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Control of the VGF Process

Part I: Feedforward and Flatness Based State Feedback

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Introduction

The Vertical-Gradient-Freeze (VGF) crystal growth process is a key technology in the production of compound semi-conductors such as Gallium-Arsenide or Indium-Phosphide. Crystallization of the material molten in an ampoule is realized by appropriately moving a temperature field along the ampoule's vertical axis. From a technological point of view, the control of this process is a challenging task because no in-situ measurement information about the controlled object is available, esp. of the growth rate. A few thermocouples placed outside of the ampoule are the only devices providing information about the process state. Moreover, the manipulated variables are of *lumped* type, i.e., the electrical current entering the heaters, while one has to properly track a *spatially distributed* temperature field in the ampoule.

Modelling

To involve all characteristic phenomena of the process such as its biphasic character or the varying domains of crystal and melt in the model while simultaneously keeping it simple enough to illustrate the general design idea of the control concept, a rigorously simplified 1D-model is used. Herein, convection in the melt and radial temperature gradients are neglected. Hence, the temperature $T(z,t)$ in the crucible depends on the time t and the height z. The boundaries at the bottom and top of the crucible are located at $z = \Gamma_0$ and $z = \Gamma_1$, respectively. Assuming piecewise constant parameters for the solid and liquid phase the plant is decomposed into two free boundary problems for $T_\mathrm{s}(z,t)$ and $T_\mathrm{l}(z,t)$:

$$\begin{array}{ll} \partial_t T_\mathrm{s}(z,t) = \alpha_\mathrm{s}\partial_z^2 T_\mathrm{s}(z,t) & \partial_t T_\mathrm{l}(z,t) = \alpha_\mathrm{l}\partial_z^2 T_\mathrm{l}(z,t) \\ k_\mathrm{s}\partial_z T_\mathrm{s}(\Gamma_0,t) = -u_\mathrm{s}(t) & k_\mathrm{l}\partial_z T_\mathrm{l}(\Gamma_1,t) = u_\mathrm{l}(t) \\ T_\mathrm{s}(\gamma(t),t) = T_\mathrm{m} & T_\mathrm{l}(\gamma(t),t) = T_\mathrm{m}. \end{array} \qquad (1)$$

Herein, the index "s" denotes the solid and the index "l" the liquid phase. The heat flows $u_\mathrm{s}(t)$ and $u_\mathrm{l}(t)$ at the bottom and top boundary are considered as the system inputs and $\alpha = \frac{k}{\rho c_\mathrm{p}}$ is the thermal diffusivity. The partial derivative of the quantity T with respect to z or t is denoted by $\partial_z T(z,t)$ or $\partial_t T(z,t)$, respectively. This setup is shown in Figure 1.

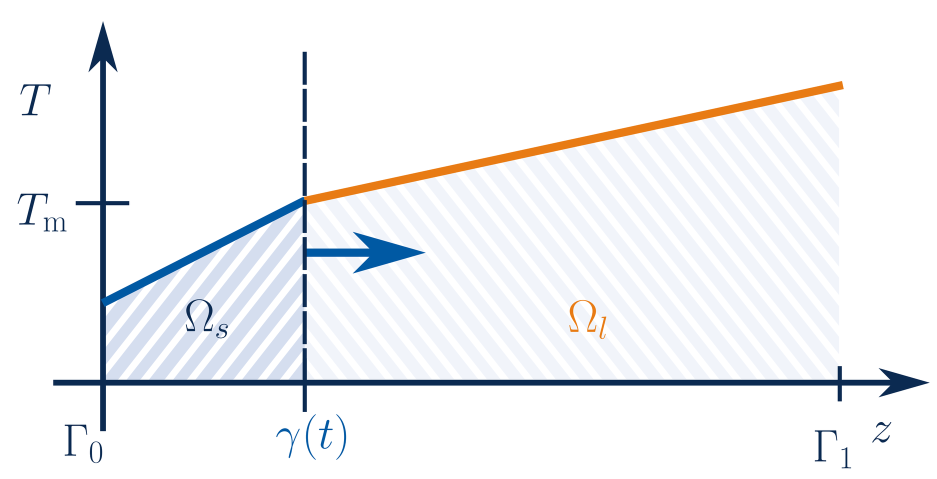

Figure 1: 1D System

Due to the moving phase boundary, latent heat is released by the solidification process. By declaring the left- ($\bullet^-$) and right- ($\bullet^+$) hand side limits respectively, this effect is described by the Stefan-Condition

$$\rho_\mathrm{m} L\dot{\gamma}(t) = k_\mathrm{s}\partial_z T_\mathrm{s}(\gamma(t)^-,t) - k_\mathrm{l}\partial_z T_\mathrm{l}(\gamma(t)^+,t) \qquad (2)$$

with the density of the solid phase at melting temperature ρ_m and the specific latent heat L.

