

# Linear Parameter Varying Controller Design For Satellite Attitude Control<sup>\*</sup>

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**Abstract:** This paper presents a systematic linear parameter varying (LPV) control approach for the 3-axis attitude control of an Earth-observation satellite in a sun-synchronous orbit. The dynamics of the satellite depend on the orientation of the solar array, which completes a full rotation every orbit, thus it is used as a scheduling parameter in the design. The satellite has two additional flexible appendages; these are 2 scatterometers. The control objective is to precisely track a given reference attitude using reaction wheels, while rejecting external torque disturbances and sensor noise. The design follows a mixed-sensitivity approach, applying a recently introduced weighting scheme. It allows traceable and effective controller tuning by using a low number of physically interpretable weights. The controller is synthesised by solving the induced  $L_2$ -norm of the closed-loop interconnection of the controller and weighted plant. Scheduling with the solar array orientation leads to an LPV notching behaviour in the controller that effectively mitigates the effects of the array's most prominent flexible modes. This behaviour enables increased performance, when compared to a linear time invariant controller, while maintaining robustness. The pointing performance of the synthesised controller over the complete satellite lifecycle is verified using the European Space Agency's standards for spacecraft attitude control.

*Keywords:* Linear parameter-varying systems, robust control, aerospace, disturbance rejection, tracking, high accuracy pointing

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## 1. INTRODUCTION

Observation satellites must fulfil stringent pointing requirements in order to produce high quality image data. For this purpose, they are equipped with attitude control systems. These control systems must operate accurately under varying system dynamics as well as complex environmental disturbances. To achieve optimal performance at all times, the controller must be able to adapt to changes in the spacecraft dynamics. Some of these changes result from measurable parameter variations, which can then be explicitly considered in the design and used for controller scheduling during operations. Moreover, the resulting controller must be robust towards uncertainty in the system dynamics over the satellite's life cycle. Given that satellite pointing performance requirements are frequency-based,  $H_\infty$  control is the logical framework to directly incorporate the requirements into the design. A linear parameter varying (LPV) framework that uses insights from classical  $H_\infty$  then provides an efficient approach to address both robustness and varying system dynamics effectively. It is able to provide an optimal robust controller scheduled with the measurable parameters in a single synthesis. Additionally, design and analysis of LPV controllers can be conducted using readily available tools such as the LPVTools toolbox in MATLAB (Hjartarson et al., 2015).

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Fig. 1. MetOp-SG Satellite (ESA, 2013).

In recent years, there is growing interest in modelling dynamical systems with respect to variable parameters, as in Kassarian et al. (2022). Within the aerospace industry, there are tools available for the modelling of uncertain flexible space structures (Alazard and Sanfedino, 2020) and methods for controlling them (Angeletti et al., 2022). There are also techniques for obtaining linear models of spacecraft that are parameterised with the motion of flexible appendages, as in Guy et al. (2014). By extension, the present paper details an LPV approach for the reaction wheel based attitude control of an industry-standard Earth-observation satellite that is parameterised with the motion of a flexible solar array.

The given satellite has a flexible solar array that rotates  $360^\circ$ , with a fixed angular rate, every orbit. It also has two additional flexible appendages and uncertain mass and dimensional properties. Eumetsat's MetOp and MetOp-SG satellites are typical examples of such spacecraft, pictured in Fig. 1. The biggest driver in the proposed design is the rotating solar array, which changes the dynamics of the spacecraft throughout the mission. Because of this, the LPV design is scheduled with the motion of the array in a grid-based approach, which is particularly suited to the proposed problem due to the slow nature of the parameter variation and resulting dynamical changes.

It is also important to consider that, in an industrial setting, the design process must be well established, versatile for different use-cases and intuitive for the design engineer to follow. To address these matters, the control design applies a recently proposed weighting scheme (Theis et al., 2020) to the grid-based LPV controller synthesis. It facilitates highly traceable and particularly easy tuning as the system requirements and constraints directly translate into the weights. The scheme's versatility has so far been demonstrated on launchers (Biertümpfel et al., 2022), flexible aircraft (Theis et al., 2020) and auto-landing problems (Theis et al., 2018). Applying this mixed-sensitivity weighting scheme using the grid-based LPV method is motivated by a combination of optimal robust control for flexible systems and traditional aircraft gain-scheduling control design. It has the advantages of  $H_\infty$  controller design with the additional benefit that the synthesised controller is automatically scheduled.

