Periodic LQG Wind Turbine Control with Adaptive Load Reduction

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Abstract: A periodic linear quadratic Gaussian (LQG) control law augmented with a reference point adaption to enable adequate rotor speed tracking and sufficient load reductions for a wind turbine is presented. The solution of the periodic LQG control problem is based on solving two periodic Ricatti differential equations in continuous time with a multiple shooting integration technique. For this, the available gridded linear time-variant description of the turbine is converted to a harmonic representation using harmonic Fourier approximation. While the periodic LQG controller provides rotor speed tracking and effective damping of the aeroelastic blade modes, the reference point adaption explicitly reduces the loads resulting from the periodic operation of the turbine rotor at the rotor rotational frequency. The performance of the proposed control system is compared against a baseline controller in realistic wind scenarios using a high fidelity nonlinear simulator. The results show a significant damage equivalent load reduction while maintaining adequate rotor speed tracking.

Keywords: Periodic control, model-based control design, adaptive control, wind turbine control.

1. INTRODUCTION

The global interest in an environmentally sustainable future and hence cleaner energy has been steadily growing over the past decade. Wind energy plays a prominent role in this context. The demand for larger wind turbines is already apparent and expected to continue, as researched by Orell et al. (2022). While upscaling brings benefits such as increased efficiency, cost-effectiveness and overall higher energy production, it also results in rising structural stresses. The increased rotor size makes the blades more susceptible to turbulence and self-induced loads caused by interactions between rotor and tower structure.

The implementation of advanced control techniques can help reduce those structural loads. State-of-the-art utility-scale wind turbines include a number of controllable components such as torque varying generators, yaw adjusting nacelles and pitch controlled blades. Individual pitch control (IPC) was introduced for wind turbines by Donham and Heimbold (1979) and has since been established as a common method to dampen out-of-plane oscillations of the blades. For instance, IPC with proportional-integral control significantly reduced the blade loads during a field study by Bossanyi et al. (2013) on two and three bladed turbines. In a similar field study by Ossmann et al. (2021) on a utility-scale wind turbine, robust control based IPC achieved an even greater load reduction. This study highlighted the significant potential of advanced control schemes, especially multi-variable ones, for IPC. Still, most modern, model-based control laws are based on a linear time-invariant (LTI) description of the wind turbine. This is enabled by the so-called multi-blade coordinates (MBC) as described in Bir (2010). However, wind turbine dynamics are inherently linear time-periodic (LTP) due to the rotor rotation. The MBC transformation and the subsequent averaging over all transformed models to create a single LTI system does not accurately capture the periodic behavior. In other words, important information on the dynamics may be lost during the transformation. While linear model-based controllers using MBC transformation commonly provide adequate performance, this may become more difficult in the future for larger turbines. One way to improve controller performances for such turbines is using the periodic system for controller design. In studies by Jakobsen et al. (2013) and Camino and Santos (2019), a significant reduction in vibrations of four-bladed rotor systems has already been achieved using periodic linear quadratic regulator (LQR) and linear quadratic Gaussian (LQG) controllers compared to time-invariant control schemes.

The contribution of this paper is an periodic LQG controller augmented with a reference point adaption (RPA) algorithm, developed for a utility-scale wind turbine. To determine the optimal state feedback and observer gains for the periodic LQG controller, two periodic Ricatti differential equations (RDEs) are solved in continuous time via a multiple shooting integration technique. For this, a gridded linear time-periodic description of the wind turbine is derived using the *Fatigue, Aerodynamics, Structure and Turbulence* (FAST) software. It is then converted to a harmonic representation using a harmonic Fourier approximation. In order to convert the gridded model to a harmonic representation and the periodic RDEs, tools provided by Varga (2013) are employed. The resulting periodic LQG controller enables effective rotor speed tracking and damping of the aeroelastic modes. Due to the periodic nature of the operating condition of the turbine rotor, it is however unable to fully alleviate the blade loads associated with this condition. A new reference point is provided adaptively during operation by a modified version of the pseudo-gradient algorithm for a load-driven cost function, as proposed by Bodson (2005). It is based on the least-mean-square (LMS) method known from adaptive control and signal processing theory described in the work of Elliott (2001).

