

## INTERACTIVE 3D-REPRESENTATION OF MATERIAL MATHEMATICAL MODELS WITH WebGL

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**ABSTRACT:** Collections of mathematical models are historical witnesses for "Anschauung" (notion, comprehension by experience), they are a cultural heritage and in recent years appreciated as scientific sources. Since the models are often hidden in single institutes, it is rewarding to make them accessible via online research. In DAMM (Digital Archive of Mathematical Models) we use the current 3D-web-technologies to compensate the lack of physical presence. This 3D approach facilitates the exploration of the material models and adds extra options like virtual cutting and the connection with results from computer algebra systems.

**Keywords:** material mathematical models, scientific collection, 3D digitization, WebGL, virtual reality

### 1. DAMM

The web project DAMM (Digital Archive of Mathematical Models [1]) at the TU Dresden aims to provide an infrastructure for multi-disciplinary research. We have created a platform not only for mathematical communication but as well for inspiration. In the near future an integration of related scientific collections of various subject areas, like Crystallography, Physics or Chemistry, is pursued.

At the beginning, the main goals were to support the preservation of mathematical artefacts as a cultural heritage, to foster the dissemination of previously hidden treasures of scientific collections and last but not least to enable a new approach to the perception of historical models. Therefore we published photographs of the mathematical models online. Within DAMM we take the next step and additionally show interactive 3D-representations. This enhances the user experience during online investigation and broadens possibilities even more than we at first expected.

Various users may be interested in this

archive: scientists of mathematics and of the above-mentioned disciplines as well as engineers, didacticians, historians, designers, and artists. The faceted search tool allows to meet the demands of mathematical experts as well as the curiosity of the general public. To list models of interest the user can select purposeful characteristics, like material, special field or catalogue. It can be seen as an extensive place for open education, because everyone has access to it and the users can choose their individual way to study the models.

Due to the worldwide connectivity the archive can be seen as a platform for exchange. Collections from different locations within one subject may be connected to one big compilation in DAMM.

### 2. MEDIABOX

We have constructed a data framework called "Mediabox", which technically is a special structure of files in a directory on a web server. It can be loaded as a whole by the website.

For every 3D-representation there is a Mediabox containing a standalone presentation with a user interface. Essentially it only needs HTML, JavaScript and WebGL to show the 3D content. Detailed information is given in chapter 3. Furthermore, the Mediabox is compatible with other techniques to create any new interactive content.

This container "Mediabox" was initially created to compensate the absence of the material models during web research. Of course we cannot imitate the tactility, but at least the models can now be viewed from all directions. Additionally, with the 3D technology there come even more possibilities that radically exceed the ideas of compensation and imitation.

At first we want to report, how we create content for the Mediabox from digitized models. After this we will show, how the user interface supports innovative investigation techniques. Therefore we demonstrate some aspects of user customization. As an outlook we introduce, how the content can be extended with additional virtual data, how several 3D-representations can be linked and finally, how the user can make use of the data for contexts like education and fine art.

### 3. CREATION OF CONTENT

The creation of content for the Mediabox is critically dependent on the material properties of the models. Thus the digitizing process forces a classification of the models in categories like solid plaster, transparent and reflecting plastic, movable and flexible, and string models. Each category demands a different workflow, which is described below. Our aim is to obtain a very high precision with very limited resources. This is why our processes are mainly built with open source components.

#### 3.1 Solid plaster

The digitizing process for solid plaster combines two techniques. For the 3D model, we use an

industrial optical 3D scanner. For the textures, we have created a setup that automatically produces a series of photographs from different angles with a full frame camera. Both datasets are then merged by means of photogrammetry and additional routines, which have been compiled during this project. In [2] the processing of plaster models is described in depth.

As a result we obtain a HTML and a X3D file with an accompanying texture file as PNG. HTML was selected to visualize the content in any of the latest webbrowsers (Fig. 1). X3D is a language to represent 3D objects independent from additional plug-ins by using X3DOM [3]. This framework is open source and uses JavaScript to create 3D scenes. Some examples of its potentials are given in 4.1.



Figure 1: Ruled surface of third order, Schilling catalogue, VII 20 (left: photograph of the historical model, right: rendering of the digitized model in the Mediabox)

In the following we describe alternative techniques which can be used to create no less accurate digital representations. This is especially relevant for models, which do not fit the optical scanning process.

