TECHNISCHE UNIVERSITÄT DRESDEN

Faculty of Electrical and Computer Engineering Institute of Control Theory

Control of the VGF Process

Part II: Flatness Based Observer Design

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Introduction

This contribution continues the feedforward and feedback controller design for a simplified model of the Vertical-Gradient-Freeze (VGF) process that has been presented in the poster *Control of the VGF Process Part I: Feedforward and Flatness Based State Feedback*. The presented control system needs values of quantities that cannot be measured directly. The observer designed in this poster will overcome this problem.

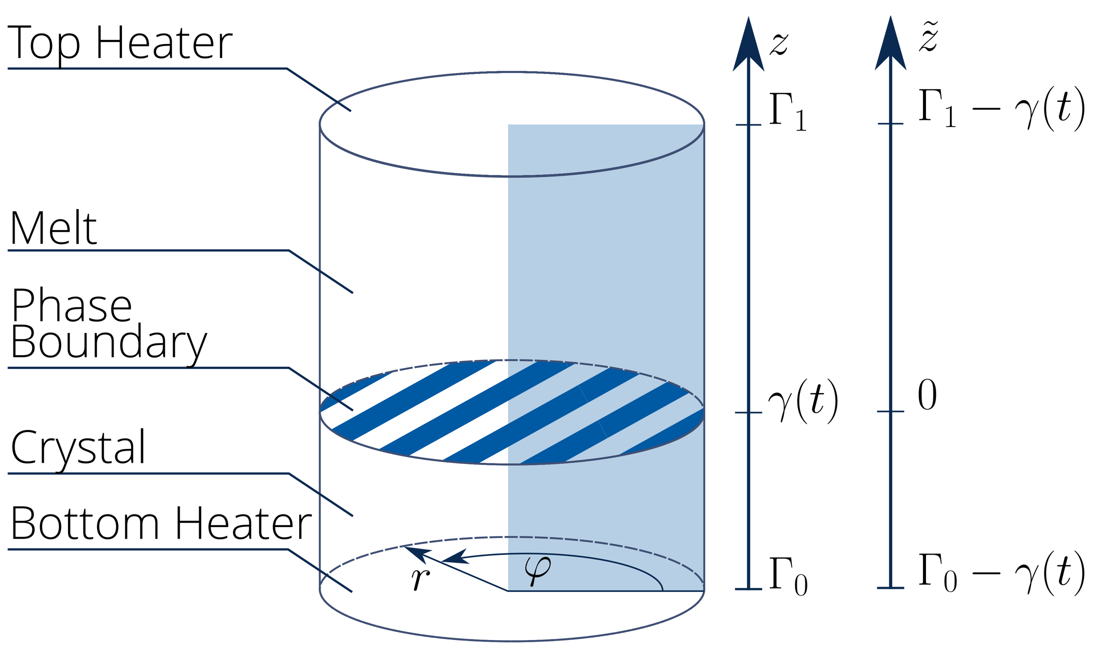

Figure 1: Schematics of the general cylindrical coordinate system (r, φ, z, t), a meridional plane (blue) and the domains of the reduced coordinates z and $\tilde{z}$.

Modelling

A very simple, one-dimensional biphasic model neglecting any convective and radiative effects of the VGF process is considered, cf. Figure 1:

$$\partial_t T(z,t) = \alpha \partial_z^2 T(z,t),\ z \in \Omega \tag{1a}$$

$$k \partial_z T(\Gamma, t) = \delta u(t) \tag{1b}$$

$$T(\gamma(t), t) = T_{\mathrm{m}} \tag{1c}$$

with the Stefan condition:

$$\rho_{\mathrm{m}} L \dot{\gamma}(t) = k_{\mathrm{s}} \partial_z T_{\mathrm{s}}(\gamma(t)^{-}, t) - k_{\mathrm{l}} \partial_z T_{\mathrm{l}}(\gamma(t)^{+}, t)\,. \tag{2}$$