The proposed control design strategy is applied to a utility scale 2.5 MW Clipper wind turbine. The operation of the wind turbine is categorized based on its rotor speed in standstill (region 1), variable speed (region 2) and constant speed (region 3). Since blade loads for this turbine are most severe in region 3, the controller in this paper is tested for this area. Verification is performed with the nonlinear simulation environment FAST using realistic, turbulent wind inputs. Subsequently, damage equivalent loads are derived, enabling verification of the load reduction capabilities. The obtained results are compared to results of a baseline controlled system, featuring proportional-integral collective blade-pitch control together with protection functions in region 3.

2. PERIODIC LQG CONTROL

Consider a nonlinear time-periodic system defined as

$$\begin{aligned} \dot{x}(t) &= f(t, x(t), u(t)) \\ y(t) &= g(t, x(t), u(t)), \end{aligned} \tag{1}$$

with the state vector $x(t) \in \mathbb{R}^n$, the input vector $u(t) \in \mathbb{R}^p$ and the output vector $y(t) \in \mathbb{R}^q$. The functions f and g are assumed to be differentiable and with time-period T, i.e.,

$$\begin{aligned} f(t + kT, x, u) &= f(t, x, u) \\ g(t + kT, x, u) &= f(t, x, u) \end{aligned} \tag{2}$$

for all integer $k > 0$. The system (1) can be linearized via Jacobian based linearization about a T-periodic reference trajectory to obtain a linear time-periodic system

$$\begin{aligned} \dot{x}_\delta(t) &= A(t)x_\delta(t) + B(t)u_\delta(t) \\ y_\delta(t) &= C(t)x_\delta(t) + D(t)u_\delta(t). \end{aligned} \tag{3}$$

The real matrices $A(t) \in \mathbb{R}^{n\times n}$, $B(t) \in \mathbb{R}^{n\times p}$, $C(t) \in \mathbb{R}^{q\times n}$ and $D(t) \in \mathbb{R}^{q\times p}$ are the dynamic, input, output and feedthrough matrices, respectively. The signals x_δ, u_δ, and y_δ denote deviations from the reference trajectory. This is an important distinction for later, as the controller for the linearized system only knows about these deviations. Hence, in the present work, a reference point adaption will be added to deal with the periodic reference trajectory while the linear controller covers the deviations.

The optimal control problem of an LQG controller applied to the LTP system in (3) is to determine a time-varying gain $F(t)$ for the full-state feedback law noted as

$$u_\delta(t) = -F(t)\hat{x}_\delta(t), \tag{4}$$

where $\hat{x}_\delta(t)$ is the estimation provided by an observer. The separation theorem described in Willems and Mitter (1971) is applicable and allows for the independent design of a linear-quadratic regulator as the control problem and a linear-quadratic estimator as the filter problem. The corresponding control and filter Ricatti differential equations are described in Lemma 1 and Lemma 2, respectively. A multiple shooting integration technique described in Varga (2013) is used for the integration of the periodic RDEs.

Lemma 1. Camino and Santos (2019). *Assume the matrices $A(t)$, $B(t)$, $Q(t) = Q^T(t) \geq 0$ and $R(t) = R^T(t) > 0$ are bounded continuous functions of time. Furthermore, assume the pair $(A(t), B(t))$ is completely controllable for every time t. Then, the optimal gain $F(t)$ for the feedback law in (4) is given by*

$$F(t) = R^{-1}(t)B^T(t)X(t), \tag{5}$$

where the positive semidefinite matrix $X(t) = X^T(t)$ satisfies the Ricatti differential equation

$$\begin{aligned} \dot{X}(t) + A^T(t)X(t) + X(t)A(t) + Q(t) \\ - X(t)B(t)R^{-1}(t)B^T(t)X(t) = 0. \end{aligned} \tag{6}$$

Moreover, the gain F(t) guarantees asymptotic stability of the closed-loop system.