#### 3.2 String models

The digitization of string models can be realised in two different ways. The first option is an interactive virtual reality tour (like QuickTime VR models), which is simulated by a series of

pre-captured photographs (3.2.1). The other option is a kind of a reverse engineering process, which is based on flatbed scans taken from the frameworks of the string models (3.2.2).

### 3.2.1 Virtual reality

With the setup used for the texturing of the solid plaster models, we take a systematic series of photographs of each model from different angles. These photographs then are integrated into the Mediabox with JavaScript. The user can change his viewpoint by clicking on arrows or mouse movements. This application suggests that the beholder is moving around the model, although he is watching 2D images (Fig. 2).

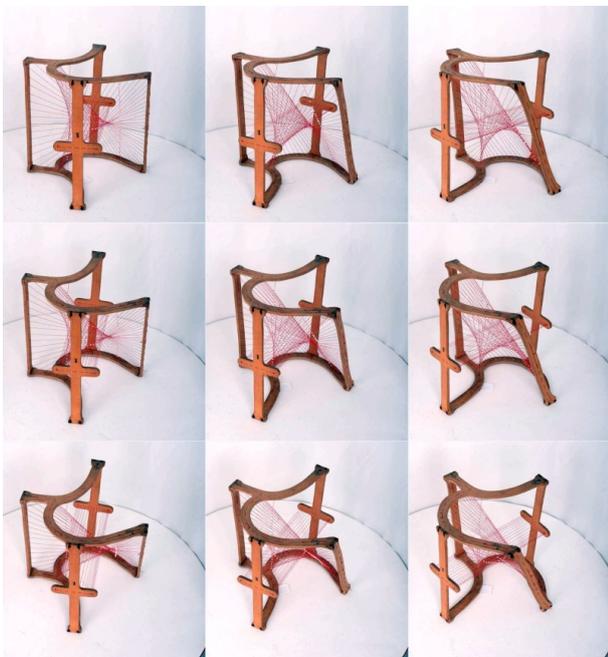


Figure 2: 9 pictures from a set of photographs used to form a VR object, Schilling catalogue, XVIII 2

### 3.2.2 3D modeling software

At first we scan every side of the metal or wooden framework of the string model with a flatbed scanner. Then we re-model the framework with a graphical 3D software, for example Blender [4]. The scans both serve as source for the measurements and as textures for the constructed 3D elements. The threads are represented by

cylinders. Each cylinder is also constructed within the modelling software and connects two corresponding holes in the framework (Fig. 3).

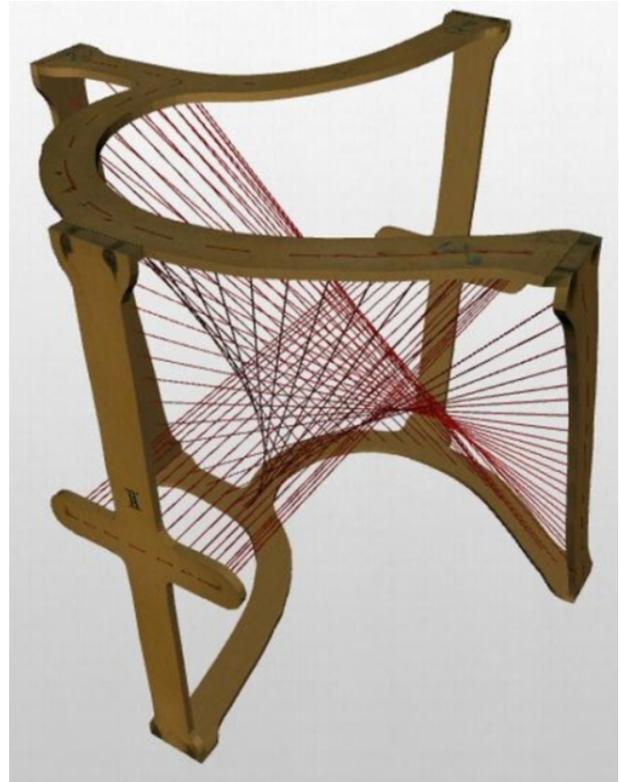


Figure 3: Flatbed scanned and remodelled framework. Cylinders in the place of the threads are also constructed with 3D software, Schilling catalogue, XVIII 2

### 3.3 Transparent and reflecting plastic, movable and flexible models

Of course, like described in 3.2.1 a virtual reality model may always be adequate for those models, which do not fit the previously described 3D processes. The advantage of VR models is that they can be produced more or less automatically. But if we want to achieve true 3D models, we have to apply techniques of reverse engineering and texturing.