## 2. LINEAR PARAMETER VARYING SYSTEMS

LPV systems are a class of systems whose state space matrices depend continuously on a time-varying parameter vector  $\rho : \mathbb{R} \rightarrow \mathcal{P}$ , where  $\mathcal{P} \in \mathbb{R}^{n_\rho}$  is a compact subset chosen based on physical considerations. In addition, the parameter rates of variation  $\dot{\rho}$  are assumed to lie within a hyper-rectangle  $\dot{\mathcal{P}}$  defined by  $\dot{\mathcal{P}} = \{\dot{\rho}(t) \in \mathbb{R}^{n_\rho} \mid |\rho_i(t)| \leq \nu_i, i = 1, \dots, n_\rho\}$ . Hence, the set of all admissible trajectories is  $\mathcal{T} = \{\rho : \mathbb{R} \rightarrow \mathcal{P} \mid \rho \in \mathcal{C}^1, \rho(t) \in \mathcal{P} \text{ and } \dot{\rho}(t) \in \dot{\mathcal{P}} \forall t \geq 0\}$ .

The state space matrices of an LPV system are continuous functions of the parameter vector, i.e.,  $A : \mathcal{P} \rightarrow \mathbb{R}^{n_x \times n_x}$ ,  $B : \mathcal{P} \rightarrow \mathbb{R}^{n_x \times n_u}$ ,  $C : \mathcal{P} \rightarrow \mathbb{R}^{n_y \times n_x}$ , and  $D : \mathcal{P} \rightarrow \mathbb{R}^{n_y \times n_u}$ . An  $n_x^{\text{th}}$ -order LPV system  $G_\rho$  is defined by

$$\begin{bmatrix} \dot{x}(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} A(\rho(t)) & B(\rho(t)) \\ C(\rho(t)) & D(\rho(t)) \end{bmatrix} \begin{bmatrix} x(t) \\ u(t) \end{bmatrix}, \quad (1)$$

where  $x(t) \in \mathbb{R}^{n_x}$  is the state,  $u(t) \in \mathbb{R}^{n_u}$  the input, and  $y(t) \in \mathbb{R}^{n_y}$  the output. The explicit dependence on  $t$  is occasionally omitted to shorten the notation.

### 2.1 Induced $L_2$ -norm controller synthesis

The performance of an LPV system can be specified in terms of its induced  $L_2$ -norm

$$\|G_\rho\| = \sup_{u \in \mathbb{L}_2 \setminus \{0\}, \rho \in \mathcal{T}, x(0)=0} \frac{\|y\|_2}{\|u\|_2}. \quad (2)$$

A generalization of the Bounded Real Lemma (Wu et al., 1996) provides a sufficient condition to upper bound  $\|G_\rho\|$ . The next theorem states this condition.

*Theorem 1.* (Wu et al., 1996):  $G_\rho$  is exponentially stable and  $\|G_\rho\| \leq \gamma$  if there exists a continuously differentiable symmetric matrix function  $P : \mathcal{P} \rightarrow \mathbb{R}^{n_x \times n_x}$  such that  $P(p) \geq 0$  and

$$\begin{bmatrix} PA + A^T P + \partial P & PB \\ B^T P & -I \end{bmatrix} + \frac{1}{\gamma^2} \begin{bmatrix} C^T \\ D^T \end{bmatrix} [C \ D] \leq 0 \quad (3)$$

hold for all  $p \in \mathcal{P}$  and  $q \in \dot{\mathcal{P}}$ , where  $\partial P$  is defined as  $\partial P(p, q) = \sum_{i=1}^{n_\rho} \frac{\partial P}{\partial \rho_i}(p) q_i$ . In (3), the dependence of the matrices on  $p$  and  $q$  has been omitted to shorten the notation.