The matrices $Q(t)$ and $R(t)$ are weightings for the system states and control inputs, respectively. They can be used during the design process to tune the periodic LQG controller for its opposing objectives state regulation and control usage, as described in Kalbat (2013). The periodic LQG controller is optimal in the sense that it minimizes the quadratic performance index

$$J_{\mathrm{LQG}} = \lim_{T\to\infty} \mathbb{E}\left\{\frac{1}{T}\int_0^T \left[x_Q(t) + u_R(t)\right] \mathrm{d}t\right\}, \tag{7}$$

with $x_Q(t) = x_\delta^T(t)Q(t)x_\delta(t)$ and $u_R(t) = u_\delta^T(t)R(t)u_\delta(t)$.

In the following Lemma the observer is defined. As described in Skogestad and Postlethwaite (2005) the process and measurment noise is taken into account via the weighting matrices $W(t) = W^T(t) \geq 0$ and $V(t) = V^T(t) > 0$, respectively.

Lemma 2. Camino and Santos (2019). *The optimal Kalman-Bucy filter gain $L(t)$ that minimizes $\mathbb{E}\left\{e^T(t)\, e(t)\right\}$, with $e(t) = x_\delta(t) - \hat{x}_\delta(t)$ the estimation error, is given by*

$$L(t) = Y(t)C^T(t)V^{-1}(t), \tag{8}$$

in which the estimation error covariance matrix $Y(t) = Y^T(t) > 0$ is the solution of the Riccati differential equation

$$\begin{aligned} -\dot{Y}(t) + A(t)Y(t) + Y(t)A^T(t) + W(t) \\ - Y(t)C^T(t)V^{-1}(t)C(t)Y(t) = 0. \end{aligned} \tag{9}$$

The state-space representation G_{obs} of the observer is then given by

$$\begin{aligned} \dot{\hat{x}}_\delta(t) &= [A(t) - L(t)C(t)]\hat{x}_\delta(t) \\ &\quad + \begin{bmatrix} B(t) - L(t)D(t) & L(t) \end{bmatrix} \begin{bmatrix} u_\delta(t) \\ y_\delta(t) \end{bmatrix}. \end{aligned} \tag{10}$$

3. WIND TURBINE CONTROL SYSTEM DESIGN

The utility-scale three-bladed Clipper Liberty 2.5 MW wind turbine is investigated within operating region 3, i.e., constant generator and rotor speed. The rotational frequency of the turbine is indicated by $\omega_{1\mathrm{P}}$. For region 3 the expected value is 15.5 rpm or 0.258 Hz. A linearized model of the wind turbine is used for the design of the periodic LQG controller. An adaptive scheme is added later on to deal with the periodic reference condition. They provide bending moment damping to reduce blade loads with the former also being designed to enable generator speed tracking. A nonlinear wind turbine model with both controllers implemented is used to evaluate their performance.

3.1 Wind turbine model

A high-fidelity nonlinear framework of the Clipper wind turbine is available with the *Fatique, Aerodynamics, Structures and Turbulence* (FAST) simulation environment documented in Jonkman and Buhl (2005). The modes considered in the nonlinear simulation include first and second flapwise-blade bending, edgewise-blade bending, the first bending of the tower in fore-aft and side-to-side direction, as well as generator speed and drive-train rotational flexibility. The model allows individual input for each blade-pitch angle and generator torque. The corresponding actuators are described as linear first-order systems. The model includes a state-of-the-art baseline controller that generates collective blade-pitch and torque commands based on the current rotor speed and provides baseline performance data.