Here one has the following parameters: melting temperature T_{m}, thermal conductivity k, thermal diffusivity $\alpha = \frac{k}{\rho c_{\mathrm{p}}}$ with specific heat capacity c_{p} and density ρ. Furthermore one has the density ρ_{m} in the melt, specific latent heat L, growth rate $\dot{\gamma}(t)$, heat conductivities k_{s} and k_{l} of the solid and the melt (at melting temperature) and the temperature gradients $\partial_z T_{\mathrm{s}}(\gamma(t)^{-}, t)$ and $\partial_z T_{\mathrm{l}}(\gamma(t)^{+}, t)$ on the solid and liquid side of the phase boundary, respectively. The direction of the heat flow is considered by $\delta = -1$ (bottom, solid boundary) and $\delta = 1$ (top, liquid boundary).

A flat approximation

The observer design is based on a lumped nonlinear system model of the following structure:

$$\dot{\boldsymbol{x}}(t) = f(\boldsymbol{x}(t), \boldsymbol{u}(t)), \quad \boldsymbol{y}(t) = h(\boldsymbol{x}(t))\,. \tag{3}$$

Hence, a lumped approximation of the systems dynamics (1) is required. For further ease of computation, the coordinate transform $\tilde{T}(\tilde{z}, t) = T(z, t)$ with $\tilde{z} := z - \gamma(t)$ is introduced which shifts the phase boundary into the origin of a new, moving reference frame as it can be seen on the right hand side of Figure 1. Hence, in the new coordinates the phase boundary is always located at $\tilde{z} = 0$. This yields the transformed generic system

$$\partial_t \tilde{T}(\tilde{z}, t) = \alpha \partial_{\tilde{z}}^2 \tilde{T}(\tilde{z}, t) + \dot{\gamma}(t) \partial_{\tilde{z}} \tilde{T}(\tilde{z}, t) \tag{4a}$$

$$k \partial_{\tilde{z}} \tilde{T}(\Gamma - \tilde{z}, t) = \delta u(t) \tag{4b}$$

$$\tilde{T}(0, t) = T_{\mathrm{m}}\,. \tag{4c}$$

The solution of the system can be expressed in terms of a power series expansion $\tilde{T}(\tilde{z}, t) = \sum_{i=0}^{\infty} c_i(t) \frac{\tilde{z}^i}{i!}$. From this a so-called flat output $\boldsymbol{\xi}(t) = \left(\partial_{\tilde{z}} \tilde{T}(0, t),\ \gamma(t)\right)^T$ is found which completely parameterize the solution $\tilde{T}(\tilde{z}, t)$ via the recursion $\partial_t c_i(t) = \alpha c_{i+2}(t) + \dot{\gamma}(t) c_{i+1}(t)$ for $i = 0, \ldots, \infty$ with $c_0(t) = T_{\mathrm{m}}$ and $c_1(t) = \tilde{T}(0, t)$. Since the temperature distribution in the whole system is given by its components and their derivatives, they serve well as components of a new approximation. Truncating the series expansion at order N therefore yields the following state

$$\boldsymbol{x} := \left(\partial_{\tilde{z}}^{(0)} \tilde{T}(0),\ \ldots,\ \partial_{\tilde{z}}^{(N/2-1)} \tilde{T}(0),\ \gamma^{(0)},\ \ldots,\ \gamma^{(N/2)}\right)^T \tag{5}$$

of dimension $N - 1$, with the input $\boldsymbol{u}(t) := \left(u_{\mathrm{s}}(t), u_{\mathrm{l}}(t)\right)^T$ and the output measurement $\boldsymbol{y}(t) := \left(\tilde{T}_{\mathrm{s}}(\Gamma_0, t),\ \tilde{T}_{\mathrm{l}}(\Gamma_1, t)\right)^T$.