This theorem forms the basis for the induced  $L_2$ -norm controller synthesis in Wu et al. (1996). In short, consider an open loop LPV system  $G_\rho$  with inputs  $[w^T, u^T]^T$  and outputs  $[z^T, y^T]^T$ . The objective is to synthesize a controller  $K_\rho$ :

$$\begin{bmatrix} \dot{x}_K \\ u \end{bmatrix} = \begin{bmatrix} A_K(\rho) & B_K(\rho) \\ C_K(\rho) & D_K(\rho) \end{bmatrix} \begin{bmatrix} x_K \\ y \end{bmatrix} \quad (4)$$

such that the induced  $L_2$ -gain of the closed loop interconnection of  $G_\rho$  and  $K_\rho$ , denoted by the lower fractional transformation  $F_l(G_\rho, K_\rho)$ , is minimized

$$\min_{K_\rho} \|F_l(G_\rho, K_\rho)\| \quad (5)$$

This optimization problem can be solved via parametrized LMI conditions, see Wu et al. (1996) for details. It should be noted that the synthesis problem involves an infinite collection of LMI constraints parametrized by  $(p, q) \in \mathcal{P} \times \dot{\mathcal{P}}$ . A remedy to this issue, which works in many practical examples, is to approximate them with finite-dimensional LMIs evaluated on a grid. Tools to solve the synthesis problem are readily available, e.g., Hjartarson et al. (2015) which are used in this paper.

### 2.2 Mixed-sensitivity control architecture

Many requirements on feedback control systems can be directly specified in terms of the induced  $L_2$ -norm of weighted sensitivity functions, e.g. disturbance attenuation levels, tracking capabilities, the frequency range of control activity, and robustness. Hence, it is common practice to design induced  $L_2$ -norm optimal LPV controllers by mixed-sensitivity loopshaping, see, e.g., Zhou et al. (1996); Skogestad and Postlethwaite (2005). Defining the output sensitivity function  $S = (I + G_\rho K_\rho)^{-1}$ , the generalized closed loop of the weighted mixed-sensitivity problem shown in Fig. 2 is

$$\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} W_e V_e^{-1} & 0 \\ 0 & W_u V_u^{-1} \end{bmatrix} \begin{bmatrix} S & S G_\rho \\ K_\rho S & K_\rho S G_\rho \end{bmatrix} \begin{bmatrix} V_e & 0 \\ 0 & V_d \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}, \quad (6)$$

where  $W_e$  and  $W_u$  denote frequency dependent weights and  $V_e$ ,  $V_u$ , and  $V_d$  constant scaling factors. A high gain in  $W_e$  reduces the sensitivity function leading to better tracking and disturbance rejection capabilities. A high gain in  $W_u$  reduces the control effort. Hence,  $W_u$  can enforce controller roll-off at high frequencies, e.g. to avoid excitation of flexible modes in a system. The static weights are used as the main tuning knobs. Good initial values are obtained based on the maximum expected disturbances ( $V_d$ ) and the maximum allowable errors ( $V_e$ ) and inputs ( $V_u$ ). Theis et al. (2020) present a comprehensive treatment of this parameterisation.

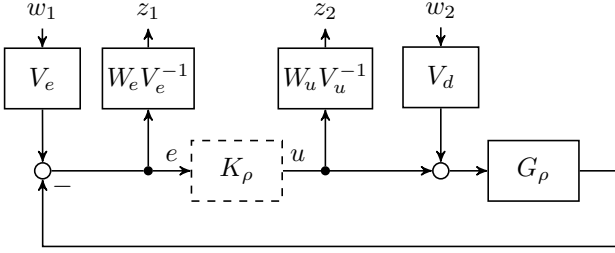


Fig. 2. Weighted four-block mixed-sensitivity problem.

### 3. SATELLITE ATTITUDE CONTROL

#### 3.1 Satellite and mission

The considered Earth-observation satellite is in an almost circular (eccentricity  $\approx 0$ ), sun-synchronous orbit with an altitude of  $817\text{km}$ , an inclination of  $99^\circ$  and an orbital period of  $\approx 102$  minutes. It orbits in a “normal” mode, with the goal of maintaining its attitude aligned to a chosen reference frame while rejecting external torque disturbances using only reaction wheels. Throughout this paper, plots that detail the dynamical behaviour of the satellite are normalised across frequency, to maintain confidentiality of the model.