Algorithms are provided within the FAST software to linearize the nonlinear wind turbine model around the rotary trajectory. A gridded time-varying description of the system dynamics at 20 m/s wind speed over one period is obtained, i.e., the periodic wind turbine dynamics are represented by a set of LTI systems. For the periodic control approach in this work, sixty gridded LTI systems of reduced-order are generated. They are of 7$^{\text{th}}$ order and include the generator speed ω and the first flapwise bending mode for each blade as states. The states are selected to enable a control design to meet the objectives of reference tracking and out-of-plane bending moment damping. The system's input is the individual blade-pitch command $\phi = [\phi_1, \phi_2, \phi_3]^T$ for each of the three blades. Its output is the three root out-of-plane bending moments $M = [M_1, M_2, M_3]^T$ at each blade and the generator speed ω.

3.2 Control Architecture

The proposed control structure is a periodic LQG and an RPA scheme working in parallel. The LQG tracks the generator speed to ensure the wind turbine's power production. It simultaneously dampens the aeroelastic modes for each blade. The RPA provides a load minimal operating condition for the blade bending moments by creating a periodic input signal with the rotational frequency $\omega_{1\text{P}}$ during constant wind conditions. Both controllers commands are superimposed such that

$$\phi(t) = \phi_{\text{LQG}}(t) + \phi_{\text{RPA}}(t). \tag{11}$$

The time-varying controller design is mapped to the rotor angle $\Psi(t) \in [0, 2\pi]$ for computational implementation. In Fig. 1 the complete control architecture is depicted. The periodic LQG part consists of the state feedback gain F, the observer G_{obs} and an integral extension for steady state tracking of the rotor speed. The RPA contains an estimation of the constant portion of the bending moments and a least-mean-square optimization to minimize the varying part of the bending moments at the rotational frequency.

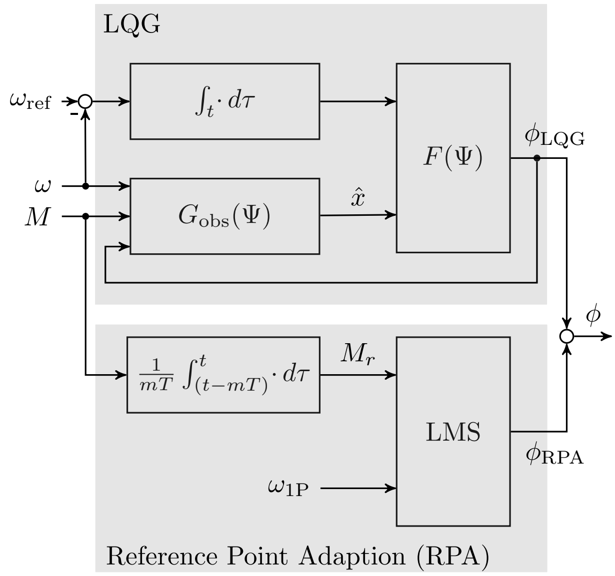

Fig. 1. Control architecture with periodic LQG and RPA in parallel

3.3 Reference Point Adaption

The reference trajectory for the out-of-plane bending moments in the nonlinear simulation includes a periodic component for constant wind. It is the result of the aerodynamic interaction between the rotor blades and the tower structure and occurs with the fundamental frequency of the system $\omega_{1\text{P}}$. Note that for improved readability this section provides a blade-wise derivation of the RPA with the index $b = \{1, 2, 3\}$ indicating the respective blade. The scheme is then applied in an identical manner for each one.

It is assumed herein that the out-of-plane bending moments can be directly measured as described in Ossmann and Theis (2017). The moments can be separated into two components and thus be described by

$$M_b(t) = M_{b,0}(t) + M_{b,r}(t), \tag{12}$$

with a time-varying mean value $M_{b,0}(t)$ and an oscillation $M_{b,r}(t)$ around the mean value. The latter is caused by self-induced stimulation through aerodynamic interaction, while the former is a result of the thrust created by the blade and shall be unaffected by the RPA. In order to determine the oscillating component of the bending moment, $M_{b,0}(t)$ is first estimated by a windowed integrator scheme described by

$$M_{b,r}(t) \approx M_b(t) - \frac{1}{mT}\int_{t-mT}^{t} M_b(\tau)\mathrm{d}\tau, \tag{13}$$

with the integer $m > 0$ defining the integration interval as a multiple of the fundamental period T of the system. Larger values of m improve performance for short wind gusts but decrease accuracy for turbulent conditions. For the proposed controller $m = 2$.