For flexible models an additional parameter is necessary to show the movement.

#### 4. USER INTERFACE

The handling of the models is quite intuitive. Three elementary functions are given by the Mediabox: the left mouse button rotates the model, the right mouse button zooms in or out and the middle mouse button moves the model.

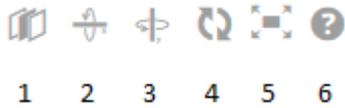


Figure 4: Interface of the Mediabox for optional sections, rotations about special axes, fullscreen view and help bubble

In addition a user menu is displayed (Fig. 4). Implemented functions are a virtual clipping plane (1) (Fig 5) as well as an optional limitation to a vertical (2) or horizontal (3) rotation axis. Button 4 restores the view to the preset, 5 starts the full screen mode and 6 gives information about the Mediabox.



Figure 5: Parabolic cyclide rendered in four positions with a clipping plane

#### 4.1 User customization

X3DOM is very powerful. Lighting and shadow effects, especially intensity and size, allow the user to modify the appearance of the Mediabox [5]. Thus the model can be displayed attractively and in different atmospherical settings. An-

other option is to apply section planes and cross-sections (Fig. 6).

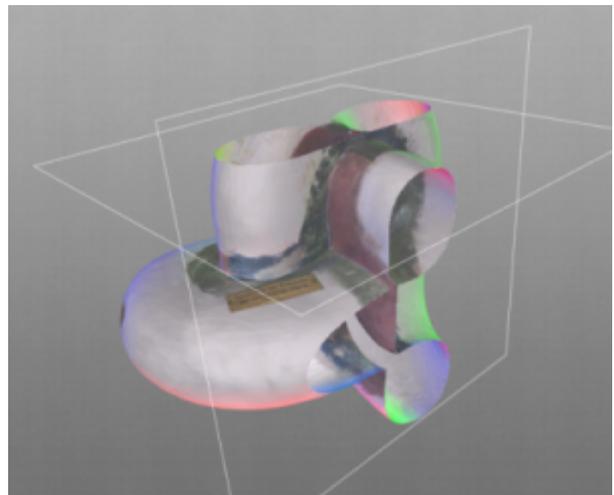


Figure 6: Schilling catalogue, XXX 2, with two virtual section planes

#### 4.2 Additional data

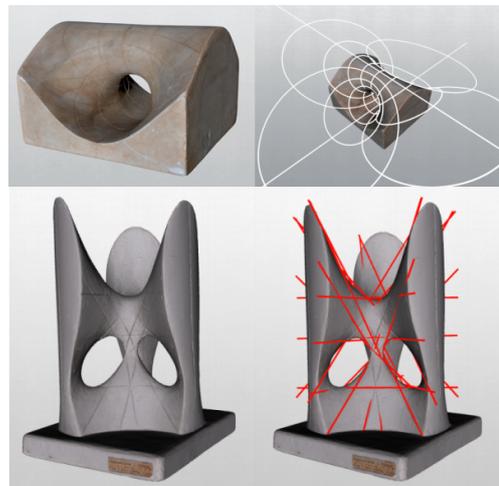


Figure 7: Two models with additional data (top: Schilling catalogue, X 5, with additional circles; bottom: Schilling catalogue, VII 1, with additional lines)

The procedures described in 3.2.2 already may suggest that it is possible to combine the results of the optical scanning process with constructed or calculated data. In fact, such overlays can be used for the visualization of scientific facts

and cross references. Thus the user can easily grasp background information, even beyond the subjects which are incorporated in the mere material model (Fig. 7).

It is possible to load two digitized models into one Mediabox. Hence the user is able to compare two models by comparing their point clouds. Another option is to investigate the intersection of two models.