Exploiting their identical structure and denoting the domains $\Omega_\mathrm{s} = (\Gamma_0, \gamma(t))$ and $\Omega_\mathrm{l} = (\gamma(t), \Gamma_1)$, the systems in (1) can be written as one generic system

$$\partial_t T(z,t) = \alpha\partial_z^2 T(z,t),\ z \in \Omega \qquad (3a)$$
$$k\partial_z T(\Gamma,t) = \delta u(t) \qquad (3b)$$
$$T(\gamma(t),t) = T_\mathrm{m} \qquad (3c)$$

with the phase dependent sign $\delta = -1$ at the solid and $\delta = 1$ at the liquid boundary as well as appropriately chosen parameters k and α.

For further ease of computation, the coordinate transform $\tilde{T}(\tilde{z},t) = T(z,t)$ with $\tilde{z} := z - \gamma(t)$ is introduced which shifts the phase boundary into the origin of a new, moving reference frame as it can be seen in Figure 2. Hence, in the new coordinates the phase boundary is always located at $\tilde{z} = 0$. This yields the transformed generic system

$$\partial_t\tilde{T}(\tilde{z},t) = \alpha\partial_{\tilde{z}}^2\tilde{T}(\tilde{z},t) + \dot{\gamma}(t)\partial_{\tilde{z}}\tilde{T}(\tilde{z},t) \qquad (4a)$$
$$k\partial_{\tilde{z}}\tilde{T}(\Gamma - \tilde{z},t) = \delta u(t) \qquad (4b)$$
$$\tilde{T}(0,t) = T_\mathrm{m}\,. \qquad (4c)$$

where $\dot{\gamma}(t)$ denotes the growth rate.

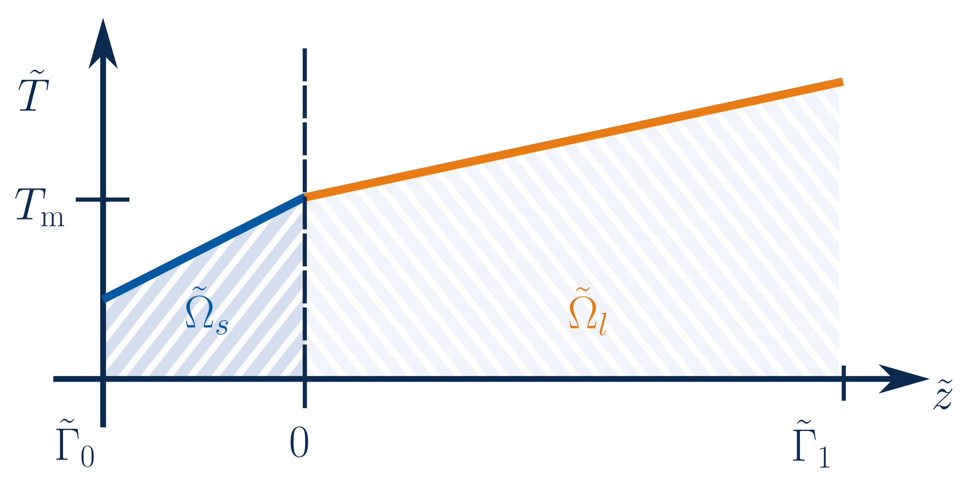

Figure 2: Transformed System

Feedforward

Expanding $\tilde{T}(\tilde{z},t)$ into a power series $\tilde{T}(\tilde{z},t) = \sum_{i=0}^{\infty} c_i(t)\frac{\tilde{z}^i}{i!}$ one obtains the recursion formula $\partial_t c_i(t) = \alpha c_{i+2}(t) + \dot{\gamma}(t)c_{i+1}(t)$ for $i = 0,\ldots,\infty$. Closer examination of this relation shows that, by providing trajectories for the temperature gradient $\partial_{\tilde{z}}\tilde{T}_\mathrm{s}(0^-,t)$ at the solid side of the interface and for the interface position $\gamma(t)$, the temperature distribution in both phases can be determined without solving any partial differential equation (PDE). Note that these are exactly the variables one is targeting in the VGF process.