The objective of the adaptive scheme is to produce an input signal for each blade to achieve a constant bending moment, i.e., $M_{b,r}(t) \to 0$. An adaptive approach has been selected to allow for an online adjustment for different operating conditions as are experienced during turbulent wind scenarios. The adapted signal is sinusoidal and shares with the wind turbine its fundamental frequency $\omega_{1\text{P}}$ but can be adjusted in terms of amplitude and phase offset. It is defined as

$$\phi_{b,\text{RPA}}(t) = \theta_{b,c}(t)\cos(\omega_{1\text{P}}t) - \theta_{b,s}(t)\sin(\omega_{1\text{P}}t), \tag{14}$$

with the adaptive parameters $\theta_{b,c}$ and $\theta_{b,s}$. It can be shown using the angle sum theorem that this structure enables the signal to represent any arbitrary sinusoidal signal with the frequency $\omega_{1\text{P}}$. Based on the LMS method described in the work of Elliott (2001) the update law for the adaptive parameters is defined to minimize the cost function $M_{b,r}^2(t)$ by the ordinary differential equation

$$\begin{bmatrix} \dot{\theta}_{b,c}(t) \\ \dot{\theta}_{b,s}(t) \end{bmatrix} = g\, M_{b,r}(t) \begin{bmatrix} \cos(\omega_{1\text{P}}t) \\ -\sin(\omega_{1\text{P}}t) \end{bmatrix}, \tag{15}$$

with the arbitrary adaptive parameter g to manipulate the convergence speed. Stability of the adaptation is guaranteed for positive but small enough values for g according to Bodson (2005). For the proposed controller $g = 10^{-4}$ for all three blades. The initial values for RPA are set to

$$\begin{aligned} \theta_{b,c}(0) &= \theta_{b,s}(0) = 0 \\ \dot{\theta}_{b,c}(0) &= \dot{\theta}_{b,s}(0) = 0. \end{aligned} \tag{16}$$

3.4 Periodic LQG Control Design

A periodic LQG controller is designed using the gridded linearized models described in Sec. 3.1. The reference value for generator speed tracking is defined as ω_{ref}. The steady state reference tracking performance is improved by extending the controller with an additional integrator state. The periodic filter and control RDEs in equations (6) and (9) are solved in continuous time by creating harmonic representations of the gridded system matrices with a harmonic Fourier approximation. Solving the RDEs requires for the weighting matrices $R(t)$, $Q(t)$, $W(t)$ and $V(t)$ to be defined.

The input weights are identical for each blade-pitch due to the rotationally symmetric design of the wind turbine. Hence, the input weighting matrix is defined as $R(t) = I$. The matrix Q weights the states in (6), including the integrator state, and is given by

$$Q(t) = \begin{bmatrix} & & 0 \\ q_m\tilde{Q}(t) & & \vdots \\ & & 0 \\ 0 & \cdots\ 0 & q_\omega \end{bmatrix}, \tag{17}$$

with the time-invariant tuning parameters q_m and q_ω. The parameters influence state regulation and control usage, which corresponds to out-of-plane bending moment damping and reference tracking, respectively. There is a direct trade off between both objectives depending on the ratio of these tuning parameters. For the implemented controller the values $q_m = 3 \times 10^3$ and $q_\omega = 2$ are selected. The quadratic matrix $\tilde{Q}(t) \in \mathbb{R}^{n\times n}$ weights the importance of the plant's states. It is calculated via $C(t)^T C(t)$ and normalized using its largest-element magnitude.

