Mixing Description Logics in Privacy-Preserving Ontology Publishing

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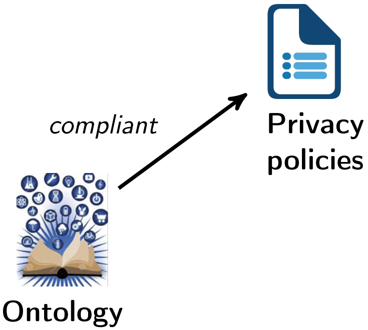
Privacy-Preserving Ontology Publishing (PPOP)



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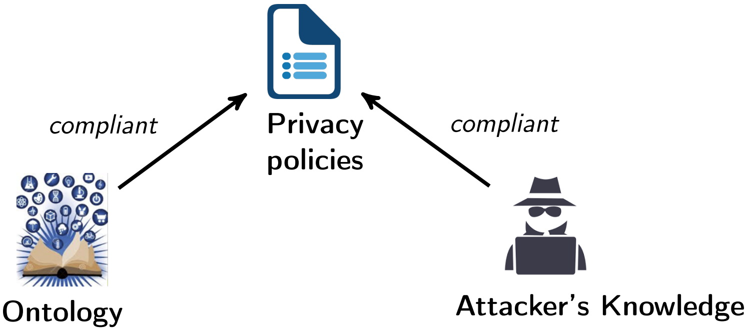

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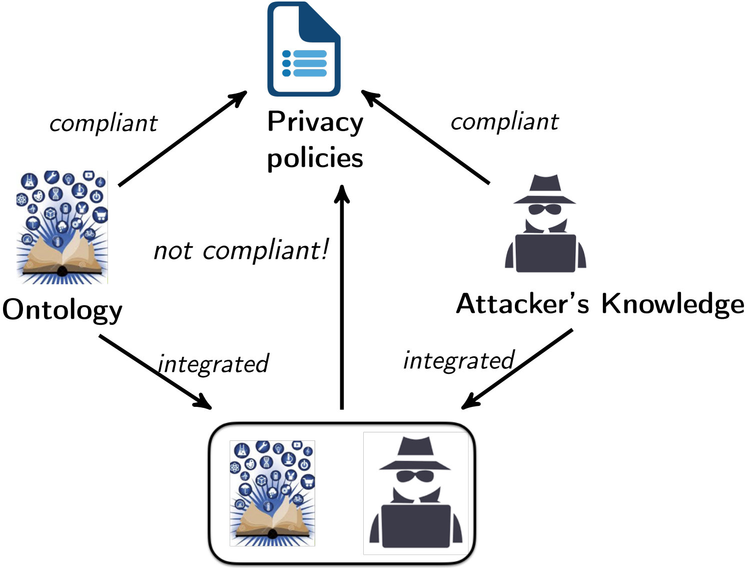

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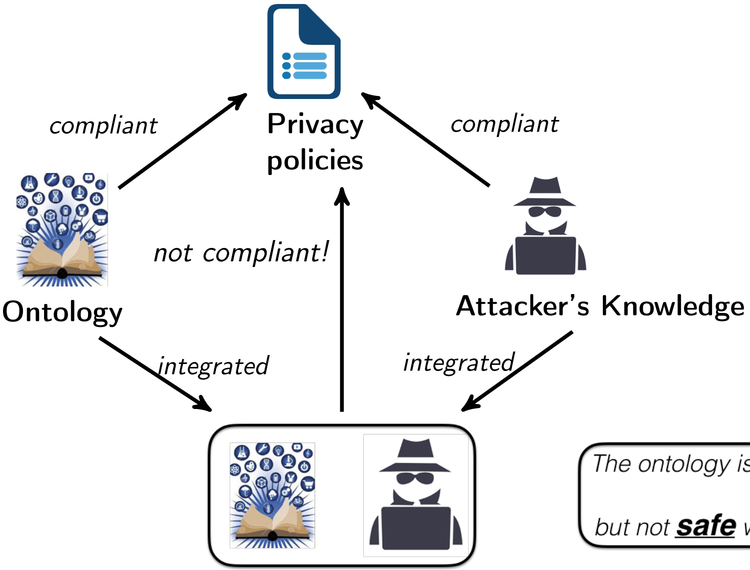

The ontology is ***compliant***,

but not ***safe*** w.r.t. policy!!