The satellite is modelled as a rigid central body with three flexible appendages; two scatterometers and the solar array which completes one full rotation each orbit to maintain its orientation to the sun. The central body is described by the linear Newton-Euler equations (7) with mass  $m_{CB}$  and inertia  $J_{CB}$ . Given the small angular rates of the problem, non-linearities are neglected.

$$\begin{pmatrix} \Sigma f_{\text{ext}} \\ \Sigma \tau_{\text{ext}} \end{pmatrix} = \begin{bmatrix} m_{CB} I_{3 \times 3} & 0_{3 \times 3} \\ 0_{3 \times 3} & J_{CB} \end{bmatrix} \begin{pmatrix} \ddot{r} \\ \dot{\omega} \end{pmatrix} \quad (7)$$

The sum of external forces  $f_{\text{ext}}$  and torques  $\tau_{\text{ext}}$  acting on the spacecraft result in a translational and rotational acceleration ( $\ddot{r}$  and  $\dot{\omega}$  respectively). The flexible appendages (FA) are each modelled as a hybrid-cantilever beam connected to the central body at a hinge point  $P$  (8).  $L_P$  describes the modal contributions of the flexible appendage. Each second-order mode (denoted  $i$ ) has damping  $\zeta_i$  and natural frequency  $\omega_i$ .

$$\begin{pmatrix} f^P \\ \tau^P \end{pmatrix} = \begin{bmatrix} m_{FA} I_{3 \times 3} & 0_{3 \times 3} \\ 0_{3 \times 3} & J_{FA} \end{bmatrix} \begin{pmatrix} \ddot{r}_{FA} \\ \dot{\omega}_{FA} \end{pmatrix} + L_P^T \ddot{\eta} \quad (8)$$

$$-L_P \begin{pmatrix} \ddot{r}_{FA} \\ \dot{\omega}_{FA} \end{pmatrix} = \ddot{\eta} + \text{diag}(2\zeta_i \omega_i) \dot{\eta} + \text{diag}(\omega_i^2) \eta$$

As the solar array rotates,  $\zeta_i$  and  $\omega_i$  change, as does the location of the centre of mass. Thus the overall spacecraft dynamics described by (7) and (8) change as a function of the solar array angular position  $\theta_{SA}$ . This describes the LPV nature of the plant. Fig. 3 demonstrates how the plant dynamics vary over a grid of solar array angles,  $0 \leq \theta_{SA} < 360^\circ$ .

The external torques on the spacecraft are a summation of the solar radiation pressure torque, aerodynamic torque, gravity gradient torque and magnetic field torque which are calculated using a detailed description of the space

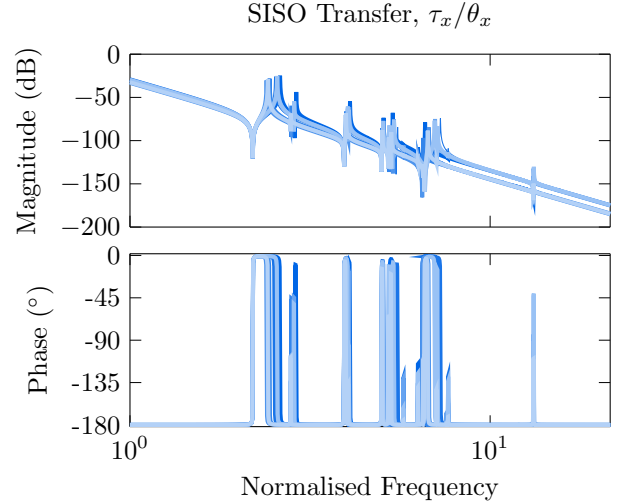


Fig. 3. Satellite dynamics as solar array rotates from minimum angular displacement (—) to maximum (—).