For the solution of (9) the weighting matrix V accounts for the measurement noise. It is a diagonal matrix with the variance of the bending moment for each blade and the rotor speed as entries. Their standard deviation is assumed as 10 and 0.1, respectively. The matrix W accounts for the realization error and is defined as

$$W(t) = B(t)\tilde{W}B(t)^T, \tag{18}$$

where $\tilde{W} \in \mathbb{R}^{p\times p}$ is a diagonal matrix of the inputs' variance. It is time-invariant as they do not changed over the system's period. For the implemented controller the blades' pitch standard deviation is set to 0.1 degrees.

4. NONLINEAR SIMULATION RESULTS

The developed control system is verified using a nonlinear simulation model of the Clipper turbine, which is implemented in the software environment FAST. A state-of-the-art collective pitch baseline controller including protection functions is used for comparison.

In the first test scenario a simple 5 s wind gust of 3 m/s about the reference hub height wind of 20 m/s, shown in the first diagram of Fig. 2, is tested. The second diagram shows the rotor speed tracking performance. Overall, the periodic LQG controlled systems show better tracking performance than the baseline controlled system, i.e., the deviation from the targeted rotation speed of 15.5 rpm is about 15% smaller than for the baseline controller, while ensuring a faster settling time. Note that the extension of the system with the RPA algorithm has only minor impact on the rotor speed tracking. In the third subplot of Fig. 2 the first blade's out-of-plane bending moment computed at the blade root is depicted. Due to the improved damping of the flapwise bending modes, provided by the periodic

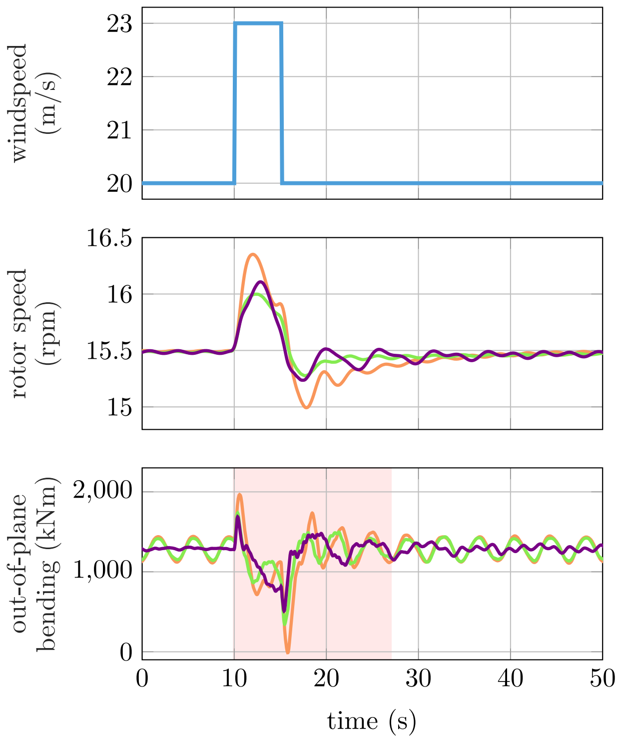

Fig. 2. Rotor speed and first blade out-of-plane bending for the baseline (——), periodic LQG (——), and periodic LQG with RPA (——) controlled system to a simulated wind gust (——).

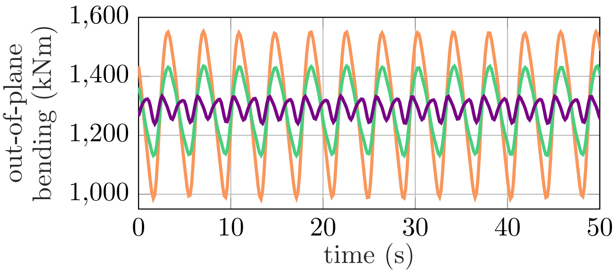

Fig. 3. First blade's out-of-plane bending moment for the baseline (——), the periodic LQG (——) and periodic LQG with RPA (——) controlled systems during a vertical wind shear.