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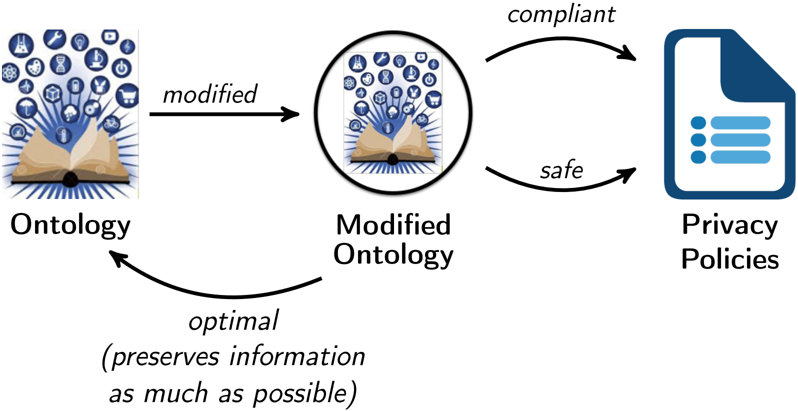

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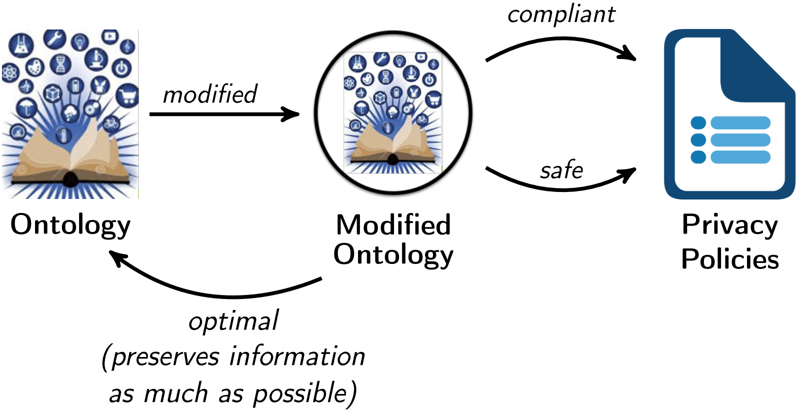

Assumption: *Ontologies are formulated in Description Logics (DLs).*

What are DLs?

Description Logics

- The **logical underpinning** of **Web Ontology Language (OWL)**
- Commonly used in **medical ontologies**
- Decidable fragments of First Order Logics

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- The **basic building blocks** are:
  - N_C: set of **concept names** A: *Female*, *Doctor*, *Patient*, ...
  - N_R: set of **role names** r: *seenBy*, *suffer*, *hasSymptom*, ...
  - N_I: set of **individual names** a: *LINDA*, *CANCER* ...

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  - N_I: set of **individual names** a: *LINDA*, *CANCER* ...
- The formal semantics is introduced by means of an **interpretation** $(\mathcal{I} = \Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$
  - $\Delta^{\mathcal{I}}$: Non-empty domain elements
  - $A^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$
  - $r^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$
  - $a^{\mathcal{I}} \in \Delta^{\mathcal{I}}$
- Using N_C, N_R, and N_I as well as necessary constructors, the notion of **DL concepts** C, D, E are built.

Description Logic Ontologies

- A DL ontology $\mathfrak{O}$ consists of a **TBox** $\mathcal{T}$ and an **ABox** $\mathcal{A}$
- A TBox $\mathcal{T}$ is a set of **General Concept Inclusions (GCIs)** $C \sqsubseteq D$
  $\rightarrow$ hierarchical relationship between concepts
- An ABox $\mathcal{A}$ is a set of **concept assertions** $C(a)$ and **role assertions** $r(a, b)$
  $\rightarrow$ knowledge about individuals

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- A DL **Instance Store** $\mathfrak{O}'$ is a DL ontology without role assertions
- A main reasoning task in DLs $\Rightarrow$ Deciding **subsumption** between concepts
- A concept C is **subsumed** by a concept D, denoted by $C \sqsubseteq D$, iff $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$ for all interpretations $\mathcal{I}$.