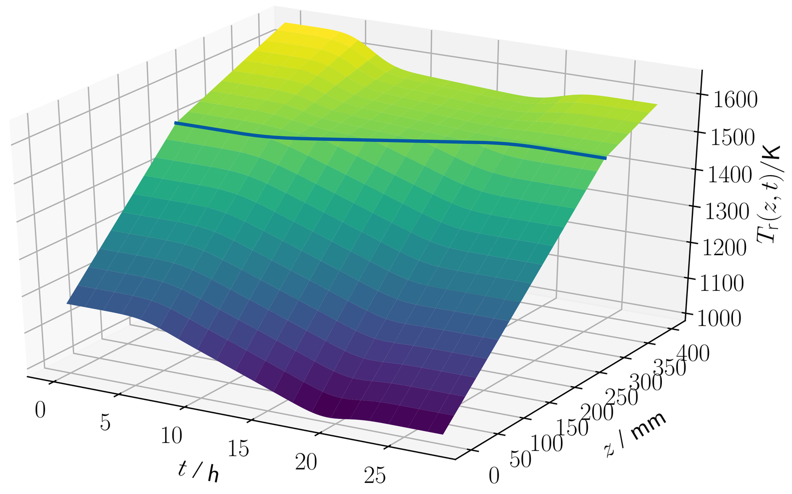

Figure 3: Reference Temperature Profile

State Feedback

Assuming that a pair of reference trajectories is given by $\boldsymbol{\xi}_\mathrm{r}(t) = \left(\partial_{\tilde{z}}\tilde{T}_\mathrm{s}(0^-),\ \gamma(t)\right)^T$ and the current state of these variables is given by $\boldsymbol{\xi}$, a tracking error can be expressed as $\boldsymbol{e}(t) = \boldsymbol{\xi}(t) - \boldsymbol{\xi}_\mathrm{r}(t)$. Thus, choosing the error dynamic as

$$\sum_{n=0}^{N}\kappa_n\boldsymbol{e}^{(n)}(t) = 0 \qquad (5)$$

with the weights κ as desired, a control law can be given with $u(t) = \Phi\left(\boldsymbol{\xi}(t),\ldots,\boldsymbol{\xi}^{(\beta-1)}(t),\boldsymbol{\xi}_\mathrm{r}(t),\ldots,\boldsymbol{\xi}_\mathrm{r}^{(\beta)}(t)\right)$.

Figure 4 shows how the controller performs compared to a pure feedforward setup.

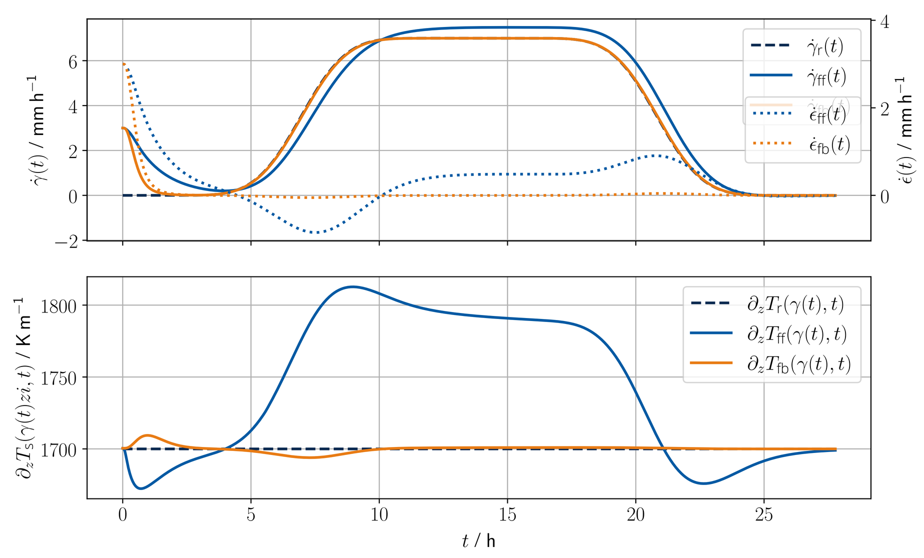

Figure 4: Results with state feedback compared to pure feedforward control

Conclusion & Acknowledgments

Flatness based methods provide an swift and straight forward way to handle feedforward and feedback design problems of the VGF process. While the tracking controller performs well, it is dependent on measurements of process internals like the growth velocity which are not feasible to attain.

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References

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