LQG controller, the transient behavior, i.e., the maximum deviations from the trim point, is improved compared to the baseline controller. Additionally, the RPA algorithm explicitly reduces the loads resulting from the periodic operation of the turbine rotor at the rotor rotational frequency (1P). With this extension, loads that arise at the 1P frequency from, e.g., vertical or horizontal wind shear or the tower shadow, can be reduced more efficiently. This is confirmed by the simulation results in the time-domain: Fig. 3 shows the out-of-plane bending moment of the first blade for a vertical windshear with a deviation of $\pm 1.4\,\mathrm{m/s}$ about the trim value of $20\,\mathrm{m/s}$ over the rotor span. The amplitudes of the moment is clearly reduced by the RPA (——) compared to the periodic LQG-only setup (——) and especially compared to the baseline controller (——).

For a more realistic verification, a turbulent wind profile of *class A*, as specified by *International Electrotechnical Commission*, with a mean hub height wind speed of $20\,\mathrm{m/s}$ is simulated. The wind field is generated using the TurbSim software documented in Jonkman (2009). The Kaiman model is selected as the turbulence spectral model. Using this turbulent wind scenario enables an evaluation of the load mitigation via the frequency dependent power spectral density (PSD) of the blade bending moments at each blade's root. The PSDs of the out-of-plane bending moments on all three blades are depicted in Fig. 4. The analysis of the baseline controlled system (——) shows the expected dominant peak around the 1P frequency of $0.258\,\mathrm{Hz}$, which is marked with a dotted line in the diagrams. While the periodic LQG controller is capable of reducing the load magnitudes over a broad frequency band (——) compared to the baseline controller, it still shows a dominant peak at the 1P frequency. Extending the periodic LQG controller with the RPA algorithm however, leads to an additional reduction of those 1P loads (——).

To quantify the improvements of the periodic controller, the gathered data from the simulations with turbulent wind conditions is used to calculate damage equivalent loads (DELs) for all three blades with the software tool MCrunch, documented in Buhl (2008). The DELs represent a measure of equivalent fatigue damage caused by the encountered load cycles, taking the material properties into account as described by Bossanyi (2003). An S-N slope of 10, representative of typical composite materials is used herein to determine the DELs for each blade. The periodic LQG controller already enables a load reduction of at least 33 % on the three blades. The periodic LQG

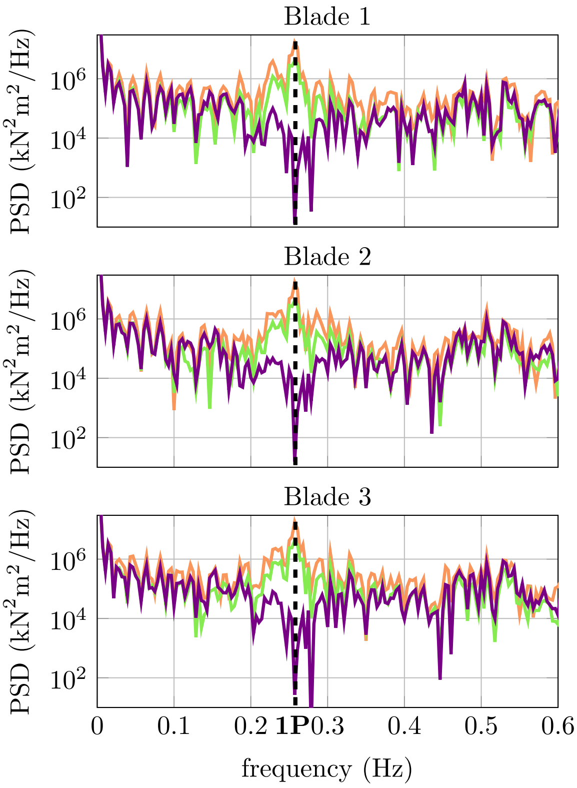

Fig. 4. PSDs of the blade bending moments for the baseline (——), the periodic LQG (——) and LQG with RPA (——) controlled systems.