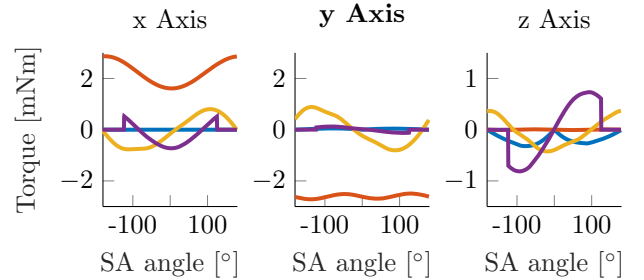


Fig. 4. Aerodynamic (—), gravitational (—), magnetic (—) and solar (—) torques experienced by the satellite during one orbit with respect to the solar array (SA) angle.

environment, for example as in Pisacane (2008). The solar radiation pressure is a summation of momentum exchange for each surface due to the photons colliding with the spacecraft. Similarly, the aerodynamic torque results from collisions with residual atmosphere particles. Both are computed using a geometrical 3-dimensional model of the spacecraft. The gravity gradient torque results from the geo-potential across the spacecraft due to the gravitational field of Earth and the magnetic torque results from the dipole moment created by the magnetic field. Each total torque experienced by the satellite in the body reference frame, as a function of the solar array angle during one orbit, is shown in Fig. 4.

The satellite navigates using star tracker fusion which introduces sensor noise described by high and low spacial frequency error (HSFE, LSFE) and temporal error (TE). The spectral errors are modelled as first-order Gauss-Markov processes and the temporal error can be considered white noise, as is common practice in industry.

#### 3.2 Control problem definition

The satellite and mission describe a multi-input multi-output (MIMO) tracking control problem. The goal is to track a 0 reference attitude described in the spacecraft

Table 1. Pointing metrics (Ott et al., 2011)

Metric	Type	Time domain
APE	absolute ( <i>abs</i> )	$\sigma_{abs}^2 = E[e(t) - \mu_{abs}]^2$
RPE	windowed variance ( <i>wv</i> )	$\sigma_{wv}^2(\Delta t) = E[\langle (e(t) - \langle e(t) \rangle_{\Delta t})^2 \rangle_{\Delta t}]$
PDE	windowed mean stability ( <i>wms</i> )	$\sigma_{wms}^2(\Delta t, \Delta t_s) = E[\langle (e(t) \rangle_{\Delta t} - \langle e(t - \Delta t_s) \rangle_{\Delta t})^2 \rangle_{\Delta t}]$

body frame,  $\theta$ , using only torque input generated by the reaction wheels,  $\tau$ , also described in the body frame. Sensor noise must be attenuated and external disturbance torques must be rejected such that the closed loop system is able to achieve the specified pointing error requirements. Three types of pointing metrics are considered, absolute performance error (APE), relative performance error (RPE) and performance drift error (PDE). For brevity, the descriptions of these metrics are summarised in Table 1. Certain requirements and limitations are also imposed on the design problem in the frequency domain to ensure sufficient robustness of the system; the gain and phase margins must exceed 6dB and 30° respectively, the peak of each flexible mode must be below -6dB and the modulus margin must be greater than 0.5.

#### 4. CONTROLLER DESIGN

Two controllers were designed, one linear time invariant (LTI) and one LPV controller, such that a reasonable comparison could be drawn. Both design procedures followed the mixed-sensitivity process described in section 2.2, while the LPV design made use of sinusoidal parameter dependent storage functions to capture the periodic nature of the plant. The plant used for design only contained the two flexible modes with the lowest frequencies, as these are most influential on the plant dynamics and the loopshaping problem. The augmented plant for the LTI controller was constructed by weighting the nominal plant ( $\theta_{SA} = 0$ ) in section 3.1, whereas the augmented plant for the LPV design encompassed a discrete grid of plants, each with a unique solar array angle ( $0 \leq \theta_{SA} < 360^\circ$ ), into one single LPV plant. This was modelled as a **pss**-object, defined in the LPVTools toolbox (Hjartarson et al., 2015). Other than the nature of the augmented plant, the design process and weighting scheme for each controller was largely the same.