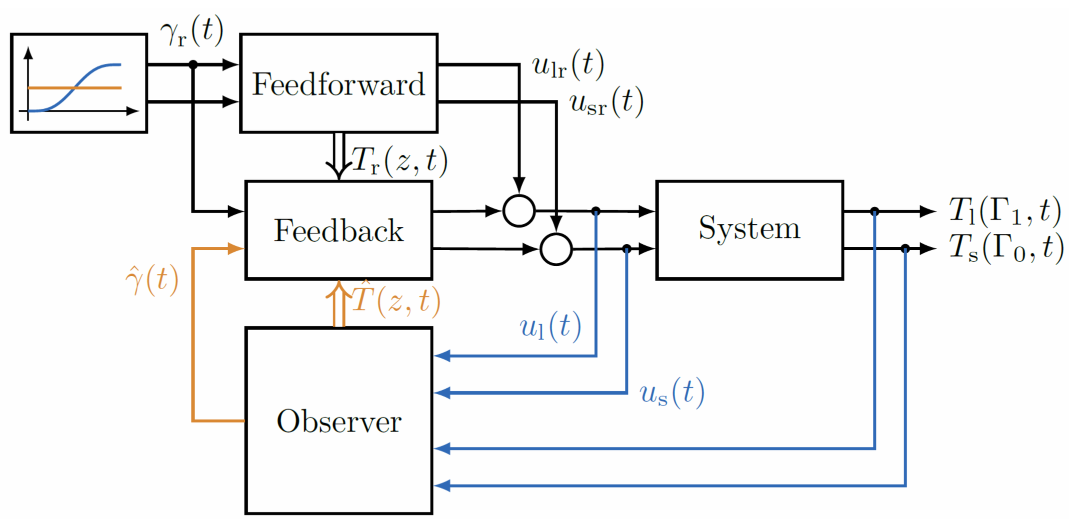

Figure 2: Structure of the complete control loop with feedforward control, feedback control and state observer.

Observer design

The derived tracking controller $u(t) = \Phi(\boldsymbol{x}(t))$ for which the observer is designed satisfies the tracking error dynamics $\sum_{n=0}^{N} \kappa_n \boldsymbol{e}^{(n)}(t) = 0$ with $\boldsymbol{e}(t) = \boldsymbol{\xi} - \boldsymbol{\xi}_{\mathrm{r}}$ and the controller parameters κ_n. This control law requires estimates $\hat{\boldsymbol{x}}(t)$ for the state $\boldsymbol{x}(t)$. They are calculated by an observer of the following structure

$$\dot{\hat{\boldsymbol{x}}}(t) = f(\hat{\boldsymbol{x}}(t), \boldsymbol{u}(t)) + \boldsymbol{L}(t) \tilde{\boldsymbol{y}}(t) \tag{6a}$$

$$\hat{\boldsymbol{y}}(t) = h(\hat{\boldsymbol{x}}(t)) \tag{6b}$$

with observer gain $\boldsymbol{L}(t) \in \mathbb{R}^{M \times 2}$ as well as the disturbances $\boldsymbol{\mu}(t)$ and $\boldsymbol{\nu}(t)$ acting on the system input and output, respectively. The calculation of the observer gain is the key point in observer design. For this purpose the dynamics of the observer error $\tilde{\boldsymbol{x}}(t) = \hat{\boldsymbol{x}}(t) - \boldsymbol{x}(t)$ is linearised around the reference trajectory $\boldsymbol{x}_{\mathrm{r}}(t), \boldsymbol{u}_{\mathrm{r}}(t)$:

$$\dot{\tilde{\boldsymbol{x}}}(t) = \boldsymbol{A}(t) \tilde{\boldsymbol{x}}(t) - \boldsymbol{B}(t) \boldsymbol{\mu}(t) + \boldsymbol{L}(t) \tilde{\boldsymbol{y}}(t) \tag{7a}$$

$$\tilde{\boldsymbol{y}}(t) = \boldsymbol{C} \left(\tilde{\boldsymbol{x}}(t) - \boldsymbol{\nu}(t)\right)\,. \tag{7b}$$