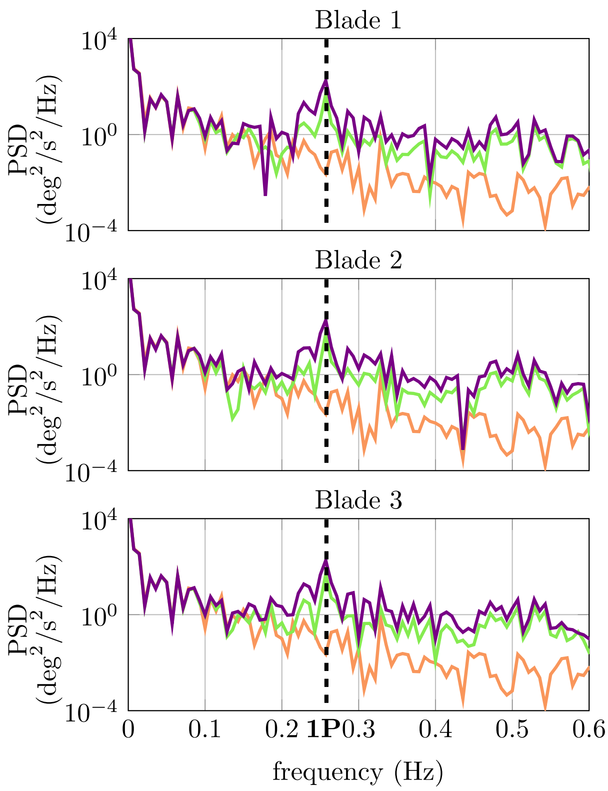

Fig. 5. PSDs of the blade pitch movement for the baseline (——), the periodic LQG (——) and periodic LQG with RPA (——) controlled systems.

controller augmented with the RPA algorithm reduces the blade loads by at least 4 % more, i.e., leading to a total load reduction of at least 37 % on the blades.These results confirm that the improved damping of the blade bending modes by the periodic LQG controller already leads to a significant fatigue reduction on the blades, without explicitly targeting the 1P loads on the blades, resulting from the turbine's periodic operation. If necessary, these 1P loads, which to a certain extend also contribute to the blade fatigue, can be further reduced by introducing the RPA algorithm.

The PSDs of the blade pitch motion during turbulent wind, depicted in Fig. 5, demonstrate how the actuation level for the periodic LQG controlled systems is increased at frequencies above 0.1 Hz compared to the baseline controlled systems. The increase in actuation is attributed to the control commands that effectively reduce loads at these frequencies. Especially around the 1P frequency the periodic LQG controller augmented with RPA shows an increased controller command activity compared to the periodic LQG-only scenario.

Finally, comparing the results presented in this paper with earlier work on active load reduction for the Clipper turbine based on the multi-blade coordinate (MBC) transformation approach Ossmann et al. (2017, 2016) reveals a significant increase in load reduction capabilities when using the periodic system directly instead of the MBC transformed linear-time invariant approximation. This finally confirms that the usage of periodic control design model as well as design methods can facilitate the improved load reduction of future wind turbines by exploiting the periodicity of the underlying dynamics.

5. CONCLUSION

A periodic LQG control design augmented with a reference point adaption algorithm has been presented. The control approach has been applied to a three-bladed IPC wind turbine model to provide reference rotor speed tracking together with a reduction of the oscillating out-of-plane bending moments. Results from nonlinear simulations confirm that the control system achieves better rotor speed tracking than a baseline controller while additionally enabling blade fatigue reductions. The gathered results build confidence that the periodic, multivariable design method will allow the design of adequate controllers even when the wind turbine control problems become more challenging, e.g., due to the increasing turbine sizes and the increasing number of control inputs.

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