All design weights are diagonal. Firstly  $W_e^{-1}$  enforces the shape of the sensitivity function. The crossover frequency was pushed as high as possible to maximise the tracking bandwidth while still ensuring the flexible modes are kept below -6dB. At high frequency the gain of  $W_e^{-1}$  is 6dB to guarantee the required modulus margin. The tracking bandwidth of the LTI controller had to be relaxed to maintain robustness to the variations in the dynamics.  $W_u$  imposes a roll-off on the controller, thus, at frequencies above the available actuator bandwidth the gain is high. The scaling weights were selected intuitively based on the physical constraints of the system.  $V_e$  is the maximum pointing error requirement and  $V_u$  the maximum torque available from the actuators.  $V_d$  was selected such that

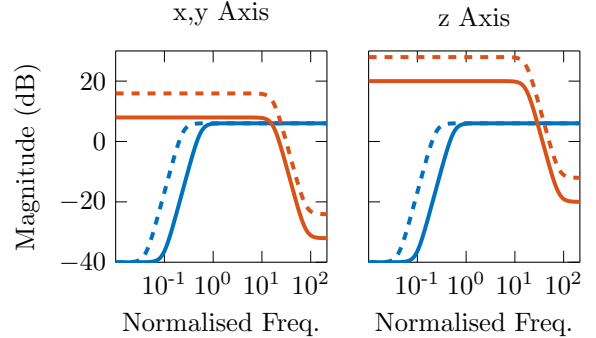


Fig. 5. Imposed requirements on  $S$  for  $K_\rho$  (—) and  $K$  (---). Imposed requirements on  $K_\rho S G_\rho$  (—) and  $K S G_\rho$  (---).

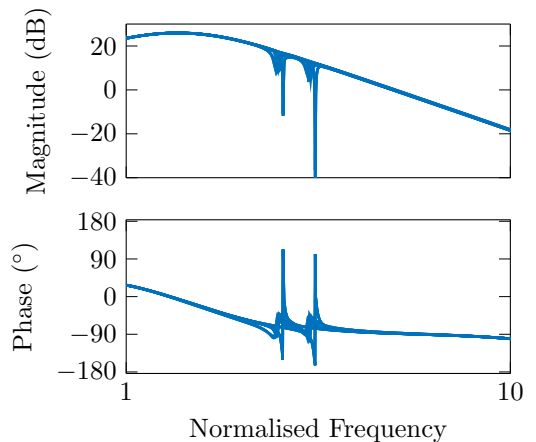


Fig. 6. LPV controller design demonstrates notching of flexible modes.

the ratio of expected disturbance to actuator effort is 20%. For the LTI design the achievable tracking bandwidth was lower in order to maintain the required flexible mode roll off for all plant uncertainties, then the ratio of  $V_u/V_d$  was increased to push the controller gain up and speed up the response, see Fig. 5.

The resulting controllers have 18 states and the LPV controller is gridded with the solar array angle at 8 points. The advantage of the LPV design is that it notches the first two flexible modes of the solar array at the corresponding solar array angle, see Fig. 6. This means the LPV controller is less sensitive to the changing frequency of the flexible modes of the solar array as it moves. Thus, while respecting design constraints, the bandwidth of the LPV controller could be pushed higher than that of the LTI controller.

#### 5. CONTROL EVALUATION

The evaluation was completed in the frequency domain using a closed-loop structure, as in Fig. 8, where  $G_\rho$  is scheduled with  $\theta_{SA}$ . To make the evaluation more representative of an industrial-scale project,  $G_\rho$  also encompasses uncertainties to cover the lifecycle of the mission, summarised in Table 2. Fig. 7 demonstrates how the dynamics of the spacecraft are affected by the uncertainties. For the frequency domain evaluation, noise and disturbances can

Table 2. Satellite parameter uncertainties.

	Parameter	Range (%)
Central body	Mass	10
	inertia	20
Fuel	Mass	23
	Inertia	scales with mass
Solar array	Mass	10
	Inertia	20
	Cantilever frequency	10
Scatterometers	Mass	10
	Inertia	20
	Cantilever frequency	10

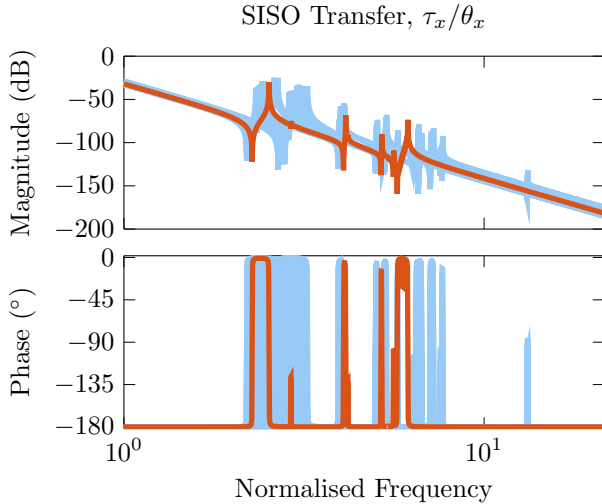


Fig. 7. Nominal plant (—) and a selection of plants over the satellite lifecycle (—).

