0\tau}{\hbar}\right). \qquad (11)$$

Eqs. (10), (11) generalize results which have been obtained earlier [1] but *without* interaction between the tunneling objects, and also for the Luttinger model with a zero-range interaction, together with a δ-function like driving electric field [24].
For V_0 much smaller than some cutoff-voltage $V_{\rm c}$ which is related to the inverse of the interaction range, $I_0 \propto V_0^{2/g-1}$. This recovers the result obtained earlier for δ-function interaction and zero-range bias electric field [28]. When $V_0 \gg V_{\rm c}$, the current becomes linear [32]. For intermediate values of V_0, I_0 exhibits a cross-over between the asymptotic regimes with a point of inflection near $V_{\rm c}$. For zero-range interaction, $I_0 \propto V_0^{2/g-1}$ for any V_0.
Figure 1 shows the currents I_0, $I_{\rm dc}$ and the differential conductance ${\rm d}I_{\rm dc}/{\rm d}V_0$ as functions of $eV_0/\hbar\Omega$ for $g = 0.9$ and $g = 0.5$ for zero-range of the driving electric field. For $g = 0.9$ one observes sharp minima in the differential conductance at integer multiples of the driving frequency in certain regions of the driving voltage V_1.

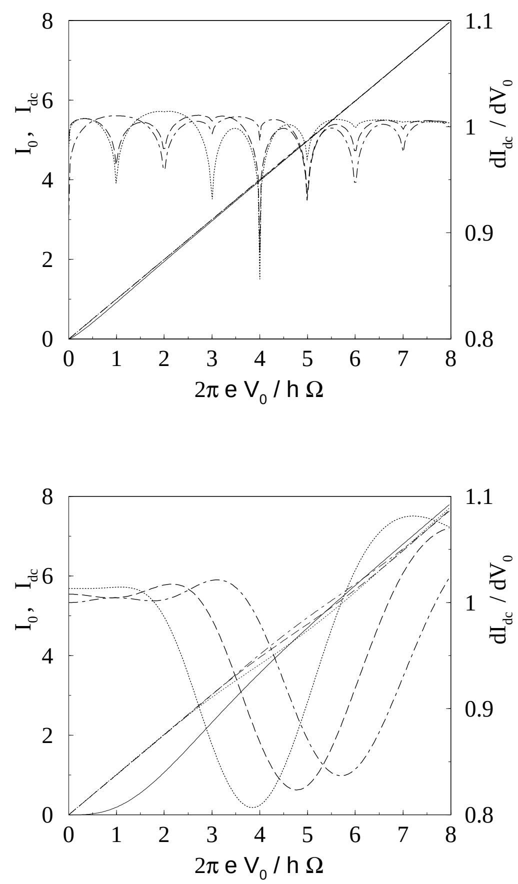

Figure 1: Currents I_0, $I_{\rm dc}$ and differential conductance $\mathrm{d}I_{\rm dc}/\mathrm{d}V_0$ at zero temperature as a function of the ratio $eV_0/\hbar\Omega$ for $g = 0.9$ (top), $g = 0.5$ (bottom) for values $\Omega = v_{\rm F}\alpha$, $a = 0$, and $eV_1/\hbar v_{\rm F}\alpha = \ell$ ($\ell = 5$ dotted, $\ell = 6$ dashed, $\ell = 7$ dash-dotted lines). Currents in units of $\hbar v_{\rm F}\alpha/eR_{\rm t}$; differential conductance in units of $R_{\rm t}^{-1}$; tunneling resistance $R_{\rm t} = 2\hbar\omega_{\rm max}^2/\pi e^2\Delta^2$.

These can be understood as follows. When the strength of the interaction is not too large, the region where $\mathrm{d}I_{\rm dc}/\mathrm{d}V_0$ is much smaller than 1 is small compared with $\hbar\Omega$ thus for $eV_0 \approx \hbar\Omega$, $\mathrm{d}I_{\rm dc}/\mathrm{d}V_0 \propto (2/g-1)\,|eV_0 - \hbar\Omega|^{2/g-2}$. Then, Eq. (10) yields near $eV_0 = m\hbar\Omega$

$$\frac{\mathrm{d}I_{\rm dc}}{\mathrm{d}V} \approx 1 - J_m^2(|z|) + \mathrm{const}\cdot J_m^2(|z|) \times |eV_0 - m\hbar\Omega|^{2/g-2}\,. \quad (12)$$

For $g > 2/3$, this yields for integer m the cusp-like structures observed in Fig. 1. For $g < 2/3$, no cusps occur anymore. In addition, the current $I_{\rm dc}$ is depleted so strongly and over such a large region of the bias voltages that the regime of almost vanishing $\mathrm{d}I_{\rm dc}/\mathrm{d}V_0$ becomes larger than $\hbar\Omega$ and in general no minima near integer multiples of the frequency exist. As can be seen in the figure, the depths of the cusps depend on the driving voltage V_1 ($\propto |z|$) which can also be understood from of Eq. (12) which shows that the values of the differential conductances at the voltages $eV_0 = m\hbar\Omega$ are approximately $1 - J_m^2(|z|)$.