Description Logic $\mathcal{FLE}$ and Its Fragments

- $\mathcal{FLE}$ concepts $C ::= \top$ (*top*) $\mid A \mid C \sqcap C$ (*conjunction*) $\mid$
  $\exists r.C$ (**existential restriction**) $\mid \forall r.C$ (**universal restriction**)
- **Semantics** of some $\mathcal{FLE}$ concepts:
  - $(\exists r.C)^{\mathcal{I}} = \{d \mid$ there is $e \in \Delta^{\mathcal{I}}$ such that $(d, e) \in r^{\mathcal{I}} \wedge e \in C^{\mathcal{I}}\}$
  - $(\forall r.C)^{\mathcal{I}} = \{d \mid$ for all $e \in \Delta^{\mathcal{I}}$ if $(d, e) \in r^{\mathcal{I}}$, then $e \in C^{\mathcal{I}}\}$
- **Fragments** of $\mathcal{FLE}$:
  - the DL $\mathcal{EL}$ (excluding value restrictions)
  - the DL $\mathcal{FL}_0$ (excluding existential restrictions)

Problem Setting: PPOP for $\mathcal{EL}$ Instance Stores

$\mathcal{EL}$ Instance Stores
without TBox

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$C_1(a), C_2(a)$ implies $(C_1 \sqcap C_2)(a)$

only one concept assertion speaking about one individual

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Published
Information

(an $\mathcal{EL}$ Concept C)

Attacker's
Knowledge

(an $\mathcal{EL}$ / $\mathcal{FL}_0$ / $\mathcal{FLE}$ Concept E)

Privacy Policy

(a finite set of $\mathcal{EL}$ concepts)
$\{D_1, \ldots, D_p\}$

Formalizing Sensitive Information in $\mathcal{EL}$ Instance Stores

- Given an $\mathcal{EL}$ concept C (published information) and an $\mathcal{EL}$ policy $\mathcal{P}$
- Given a quantifier symbol $Q \in \{\exists, \forall, \forall\exists\}$ and a DL $\mathcal{L}_\exists = \mathcal{EL}, \mathcal{L}_\forall = \mathcal{FL}_0, \mathcal{L}_{\forall\exists} = \mathcal{FLE}$

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Compliance, Safety, Optimality

1. the $\mathcal{L}_Q$ concept C' is **compliant with** $\mathcal{P}$ if $C' \not\sqsubseteq D_i$ for all $i = 1, \ldots, p$,

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   - a **Q-safe generalization** of C for $\mathcal{P}$ if $C \sqsubseteq C'$ and C' is Q-safe for $\mathcal{P}$,

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   - a **Q-safe generalization** of C for $\mathcal{P}$ if $C \sqsubseteq C'$ and C' is Q-safe for $\mathcal{P}$,
   - an **optimal Q-safe generalization** of C for $\mathcal{P}$ if
     - C' is a Q-safe generalization of C for $\mathcal{P}$ and
     - there is no Q-safe generalization C'' of C for $\mathcal{P}$ s.t. $C'' \sqsubset C'$.

Privacy Policy $\mathcal{P} = \{D\}$ about *LINDA*

$$D = Patient \sqcap \exists seenBy.(Doctor \sqcap \exists worksIn.Oncology)$$

Original **Published Information** C about *LINDA*

$$C = Patient \sqcap Female \sqcap \exists seenBy.(Doctor \sqcap Male \sqcap \exists worksIn.Oncology)$$

Note C is not **compliant with** and Q-**safe** for D for $Q \in \{\exists, \forall, \forall\exists\}$

An Illustration on Privacy Attacks in $\mathcal{EL}$ Instance Stores

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Modification

$$C_1 = Female \sqcap \exists seenBy.(Doctor \sqcap Male \sqcap \exists worksIn.Oncology)$$

Note $C \sqsubseteq C_1$ and C_1 **complies with** D

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$\exists$-Attacker is Coming!

$$C_1 = Female \sqcap \exists seenBy.(Doctor \sqcap Male \sqcap \exists worksIn.Oncology)$$

He knows $Patient(LINDA)$

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Original **Published Information** C about *LINDA*

$$C = \mathit{Patient} \sqcap \mathit{Female} \sqcap \exists \mathit{seenBy}.(\mathit{Doctor} \sqcap \mathit{Male} \sqcap \exists \mathit{worksIn}.\mathit{Oncology})$$

Note C is not **compliant with** and Q-**safe** for D for $Q \in \{\exists, \forall, \forall\exists\}$

Linked and Revealed!

$$C_1 = \mathit{Female} \sqcap \exists \mathit{seenBy}.(\mathit{Doctor} \sqcap \mathit{Male} \sqcap \exists \mathit{worksIn}.\mathit{Oncology}) \sqcap \textbf{\textit{Patient}}$$