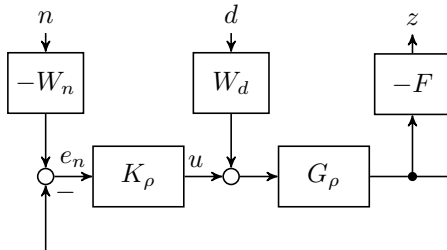


Fig. 8. Block diagram of structure used for analysis.

be approximated as white noise signals passing through filters  $W_n$  and  $W_d$  respectively.  $W_n$  combines the effects of HSFE, LSFE and TE as  $W_n = [W_{\text{HSFE}}, W_{\text{LSFE}}, W_{\text{TE}}]$ . The spatial error Gauss-Markov processes are equated to first order low pass filters, while the temporal error filter is simply a gain. In comparison, the disturbance filter,  $W_d$ , is a rational transfer function approximated from the power spectral density of simulated torque data over the course of three orbits. Similarly, for each error metric in Table 1, a rational transfer function  $F_i$  exists that transfers the error signal  $e$  into its respective error metric  $z_i$  (Pittelkau and McKinley, 2012). This frequency domain transformation is represented by  $F$  in Fig. 8.

In terms of robustness to uncertainties, both controllers meet the requirements with similar distribution. Fig. 9 shows the mean and range of the margins relative to their respective requirements across a grid of plants covering

Table 3. Worst-case achieved pointing error metrics for both  $K$  and  $K_\rho$  across all uncertainties in the plant (68% confidence level,  $1\sigma$ ).

Metric	Req.	$K$	$x/y/z$	$K_\rho$	$x/y/z$
$\Delta t_1/\Delta t_2/\Delta t_s$ [s]	[ $\mu\text{rad}$ ]	[ $\mu\text{rad}$ ]		[ $\mu\text{rad}$ ]	
APE	300	361/360/639		50.7/49.6/41.9	
RPE	150	334/333/589		49.7/48.6/41.1	
RPE	150			0.24/0.23/0.34	0.13/0.11/0.09
				1/-/-	
PDE	$x = 7$	0.08/0.08/-		0.04/0.04/-	
	$y = 3$				
PDE	150	0.08/0.08/0.12		0.04/0.04/0.03	
				0/0/0.1	

the full range of uncertainties. The LTI design is generally more conservative, although its limiting factor is keeping the flexible mode peaks below  $-6\text{dB}$  for all uncertainty cases. This is due to its lack of LPV notching behaviour.

The pointing performance was assessed in frequency domain using the principal that the standard deviation  $\sigma_e$  of a signal  $S_e$  is equal to the  $H_2$ -norm of the transfer  $H(s)$  from a white noise input to  $S_e$

$$\begin{aligned} \sigma_e &= \left( \frac{1}{2\pi} \int_{-\infty}^{\infty} S_e(\omega) d\omega \right)^{0.5} \\ &= \left( \frac{1}{2\pi} \int_{-\infty}^{\infty} |H(\omega)|^2 d\omega \right)^{0.5} \\ &= \|H(\omega)\|_2 \end{aligned} \quad (9)$$

By assessing the  $H_2$ -norm of the transfer function from the input  $d$  to the output  $z$  as in Fig. 8, the pointing error metrics are obtained (the transfer from noise  $n$  was neglected as its effect was negligible). Similarly the  $3\sigma$  values corresponding to the estimated maximum torque commands were calculated from the  $H_2$ -norm of the transfer from  $d$  to  $u$ . Pointing metric and torque command performance were verified against simulations of a parameter-varying orbit with accurate torque data (as in Fig. 4) and modelled sensor noise. Fig. 10 shows the plotted APE of one simulated orbit. After the initialisation transient, the APE remains within the required bounds, notice there is a second transient when the satellite enters eclipse. Similar results were seen with other performance metrics. All pointing metrics were far exceeded with the LPV design (see Table 3)) while respecting the robustness margins, whereas the LTI design was unable to achieve the APE and RPE pointing requirements. The maximum torque commands for both controller designs were very close, in  $x$  and  $y$  they are at approximately 60% saturation and in  $z$  they are at less than 30% saturation for all plants.