By means of the linearised system (7), the observer gain $\boldsymbol{L}(t)$ ensures that the error state $\tilde{\boldsymbol{x}}(t)$ converges to zero. When designing the observer as a Linear Quadratic Estimator (LQE), this is achieved if a trajectory $\tilde{\boldsymbol{x}}(t)$ minimizes the cost functional

$$J = \tilde{\boldsymbol{x}}^T(0) \boldsymbol{S} \tilde{\boldsymbol{x}}(0) + \int_0^t \boldsymbol{\mu}^T(t) \boldsymbol{R} \boldsymbol{\mu}(t) + \tilde{\boldsymbol{y}}^T(t) \boldsymbol{Q} \tilde{\boldsymbol{y}}(t)\, \mathrm{d}t \tag{8}$$

where $\boldsymbol{S} \in \mathbb{R}^{M \times M}$ and $\boldsymbol{R}, \boldsymbol{Q} \in \mathbb{R}^{2 \times 2}$ denote penalties concerning the initial error as well as the disturbances on input and output, respectively.

Results

The presented approach has been evaluated in a closed loop simulation of the systems utilizing a flatness based feedforward and feedback control as presented in the poster *Control of the VGF Process Part I*.

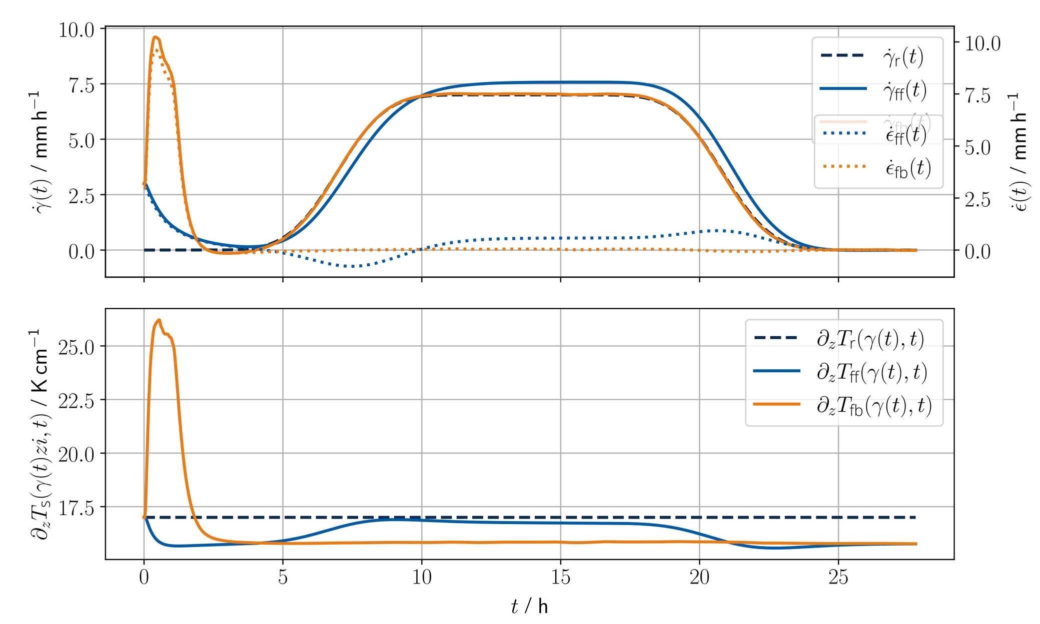

Figure 3: Comparison: Feedforward control only (blue) vs. feedforward & feedback control with observer (orange).

Conclusion & Acknowledgments

A boundary value control regime, utilizing estimates provided by a state observer has been successfully applied to a very simple model of the VGF process. Currently, work is focused on the transfer of the result to more realistic models of the process, especially including a 2D modelling of the plant.

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