Note $D(\mathit{LINDA})$ is **revealed**

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Modification

$$C_2 = Female \sqcap \exists seenBy.(Doctor \sqcap Male \sqcap \exists worksIn.\top) \sqcap \exists seenBy.(Male \sqcap worksIn.Oncology)$$

C_2 is the (unique) **optimal** $\exists$-**safe generalization** for D

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Note C is not **compliant with** and Q-**safe** for D for $Q \in \{\exists, \forall, \forall\exists\}$

$\forall$-**Attacker is Coming!**

$$C_2 = Female \sqcap \exists seenBy.(Doctor \sqcap Male \sqcap \exists worksIn.\top) \sqcap \exists seenBy.(Male \sqcap worksIn.Oncology)$$

He knows $(Patient \sqcap \forall seenBy.\forall worksIn.Oncology)(LINDA)$

Privacy Policy $\mathcal{P} = \{D\}$ about *LINDA*

$$D = Patient \sqcap \exists seenBy.(Doctor \sqcap \exists worksIn.Oncology)$$

Original **Published Information** C about *LINDA*

$$C = Patient \sqcap Female \sqcap \exists seenBy.(Doctor \sqcap Male \sqcap \exists worksIn.Oncology)$$

Note C is not **compliant with** and Q-**safe** for D for $Q \in \{\exists, \forall, \forall\exists\}$

Linked and Revealed!

$$C_2 = Female \sqcap \exists seenBy.(Doctor \sqcap Male \sqcap \exists worksIn.\top) \sqcap \exists seenBy.(Male \sqcap worksIn.Oncology) \sqcap \mathbf{Patient \sqcap \forall seenBy.\forall worksIn.Oncology}$$

$D(LINDA)$ is revealed again

An Illustration on Privacy Attacks in $\mathcal{EL}$ Instance Stores

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$$D = Patient \sqcap \exists seenBy.(Doctor \sqcap \exists worksIn.Oncology)$$

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Note C is not **compliant with** and Q-**safe** for D for $Q \in \{\exists, \forall, \forall\exists\}$

Modification

$$C_3 = Female \sqcap Patient \sqcap \exists seenBy.(Doctor \sqcap Male)$$

Note C_3 is an **optimal** $\forall$-**safe generalization** for D

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$\forall\exists$-**Attacker is Coming!**

$$C_3 = Female \sqcap Patient \sqcap \exists seenBy.(Doctor \sqcap Male)$$

He knows $(\forall seenBy.\exists worksIn.Oncology)(LINDA)$

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$D(\mathit{LINDA})$ is **revealed again**

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Note C is not **compliant with** and Q-**safe** for D for $Q \in \{\exists, \forall, \forall\exists\}$

Modification

$$C_4 = Female$$

Note C_4 is the **optimal** $\forall\exists$-**safe generalization** for D!

Our Decision and Computational Problems

Given $Q \in \{\forall, \forall\exists\}$, a published information ($\mathcal{EL}$ concept) C, an $\mathcal{EL}$ policy $\mathcal{P}$.

Decision Problems

- ***Q*-Safety**:
  Is an $\mathcal{EL}$ concept C_1 Q-safe for a policy $\mathcal{P}$?
- ***Q*-Optimality**:
  Is an $\mathcal{EL}$ concept C_1 an optimal Q-safe generalization of C for $\mathcal{P}$?

Computational Problem

Find an $\mathcal{EL}$ concept C_1 s.t C_1 is an optimal Q-safe generalization of C for $\mathcal{P}$!

compliance, **$\exists$-safety** and **$\exists$-optimality** have been investigated by (Baader, Kriegel, Nuradiansyah in JELIA 2019)

Complexity Results

Decision Problems	$\mathbf{Q} = \exists$	$\mathbf{Q} = \forall$	$\mathbf{Q} = \forall\exists$
Q-safety	PTime*	PTime	PTime
Q-optimality	coNP* and Dual-hard*	coNP and Dual-hard	PTime

Table: Complexity results of decision problems on PPOP for $\mathcal{EL}$ instance stores

Computational Problems	$\mathbf{Q} = \exists$	$\mathbf{Q} = \forall$	$\mathbf{Q} = \forall\exists$
Optimal Q-safe Generalization(s)	ExpTime*	ExpTime	PTime

Table: Complexity of computing one/all optimal Q-safe generalizations for $\mathcal{P}$