## 6. CONCLUSION

This paper presented a comprehensive, linear parameter varying (LPV) control design method for the 3-axis attitude tracking of an Earth-observation satellite. The controller is scheduled with the positional angle of a flexible solar array that rotates around the body of the satellite each orbit. The control method demonstrated improved pointing performance when compared to a linear time invariant approach for the same satellite, assessed according to industrial standards. This improvement was due



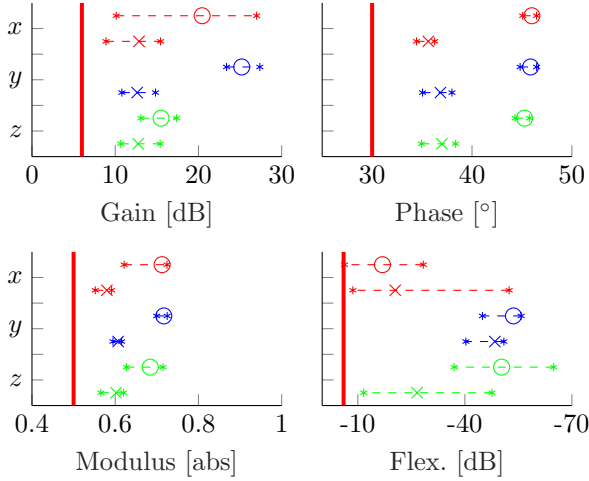


Fig. 9. SISO margin distributions against requirements (—). LTI mean values in  $x$ ,  $y$  and  $z$  (○, ○ and ○) compared to the LPV mean values (×, × and ×).

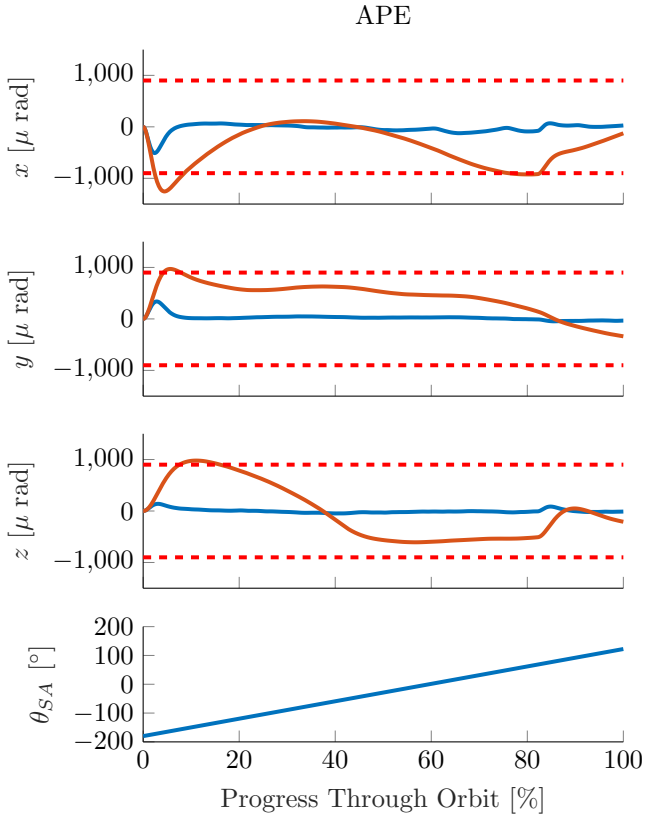


Fig. 10. Pointing performance of LPV (—) and LTI (—) controllers compared to  $3\sigma$  requirement (---).

to the automatic parameter varying notching of the solar array's flexible modes. The presented design process may be developed further for application to automated mode transition during satellite operations by considering modes together as one LPV system.

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