Moreover, broken or uncomplete models can be repaired or reconstructed digitally. Of course it lies in the responsibility of those who produce the 3D data to document any manipulations or additions.

## 5. GEOMETRY SOFTWARE

The Mediabox framework is open to include third party geometry software like Surfer [6]. Thus the user of DAMM can even experiment with equations to create and render algebraic surfaces and compare them to material models on one platform (Fig. 8).

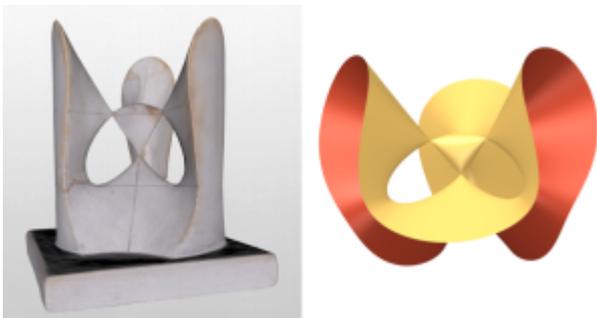


Figure 8: Surface of third order, Schilling catalogue, VII 2 (left: digitized, right: rendered with the software Surfer)

With the help of these tools the linkages between different models can be visualized. This may for example prove graphically, if two models belong to the same family (Fig. 9).

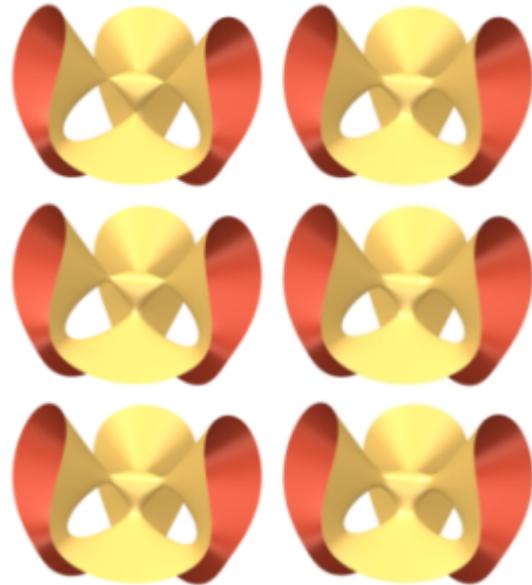


Figure 9: A real cubic surface morphs into another real cubic surface, rendered with Surfer

Furthermore, the user can check the accuracy of the historical models by superimposing the digitized and the calculated 3D models.

## 6. USAGE OF THE DATA

Every dataset combines drawings, photographs, digitized 3D models, and metadata with digitized and edited grey literature as well as mathematical and historical background information. It encourages especially mathematical writing and journalism, links current mathematical communicators and offers a broad range of information and aid. Thus the archive can also be seen as a source for open online courses and exchange.

The offered information may be used for data analysis. Provided downloads furthermore enable the reproduction of models and other research projects.

It is undeniable that the digitization process adds something to the collection, which again produces metadata. It needs awareness to deal with the implications of the new content.

A natural effect is the necessity to scale up the database.

The previous remarks also imply that the digitization process fosters a focus shift away from the mere substance of the collections towards other disciplines.

## 7. CONCLUSIONS

Within the DAMM project we create 3D-representations of material mathematical models and provide them via Internet. The latter is realized by the utilization of current web technologies like WebGL. The digitization process has mainly to deal with difficulties, which arise from the great material differences of the models. We try to cope with these challenges as we apply different digitization processes according to the material categories.

The benefits of the 3D-representations are definitely worth the efforts. The 3D models foster the accessibility of the collection and ease the spatial exploration. In fact, only the virtual representations provoke and allow completely new research questions. Thus the digitization adds new quality to the collection and as such supports two fundamental concepts: the collection shall serve as research infrastructure and as a source of inspiration.

## ACKNOWLEDGMENTS

The project DAMM is funded by the German Research Foundation (DFG) and executed in collaboration with the Chair of Media Design at the TU Dresden, Prof. Rainer Groh. The presented results are partly based on the work of Martin Bornemann and others, who worked for DAMM during the period from 2012 to 2016.

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- [4] <https://www.blender.org>
- [5] <http://www.x3dom.org/examples>
- [6] <https://imaginary.org/de/program/surfer>

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