* investigated by (Baader, Kriegel, and Nuradiansyah in JELIA 2019)

Complexity Results

Decision Problems	$\mathbf{Q} = \forall$	$\mathbf{Q} = \forall$	$\mathbf{Q} = \forall\exists$
Q-safety	PTime*	PTime	PTime
Q-optimality	coNP* and Dual-hard*	coNP and Dual-hard	PTime

Computational Problems	$\mathbf{Q} = \exists$	$\mathbf{Q} = \forall$	$\mathbf{Q} = \forall\exists$
Optimal Q-safe Generalization(s)	ExpTime*	**ExpTime**	PTime

Reasons:

- Given an $\mathcal{EL}$ concept D, $\texttt{con}(D)$ is the **set of all atoms** (A or $\exists r.D'$) in the top-level conjunction of D.
- Computing all **minimal hitting sets** of $\texttt{con}(D_1), \ldots, \texttt{con}(D_p)$, where $\mathcal{P} = \{D_1, \ldots, D_p\}$.
- The computation is performed recursively on the **role depth** of the published information C

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Computational Problems	$\mathbf{Q} = \exists$	$\mathbf{Q} = \forall$	$\mathbf{Q} = \forall\exists$
Optimal Q-safe Generalization(s)	ExpTime*	ExpTime	PTime

Reasons:

- Check if C_1 is an $\forall$-safe generalization of C for $\mathcal{P}$
- Check if there is C_2 s.t. $C \sqsubseteq C_2 \sqsubset C_1$, where C_2 is a not $\forall$-safe generalization of C for $\mathcal{P}$
- There is an NP algorithm to **guess** such concept C_2 (Baader, Kriegel, Nuradiansyah in JELIA 2019)

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Decision Problems	$\mathbf{Q} = \exists$	$\mathbf{Q} = \forall$	$\mathbf{Q} = \forall\exists$
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Q-optimality	coNP* and Dual-hard*	coNP and **Dual-hard**	PTime

Computational Problems	$\mathbf{Q} = \exists$	$\mathbf{Q} = \forall$	$\mathbf{Q} = \forall\exists$
Optimal Q-safe Generalization(s)	ExpTime*	ExpTime	PTime

Reasons:

- $\forall$-Optimality is coNP-hard? Don't know yet
- There is a polynomial reduction of Dual problem to $\forall$-optimality

  Given two **families of inclusion-comparable sets** $\mathcal{G}$ and $\mathcal{H}$, Dual asks whether $\mathcal{H}$ consists exactly of the minimal hitting sets of $\mathcal{G}$.

Complexity Results

Decision Problems	**Q** $= \exists$	**Q** $= \forall$	**Q** $= \forall\exists$
Q-safety	PTime*	PTime	**PTime**
Q-optimality	coNP* and Dual-hard*	coNP and Dual-hard	**PTime**

Computational Problems	**Q** $= \exists$	**Q** $= \forall$	**Q** $= \forall\exists$
Optimal Q-safe Generalization(s)	ExpTime*	ExpTime	**PTime**

Reasons:

$\forall\exists$-Safety and $\forall\exists$-Optimality

C is **$\forall\exists$-safe for $\mathcal{P}$** iff

1. $A \notin \mathsf{con}(C)$ for all concept names $A \in \mathsf{con}(D_1) \cup \ldots \cup \mathsf{con}(D_p)$, and
2. for all existential restrictions $\exists r.D' \in \mathsf{con}(D_1) \cup \ldots \cup \mathsf{con}(D_p)$, there is no concept of the form $\exists r.E \in \mathsf{con}(C)$

Conclusions and Future Work

Conclusions:

- Investigate **PPOP for $\mathcal{EL}$ Instance Stores**
- Considering **attacker's knowledge** to be given by an $\mathcal{FL}_0$ or $\mathcal{FLE}$ concept
- Deciding Q-**safety** and Q-**optimality**, where $Q \in \{\forall, \forall\exists\}$.
- Computing **optimal** Q**-safe generalizations** of $\mathcal{EL}$ concepts for $\mathcal{P}$

Note: *the stronger the attacker's knowledge, the more radical we need to change the concept to make it safe*

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Future Work:

- PPOP in $\mathcal{EL}$ ABoxes, including role assertions (Ongoing!)
- PPOP in $\mathcal{EL}$ Instance Stores w.r.t. (General) TBoxes
- Playing with more different or expressive DLs

Thank You

ROSI