It is therefore instructive to look into the behavior of $|z|$ as a function of the frequency. Figure 2 shows the scaling exponent ν determined from

$$\nu = -v_{\rm F}\alpha\frac{\mathrm{d}\log|z|}{\mathrm{d}\log\Omega}. \quad (13)$$

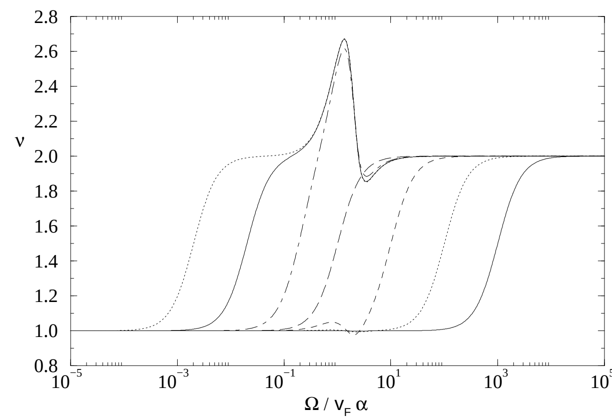

Figure 2: Scaling exponent ν of the argument $|z|$ of the Bessel functions as a function of Ω, for ranges of the driving field (curves from right to left) $\alpha a = 10^{-3}$, 10^{-2}, 10^{-1}, 1, 10, 10^2, 10^3, and $g = 0.5$.

We observe a non-universal cross-over between $|z| \propto \Omega^{-1}$, the case discussed by Tien and Gordon [1] which corresponds to a driving field of zero-range ($a \to 0$), and $|z| \propto \Omega^{-2}$ which is obtained for a homogeneous external field ($a \to \infty$) [18]. Although the behavior of z depends strongly on the parameters of the model in the cross–over regime, this does not influence qualitatively the occurrence of the cusps. Their existence depends crucially on the finite range of the interaction, and the condition $g > 2/3$. However, by varying $|z|$, the depths of the minima are changed due to the variation of $J_m^2(|z|)$.

Finally, we have demonstrated that the result which has been obtained by Tien and Gordon for tunneling of non-interacting quantum objects in 1D driven by a mono-chromatic field localized at the tunnel barrier remains valid even in the presence of interactions of arbitrary range and shape, and for an arbitrary shape of the mono-chromatic driving field. The central point is that the frequency driven current is completely given by a linear superposition of the current-voltage characteristics at integer multiples of the driving frequency, weighted by Bessel functions.

The argument of the latter contains the amplitude of the driving voltage only linearly but the dependence of the argument on the frequency and the range of the driving field is determined by its spatial shape. However, one can easily identify regions where the dependence on the frequency becomes very simple. For a driving field which is localized near the tunnel barrier, the integral in Eq. (7) can be evaluated approximately by noting that $r(x,\Omega)$ varies only slowly with x and can be taken out of the integral. Then, $|z| = eV_1/\hbar\Omega$ which corresponds to the result of Tien and Gordon [1]. In the other limit of an almost homogeneous electric field, $E_1 = V_1/a$, one needs to calculate the spatial average of $r(x,\Omega)$ [26]. This gives $\sigma(k = 0,\Omega)/\sigma(x = 0,\Omega) \approx \Omega^{-1}$, since $\sigma(x = 0,\Omega) \approx \mathrm{const}$. This implies $|z| \propto \Omega^{-2}$. Such a frequency dependence has been discussed earlier for non-interacting particles [18]. Here, we see that it is valid under quite general assumptions also for interacting particles. A possible method to detect this behavior experimentally is to investigate the real part of the first harmonic of the current through the tunnel contact and to determine the *current responsivity* which is given by the ratio of the expansions of I_{dc} and the first harmonic to second and first order in $|z|$, respectively [2].

Given the above result for the driven dc-current, the general behavior of the differential conductance as a function of $eV_0/\hbar\Omega$ can be straightforwardly obtained. Of special interest is the occurrence of cusps at $eV_0/\hbar\Omega = m$ (m integer) which appear to be quite stable against changes in the model parameters. A similar result has been discussed earlier [33], but for a small potential barrier between fractional quantum Hall edge states which implies zero-range interaction. In the case discussed here, the finite range of the interaction is crucial for obtaining the cusps, due to the absence of a linear contribution towards the current for small voltage which is characteristic of tunneling in 1D dominated by interaction. The cusps could be used to frequency-lock the dc part of the driving voltage.

To summarize, we have shown how the electron correlation and the spatial distribution of a driving field determine the anomalous scaling of the photo–induced current and the mode locking patterned structure of the nonlinear differential conductance.

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